Your Name: SOLUTIONS
Perm#:

ECE 2A–Spring 2007
Prof. K. Banerjee

University of California, Santa Barbara

Department of Electrical and Computer Engineering

MIDTERM EXAMINATION

Room 1111 North Hall, May 9, 3:30-4:45 PM
(Lecture notes and text book allowed. Calculators OK)

Include all your answers in locations specified on these pages. Show ALL WORKING used to arrive at answers. Use space provided for all working. Use the back sides if necessary. There are **8 pages** including the cover page and two intentional blank pages. Be sure to write Your **NAME/Perm#** on **EVERY PAGE**.

Question	Scores
#1	/20
#2	/25
#3	/30
#4	/25
TOTAL	/100

Good Luck!

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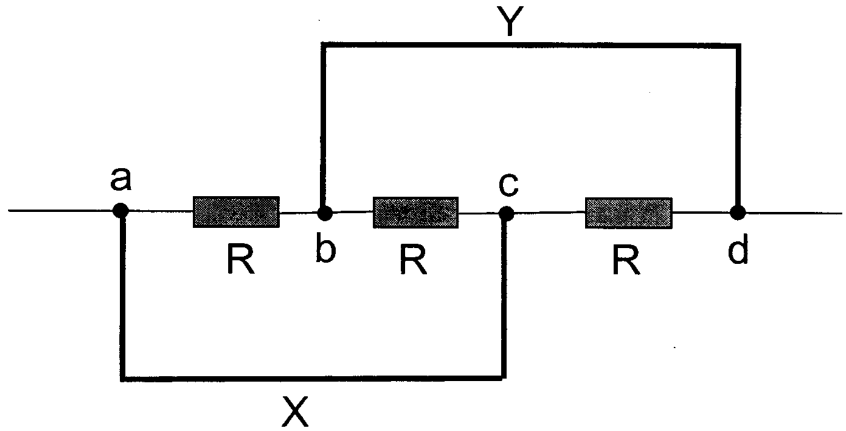
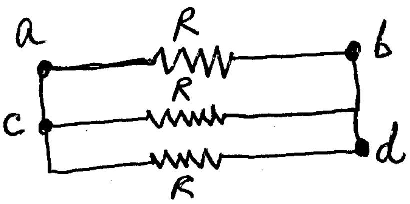
1. **(20 points) This question has two different parts.**
   (a) **(10 pts)** What is the effective resistance across points i) "a" and "d" in the circuit shown below? Disregard the resistance of the connecting wires "aXc" and "bYd".



Points "a" and "c", and "b" and "d" have the same potentials since they are connected by a wire whose resistance can be neglected. Thus the equivalent diagram is shown below.

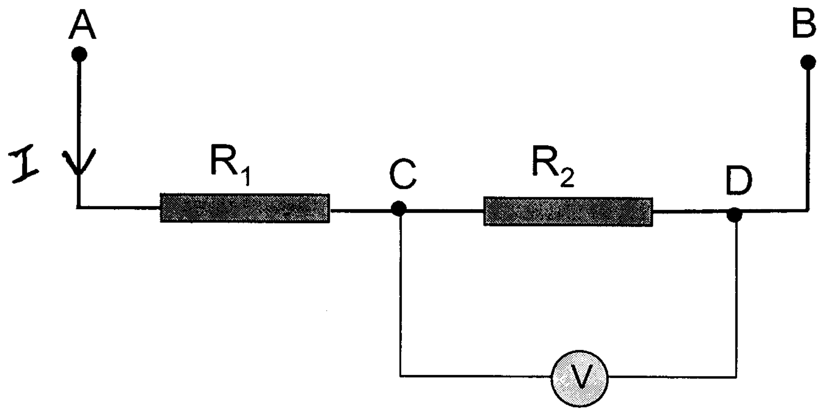


$$\therefore \; 1/R_{eq} = 1/R + 1/R + 1/R = 3/R$$

$$\Rightarrow R_{eq} = R/3.$$

Ans: $\boxed{R_{eff} = R/3}$

(b) **(10 pts)** Resistances R_1 and R_2, each 60 Ω, are connected in series. The potential difference between points A and B is U = 120 V. Find the reading U_1 in the voltmeter connected across points C and D if its internal resistance r = 120 Ω.



