

Circuit Theorems: Superposition, Thévenin

Objective: To investigate the application of basic laws of linear circuits using DC sources and resistive circuits.

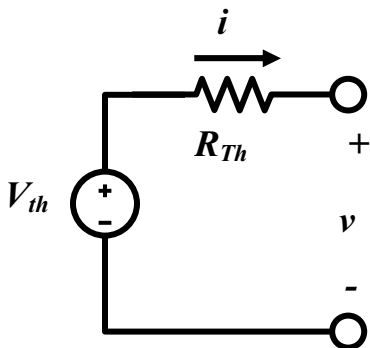
Background Information

Kirchoff's Current Law: The algebraic sum of the currents at a node is zero at every instant of time. Alternately put, the sum of the currents entering a node equals the sum of the currents leaving the node.

Kirchoff's Voltage Law: The algebraic sum of the voltages around a loop is zero at every instant of time.

Superposition Theorem: The total response, or value, of a circuit variable with all independent sources turned on is the algebraic sum of the responses caused by each source acting alone. Mathematically we find the response due to an individual source by setting all the others to zero. In a physical circuit, we could do the same thing by re-adjusting the source values – the power supply outputs in this case – but that requires care to make sure the sources are then returned to their exact previous values. A simpler approach is to disconnect the source and replace it with a zero-valued source. *Important point: a zero-valued voltage source is a short circuit; a zero-valued current source is an open circuit.*

Thévenin's Theorem: If the source circuit in a two-terminal interface is linear, then the interface signals v and i do not change when the source circuit is replaced by its Thévenin equivalent. The Thévenin equivalent circuit comprises a single voltage source in series with a single resistor. The value of the voltage source is that seen at the open-circuited terminals. The value of the resistor is the input resistance seen at the pair of nodes when all the sources of the original circuit are set to zero.

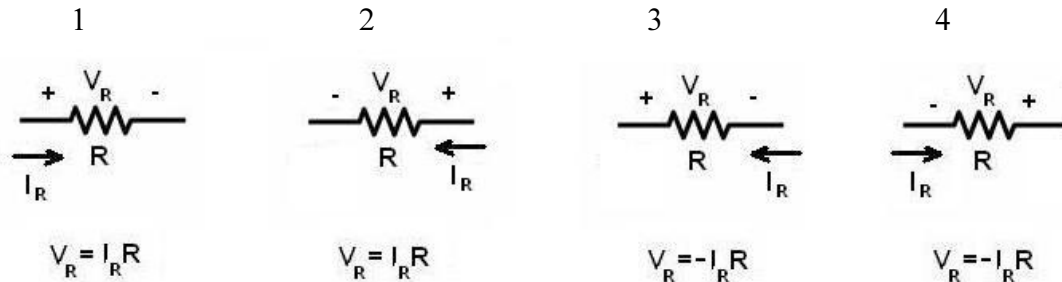


Thévenin equivalent circuit

$$V_{th} = v_{oc}$$

$$R_{Th} = v_{oc} / i_{sc}$$

Ohm's Law: The current in an ideal resistor is exactly proportional to the voltage across it: $V=IR$. This relation assumes the “passive sign convention”, as shown below in cases 1 and 2. Positive current flows into the positive reference terminal. Cases 3 and 4 would never be used because that would require adding a negative sign to Ohm's law.



Note that the resistor itself has no preferred polarity (unlike diodes and some capacitors). So, it can be oriented either way when analyzing or building circuits. KCL and KVL will still work if you are consistent with signs when writing your current or voltage equations.

References: The Analysis and Design of Linear Circuits, Thomas and Rosa, Chapters 2 and 3.

Also refer to the background information on use of breadboards in Lab 1.

Experiment

Part 1: KCL and KVL

We will be analyzing the circuit of figure 1. Before starting the procedure, draw arrows on the schematic to define the currents in each branch and label them. Likewise, mark with a + and – the reference polarity of the voltage drops on the resistors and label them. Notice that the nodes in the schematic have been numbered. The voltage drop on R_1 , for example, could be labeled either V_{R1} or V_{1-2} .

Also notice that source V_2 is shown with a positive value: 3V. The reference polarities indicate that this will in fact create a negative potential at node 4 with respect to COM (node 0). We will use the -20V output of the power supply for V_2 .

1. Set the +18V output to +7.0V, and the -20V output to -3.0V.
2. Gather the six resistors shown in figure 1. Measure and record the actual resistances.
3. Connect the circuit in figure 1 on your breadboard. Hint: use one of the red strips for +7 and one of the blue for COM. Spread out the resistors and arrange them similarly to the schematic. Bend the leads 90 degrees to insert them into the holes on the breadboard so that the resistors will be flat against the surface of the board.

