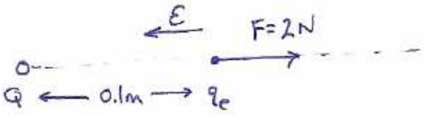
ECE 2A HOMEWORK 1 SOLUTIONS

1. ASSUME THE RELATIVE POSITIONS SHOWN:



i) $\mathcal{E} = \frac{F}{q_e} = \frac{2N}{-1.6\times10^{-19}\,C} = \underline{-1.25\times10^{19}\ N/C}$ OR V/m

ii) SINCE THE FORCE IS REPULSIVE, THE TWO CHARGES MUST HAVE THE SAME SIGN: NEGATIVE.

$$\mathcal{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \Rightarrow Q = \frac{\mathcal{E}r^2}{\left(1/4\pi\epsilon_0\right)} = \frac{-1.25\times10^{19}(N/C)\cdot(0.1m)^2}{8.89\times10^{9}\ Nm^2/C^2}$$

$$= \underline{-1.41\times10^{7}\,C}$$

iii) $\Delta V = V_{0.1m} - V_{\infty}$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{r}\Bigg|_{r=\infty}^{r=0.1m} = -1.25\times10^{18} - 0 = \underline{-1.25\times10^{18}\,V}$$

iv) $\Delta P.E. = P.E._{0.1m} - P.E._{\infty} = \frac{1}{4\pi\epsilon_0}\frac{Qq_e}{r}\Bigg|_{r=\infty}^{r=0.1m} = 0.201 - 0\ J$

ALTERNATELY, $\Delta P.E. = \Delta V\cdot q_e = \underline{0.201\ J}$

v) $W = -\Delta P.E. = \underline{-0.201\ J}$

INTUITIVELY THE POTENTIAL ENERGY MUST BE GREATER (MORE POSITIVE) WHEN THE TWO CHARGES ARE IN CLOSER PROXIMITY, SINCE THE FORCE IS REPULSIVE. WHATEVER FORCE BRINGS THEM TOGETHER WORKS AGAINST THE NATURAL REPULSION, SO THE WORK DONE BY THE FIELD OF Q IS NEGATIVE.

2. $V = IR$

$R = \frac{\rho L}{A} \Rightarrow V = I\frac{\rho L}{A} = \left(\frac{I}{A}\right)\rho L = J\rho L$

BUT $\frac{V}{L} = \mathcal{E} \Rightarrow \mathcal{E} = J\rho$ OR $\rho = \frac{\mathcal{E}}{J}$

3. $P = VI$

FOR A RESISTOR $V = IR$, SO $P = I^2R$

$R = \frac{\rho L}{A}$, SO $P = I^2\frac{\rho L}{A}$

$\rho = \frac{PA}{I^2 L}$

$A = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{1.63\,mm}{2}\right)^2 = 2.087\times10^{-6}\,m^2$

$L = 2.35\,m$

$\rho = \frac{48.5\,mW \cdot \cancel{2.086\,m^2}\ 2.086\times10^{-6}\,m^2}{(1.24A)^2 \cdot 2.35\,m}$

$\rho = 2.8\times10^{-8}\ \Omega\cdot m$

THE MATERIAL IS ALUMINUM

NOTE THAT WE CAN IDENTIFY A MATERIAL BY ITS RESISTIVITY.