The total resistance across A and B = R_{AB}

and $R_{AB} = R_1 + R_{CD}$ — (1)

$$R_{CD} = \frac{R_2 \cdot r}{R_2 + r} = 40\,\Omega \quad \text{— (2)}$$

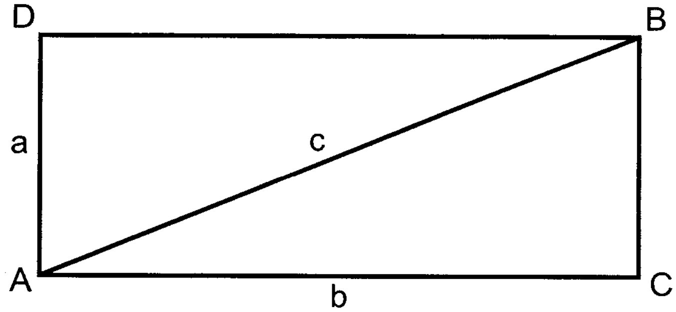
$\Rightarrow R_{AB} = 100\,\Omega$.

$\therefore$ The Current, $I = \dfrac{U}{R_{AB}} = \dfrac{120\text{ V}}{100\,\Omega} = 1.2\text{ A}$

Hence, the voltage drop across section CD $(= U_1)$ is given by, $U_1 = I \cdot R_{CD} = 1.2 \times 40 = 48\text{ V}$

Ans: $\boxed{U_1 = 48\text{ V}}$

2. **(25 points)** Wires of identical cross-section (A) and resistivity (ρ) are soldered into a rectangle ADBC, as shown below, with the diagonal AB of the same cross-section and material. Find the resistance between i) points A and B, and ii) points C and D. The lengths of the sides AD=BC=a, AC=BD=b, and AB=c. Express your answers only in terms of a, b, A and ρ.



Segment Resistances:

$$R_{AD} = R_{BC} = \rho\frac{a}{A}$$

$$R_{AC} = R_{DB} = \rho\frac{b}{A}$$

$$R_{AB} = \rho\frac{c}{A} = \rho\frac{\sqrt{a^2+b^2}}{A}$$

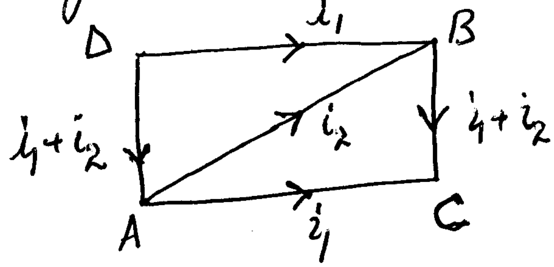
(i) $$R_{AB}^{eff} = (R_{AD}+R_{DB}) \parallel R_{AB} \parallel (R_{AC}+R_{BC})$$

$$= \left(\rho\frac{a}{A}+\rho\frac{b}{A}\right) \parallel \frac{\rho\sqrt{a^2+b^2}}{A} \parallel \left(\rho\frac{b}{A}+\rho\frac{a}{A}\right)$$

$$= \frac{1}{\frac{1}{\rho/A(a+b)} + \frac{1}{\rho/A\sqrt{a^2+b^2}} + \frac{1}{\rho/A(a+b)}} = \frac{1}{\frac{2}{\rho/A(a+b)} + \frac{1}{\rho/A\sqrt{a^2+b^2}}}$$

$$\Rightarrow R_{AB}^{eff} = \frac{1}{\frac{2\sqrt{a^2+b^2}+(a+b)}{\rho/A(a+b)\sqrt{a^2+b^2}}} = \frac{\rho/A(a+b)\sqrt{a^2+b^2}}{2\sqrt{a^2+b^2}+(a+b)}$$

(ii) It is obvious from considerations of symmetry that the currents in conductors DB and AC, and also in AD and BC are equal. Also, since (from KCL) sum of the currents at any junction (such as A) is equal to zero, we define the currents as,



Note: $2i_1+i_2$ enters D and $2i_1+i_2$ leaves C.