4. Check the currents on the power-supply ammeter. Both supplies should be sourcing under 0.1A. Checking for reasonable supply currents is a good idea any time power to a circuit is turned on.

5. Verify KCL at node 2: Since there is a resistor in series with every branch connected to this node, we can calculate each current from a voltage measurement. This saves us having to break the circuit to introduce an ammeter. Using the DMM as a voltmeter, measure the drop on R_1 , R_2 and R_5 . Calculate the currents using the actual resistance values. Is KCL satisfied, within measurement accuracy?

6. Verify KCL at node 3: Measure the drops on R_3 , R_4 , R_5 and R_6 . Calculate all the currents at node 3. Is KCL satisfied here?

7. KVL: Measure the following drops:

node 1 with respect to node 0,

node 2 with respect to node 1,

node 3 with respect to node 2,

node 0 with respect to node 3. Is KVL satisfied in traversing the path from node 0 to 1 to 2 to 3 and back to 0?

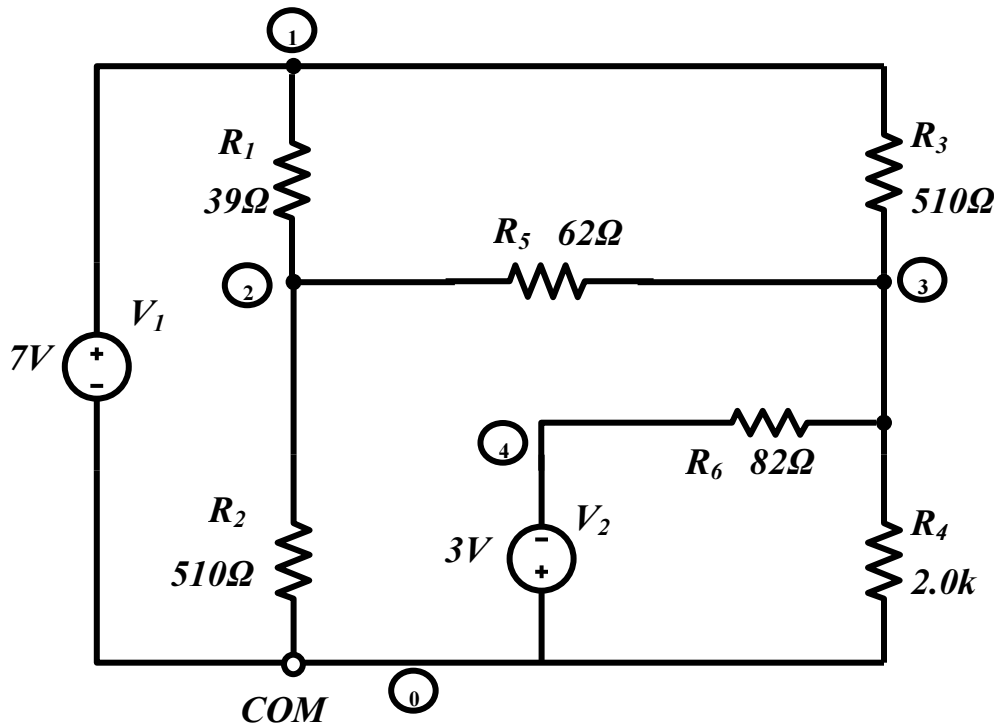


figure 1

Part 2: Superposition

1. Turn off power supply. Break the connection of R5, the 62Ω resistor, to node 3 and insert an ammeter in series with it, as shown in figure 2. An easy way to do this is to move one leg of R5 into a different row of sockets on the breadboard. Clip one ammeter lead to that leg of R5, and the other to a lead of one of the remaining resistors at node 3. Use the benchtop DMM as the ammeter and connect to the “A” terminal. Note the reference direction you defined for this current and connect the ammeter accordingly: positive current flows into the A terminal and out of the COM terminal on the DMM.

2. If you’re sure you’ve got it right, turn on power. Record I , the current in R_5 . This is the total current due to both sources V1 and V2.

3. Record the current due to each of the power supply outputs acting independently. To effectively replace the +7V supply with a short circuit, pull the banana lead out of the +18V terminal at the power supply and plug it into the back end of the banana lead already in the COM terminal. With the short in place, record the ammeter current.