4: $i = e^v - 10A$. P = $i.v$

For v = -2.2 , i = -9.889 and so P = (-9.889)X(-2.2) = 21.75 W Absorbing

For v = 3.0 , i = 10.085 and so P = (10.085)X(3) = 30.2566 W Absorbing

5:

[a] $p = vi = 200\cos(500\pi t)4.5\sin(500\pi t) = 450\sin(1000\pi t)$ W

Therefore, $p_{\max} = 450$ W

[b] $p_{\max}(\text{extracting}) = 450$ W

[c]

$$\begin{aligned} p_{\text{avg}} &= \frac{1}{4\times10^{-3}}\int_0^{4\times10^{-3}} 450\sin(1000\pi x)\,dx = \frac{450}{4\times10^{-3}}\left[\frac{-\cos 1000\pi t}{1000\pi}\right]_0^{4\times10^{-3}} \\ &= \frac{-450}{4\pi}[\cos 4\pi - \cos 0] = \frac{-450}{4\pi}[1-1] = 0 \text{ W} \end{aligned}$$

[d] $p_{\text{avg}} = \frac{-450}{4\pi}[\cos 15\pi - \cos 0] = \frac{-450}{4\pi}[-1-1] = \frac{900}{4\pi} = 71.62$ W

6:

[a] We can find the time at which the power is a maximum by writing an expression for $p(t) = v(t)i(t)$, taking the first derivative of $p(t)$ and setting it to zero, then solving for t. The calculations are shown below:

$$\begin{aligned} p &= 0 \quad t < 0, \qquad p = 0 \quad t > 40 \text{ s} \\ p &= vi = (t - 0.025t^2)(4 - 0.2t) = 4t - 0.3t^2 + 0.005t^3 \text{ W} \qquad 0 < t < 40 \text{ s} \\ \frac{dp}{dt} &= 4 - 0.6t + 0.015t^2 = 0 \end{aligned}$$

Use a calculator to find the two solutions to this quadratic equation:

$t_1 = 8.453$ s; $\qquad t_2 = 31.547$ s

Now we must find which of these two times gives the minimum power by substituting each of these values for t into the equation for $p(t)$:

$$\begin{aligned} p(t_1) &= (8.453 - 0.025(8.453)^2)(4 - 0.2\cdot 8.453) = 15.396 \text{ W} \\ p(t_2) &= (31.547 - 0.025(31.547)^2)(4 - 0.2\cdot 31.547) = -15.396 \text{ W} \end{aligned}$$

Therefore, maximum power is being delivered at $t = 8.453$ s.

[b] The maximum power was calculated in part (a) to determine the time at which the power is maximum: $p_{\max} = 15.396$ W (delivered)

[c] As we saw in part (a), the other "maximum" power is actually a minimum, or the maximum negative power. As we calculated in part (a), maximum power is being extracted at $t = 31.547$ s.

[d] This maximum extracted power was calculated in part (a) to determine the time at which power is maximum: $p_{\text{maxext}} = 15.396$ W (extracted)

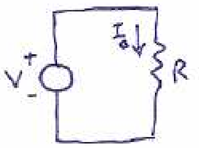
[e] $w = \int_0^t p\,dx = \int_0^t (4x - 0.3x^2 + 0.005x^3)dx = 2t^2 - 0.1t^3 + 0.00125t^4$

$$\begin{aligned} w(0) &= 0 \text{ J} & w(30) &= 112.50 \text{ J} \\ w(10) &= 112.50 \text{ J} & w(40) &= 0 \text{ J} \\ w(20) &= 200 \text{ J} \end{aligned}$$

To give you a feel for the quantities of voltage, current, power, and energy and their relationships among one another, they are plotted below:

7.

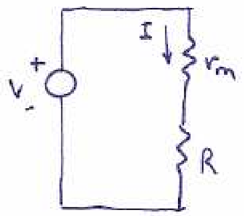
WITHOUT THE AMMETER



$V = I_0 R$

$I_0 = V/R$

WITH THE AMMETER



$V = I(R + r_m)$

$I = \dfrac{V}{R + r_m}$

$$\text{ERROR} = \frac{I_0 - I}{I_0} = \frac{\frac{V}{R} - \frac{V}{R + r_m}}{\frac{V}{R}} = 1 - \frac{R}{R + r_m} = \frac{r_m}{R + r_m} = \underline{\frac{1}{R/r_m + 1}}$$

THE ERROR IS NEGLIGIBLE IF $R \gg r_m$