For section DAC,

$$(i_1 + i_2)R_{AD} + i_1 R_{AC} = V_{DC}$$

and on section DABC,

$$2(i_1 + i_2)\cdot R_{AD} + i_2 R_{AB} = V_{DC} \quad \left\{ \because R_{AD} = R_{BC} \right.$$

Hence,
$$i_1 = \frac{R_{AD} + R_{AB}}{2R_{AD}R_{AC} + R_{AD}\cdot R_{AB} + R_{AC}\cdot R_{AB}} V_{DC} \quad \text{— (1)}$$

$$i_2 = \frac{R_{AC} - R_{AD}}{2R_{AD}\cdot R_{AC} + R_{AD}R_{AB} + R_{AC}\cdot R_{AB}} V_{DC} \quad \text{— (2)}$$

Hence the effective resistance across points C and D

$$= R_{CD}^{eff} = \frac{\text{Voltage across C and D}}{\text{Net Current flowing from D to C.}}$$

$$= \frac{V_{DC}}{i_1 + (i_1 + i_2)} = \frac{V_{DC}}{2i_1 + i_2}$$

$$\Rightarrow R_{CD}^{eff} = \frac{2R_{AD}R_{AC} + R_{AB}\cdot(R_{AD} + R_{AC})}{R_{AD} + R_{AC} + 2R_{AB}}$$

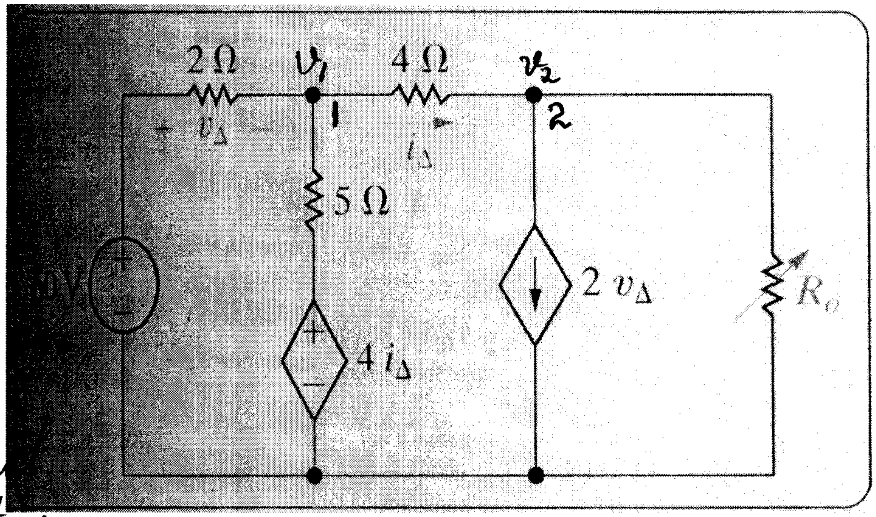
$$= \frac{2\cdot \rho\frac{a}{A}\cdot \rho\frac{b}{A} + \rho\frac{\sqrt{a^2+b^2}}{A}\left(\rho\frac{a}{A} + \rho\frac{b}{A}\right)}{\rho\frac{a}{A} + \rho\frac{b}{A} + 2\rho\frac{\sqrt{a^2+b^2}}{A}}$$

$$R_{CD}^{eff} = \frac{\frac{\rho}{A}\left[2ab + \sqrt{a^2+b^2}\,(a+b)\right]}{a + b + 2\sqrt{a^2+b^2}}$$

3. **(30 points)** The variable resistor R_0 in the circuit shown below is adjusted until it absorbs maximum power from the circuit.
   i) Find the value of R_0
   ii) Find the maximum power
   iii) Find the percentage of the total power developed in the circuit that is delivered to R_0