4. Restore the +7V connection and replace the -3V supply with a short in the same way – pull out the lead from the -20V terminal and plug it into the lead COM terminal.

5. Do the two currents measured in steps 3 and 4 add to the current of step 2? i.e. does Superposition apply, within measurement accuracy?

6. Turn off the power. Make sure both the +7V and -3V leads are plugged into their respective supply outputs.

7. Replace the 62Ω resistor with the small lamp. The leads of the lamp are stripped and tinned, so the lead connected to node 2 can be inserted into a socket of the breadboard. The other lead can be clipped directly by the ammeter lead.

8. Turn on the power and record the ammeter current.

9. Repeat steps 3 and 4 and record the currents due to the two supply outputs acting independently.

10. Do the individual currents add up to the measured total current in this case?

Explanation: superposition applies to linear systems. The light bulb is basically resistive, but is a nonlinear element. The current does not go up proportionally with applied voltage. What would you expect the total current to be from superposition? Is the actual current less or more? Does this suggest the resistance of the lamp increases or decreases with applied voltage? Can you think of a physical explanation?

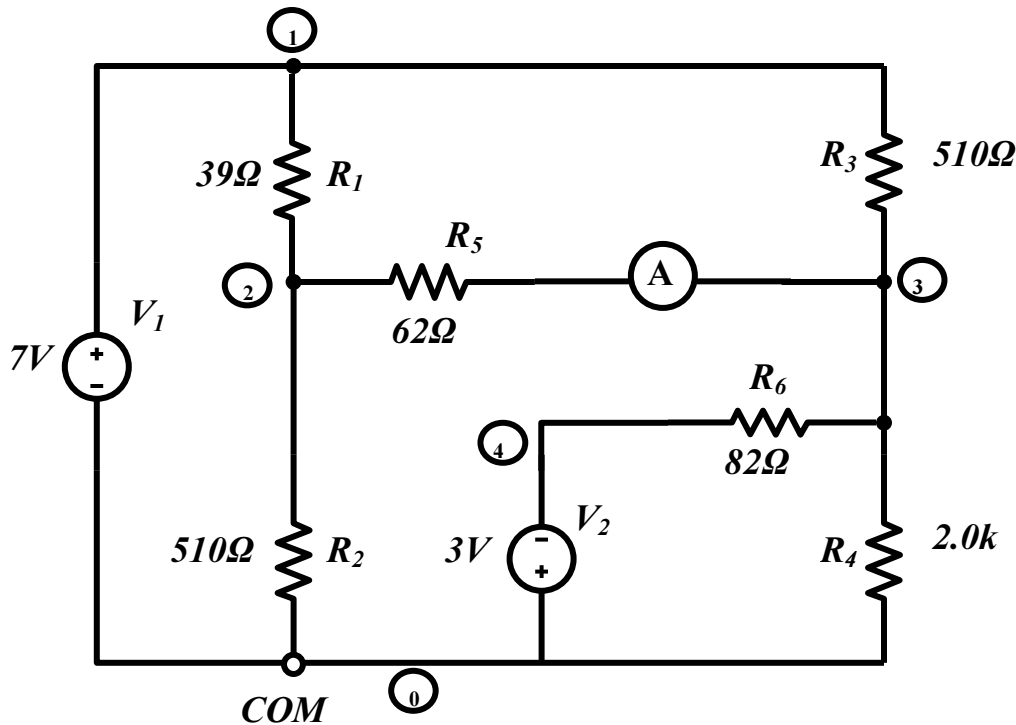


figure 2

Part 3: Thévenin Equivalent

We will consider nodes 3 and 0 as the output terminals of our circuit and find the Thévenin equivalent, as seen from those terminals.

1. Turn off the power.
2. Remove the lamp and replace the 62 ohm resistor as in fig 1.
3. Carefully lay aside the meter leads, remembering the DMM is still configured as an ammeter.
4. Turn on the power. Measure the short circuit current, I_{sc} , by connecting the ammeter between nodes 3 and 0, as shown in fig 3.

Caution: Only measure short-circuit current as we are here if the effective series resistance is known to be large enough to limit I_{sc} to within the meters specified limit.