Recall that for R_o to absorb maximum power $R_o = R_{th}$

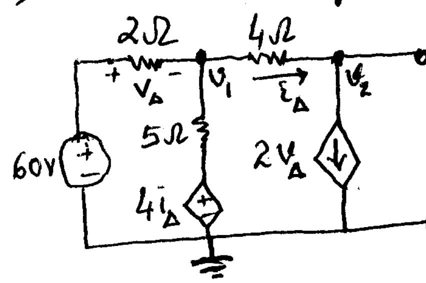
Hence, we need to find the Thévenin Equivalent voltage and resistance.



(1) Define two node voltages v_1 an $v_2\ (= V_{th})$ with R_o removed.
Apply NV Equations:

$$\frac{v_1 - 60}{2} + \frac{v_1 - 4i_\Delta}{5} + \frac{v_1 - v_2}{4} = 0 \quad \text{— (1)}$$

$$\frac{v_2 - v_1}{4} + \underbrace{2V_\Delta}_{\text{already away from node 2}} = 0 \quad \text{——— (2).}$$



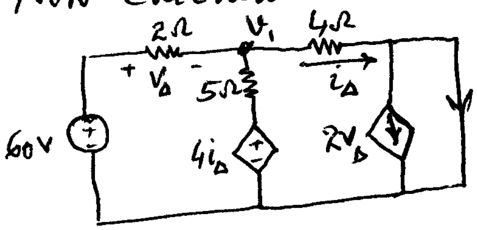
Also, the Constraint equations are:

$$\left.\begin{aligned} V_\Delta &= 60 - V_1 \\ \text{and } i_\Delta &= \frac{V_1 - V_2}{4}. \end{aligned}\right\} \text{— (3).}$$

Substitute (3) in (1) and (2) to get rid of i_Δ and V_Δ and solve.

$\Rightarrow v_1 = 20V,\ v_2 = -300V,\ i_\Delta = 80A,\ V_\Delta = 40V$

Now calculate the short-circuit current $i_{s.c}$.



Apply NV Eqn:

$$\frac{v_1 - 60}{2} + \frac{v_1 - 4i_\Delta}{5} + \frac{v_1}{4} = 0 \quad —(1)$$

The constraint Eqn: $i_\Delta = v_1/4 \quad —(2)$

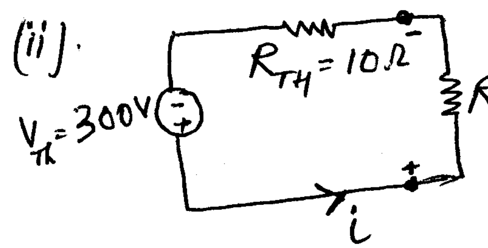
Solving (1) and (2) $\Rightarrow v_1 = 40V,\ i_\Delta = 10A$

$$v_\Delta = 60 - 40 = 20V$$

and $i_{s.c} = i_\Delta - 2V_\Delta = 10 - 40 = -30A$

Thus $R_{TH} = V_{TH}/i_{s,c} = \frac{-300}{-30} = 10\Omega.$

(ii).

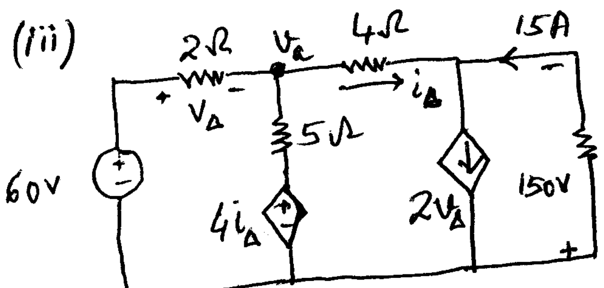


Voltage across $R_L = i \cdot R_L = V_{R_L}$

$$V_{R_L} = \left(\frac{300}{20}\right) \cdot 10 = 150V$$

$$\Rightarrow P_{max} = \frac{V_{R_L}^2}{R_L} = \frac{(150)^2}{10} = 2250W$$

(iii)