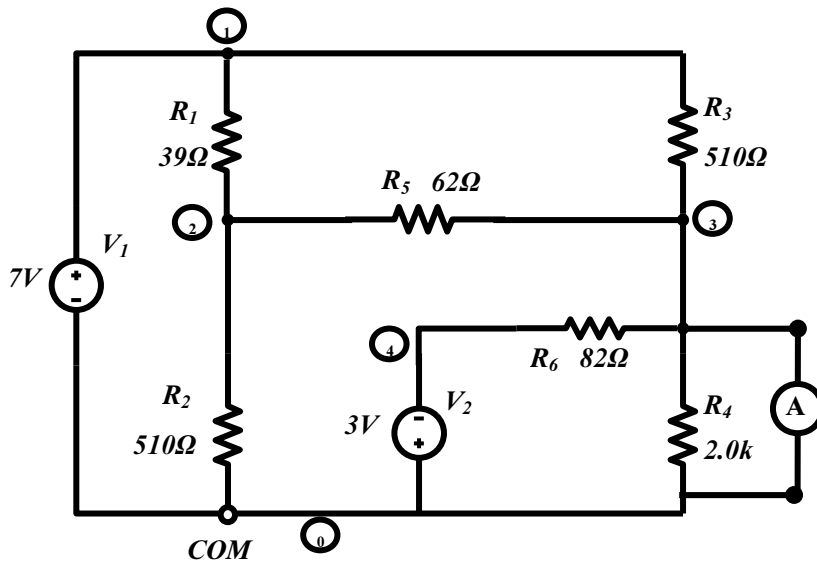


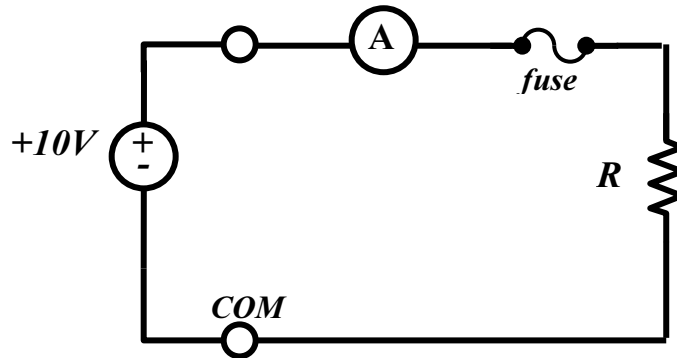
figure 3

5. Reconfigure the DMM to be a voltmeter. Don't forget to move the red lead from the A terminal to the V/ Ω terminal.
6. Measure the open-circuit output voltage, V_{oc} , from node 3 to 0. Select a range with 10mV resolution.
7. Calculate the Thévenin resistance as V_{oc}/I_{sc} .
8. An alternate method for measuring the Thévenin resistance, replace both power supply outputs with short circuits the way we did previously – by pulling the leads out of the +18 and -20 terminals and adding them to the COM terminal – and put the DMM in ohmmeter mode. How close is the resistance measured this way to what you calculated previously? (pick a range appropriate for your expected value)
9. Restore the two supplies. What does the ohmmeter read now? Put the DMM back into voltmeter mode and verify that V_{oc} hasn't changed.
10. Connect a load resistance R_L from node 3 to node 0, using the decade resistor box as R_L . Hint: before connecting the decade box to anything, set it to some high value, like 10k, just to make sure current is limited until you are sure the connection is correct.
11. For $R_L = 1k$, 100, and 50 ohms, record the output voltage (node 3 with respect to 0).
12. Find the value of R_L which makes the output voltage exactly (or as nearly as possible) half of V_{oc} .

Part 4: An Exercise in Fuse Blowing

And now for something completely different...

1. Get a fuse from the TA.
2. Set the decade resistance box to $1\text{k}\Omega$ and the power supply to 10V . Connect the series circuit shown.



3. Verify that the current is approximately 10mA .
5. Starting at 150 ohm lower R until fuse blows. When changing the decade resistance box, be careful to avoid setting it to 0. For example, to change from 1k to 150 , first dial it up to 1150 , and then down to 150 .
6. What was the max current before the fuse blew?

Lab report. It will be important to present the results of your measurements in a well organized manner in the lab report. Don't make the reader guess what you did or have to flip between pages to see figures or data at the back of the report. Refer to the Report Guidelines on the course website for details of what is expected in a report.

Discussion questions (to be answered in the lab report):

1. From your measurements of the ohmmeter in lab 1, why wouldn't the ohmmeter measure the correct Thévenin resistance with the power supplies turned on?
2. For each RL used in steps 11 and 12 of part 3, calculate the Thevenin resistance using the measured output voltage and the voltage-divider formula. For reference, the voltage-divider formula is given in general form as equation 2-31 on page 39 of Thomas and Rosa, and more specifically for a two-resistor circuit in the equation of example 2-15.

It should go without saying that if these various calculations of the Thevenin resistance don't match, you need to search for an explanation why.