Apply NV Equation:

$$\frac{v_a - 60}{2} + \frac{v_a - 4i_\Delta}{5} + \frac{v_a + 150}{4} = 0 \quad —(1)$$

The Constraint Eqn:

$$i_\Delta = \frac{v_a + 150}{4} \quad —(2).$$

from (1) and (2)
$\Rightarrow v_a = 30V$ and $i_\Delta = 45A$

Now calculate all power values:

$i_{60V} = \frac{v_a - 60}{2} = -15A \Rightarrow P_{60V} = 60(-15) = -900W$

$i_{CCVS} = \frac{v_a - 4i_\Delta}{5} = -30A \Rightarrow P_{CCVS} = 4(45)\cdot(-30) = -5400W$

$P_{VCCS} = (-150)[2(30)] = -9000W$ Thus % delivered $= \frac{2250}{15{,}300} \times 100 = 14.7\%$

$\Rightarrow \sum P_{dev} = 900 + 5400 + 9000 = 15{,}300W$

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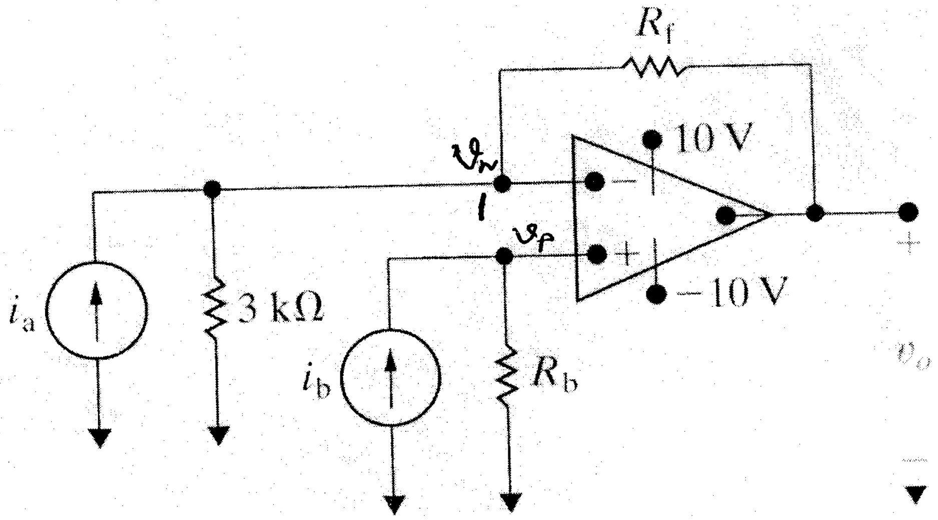
4. **(25 points)** Select the values of R_b and R_f in the circuit shown below so that: $v_0 = 2000\,(i_b - i_a)$. Assume that the OP-AMP is ideal.



$$v_p = v_n = i_b R_b$$

Applying Node Voltage Eqn. at node 1:

$$\frac{v_n - v_a}{3000} + \frac{v_n - v_o}{R_f} = 0$$

Note: $v_a = 3000\, i_a$

$$\Rightarrow \frac{R_b i_b - 3000\, i_a}{3000} + \frac{R_b i_b - v_o}{R_f} = 0$$

$$\Rightarrow \left(\frac{R_b}{3000} + \frac{R_b}{R_f}\right)(i_b) - i_a = \frac{v_o}{R_f}$$

$$\Rightarrow v_o = \underbrace{\left[\frac{R_b \cdot R_f}{3000} + R_b\right]}_{2000} i_b - \underbrace{R_f}_{2000} \cdot i_a \Rightarrow R_f = 2000\,\Omega$$

$$\left(\text{Since, } v_o = 2000\, i_b - 2000\, i_a\right)$$

$$\therefore \; 2/3\, R_b + R_b = 2000$$

$$\Rightarrow R_b = 1200\,\Omega.$$