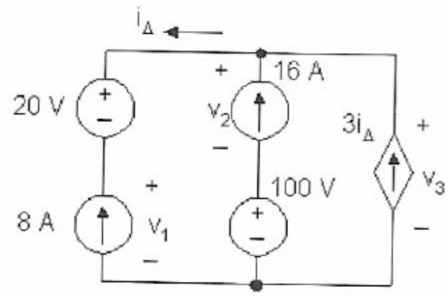
ECE2A SPRING-2008
Homework#2 Solutions

1.)

[a] Yes, Kirchhoff's laws are not violated. (Note that $i_\Delta = -8$ A.)

[b] No, because the voltages across the independent and dependent current sources are indeterminate. For example, define v_1, v_2, and v_3 as shown:



Kirchhoff's voltage law requires

$$v_1 + 20 = v_3$$

$$v_2 + 100 = v_3$$

Conservation of energy requires

$$-8(20) - 8v_1 - 16v_2 - 16(100) + 24v_3 = 0$$

or

$$v_1 + 2v_2 - 3v_3 = -220$$

Now arbitrarily select a value of v_3 and show the conservation of energy will be satisfied. Examples:

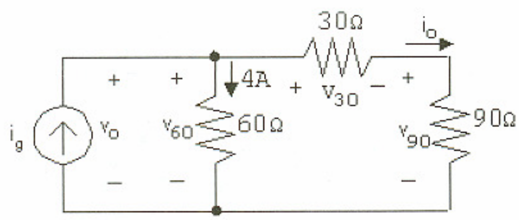
If $v_3 = 200$ V then $v_1 = 180$ V and $v_2 = 100$ V. Then

$$180 + 200 - 600 = -220 \text{ (CHECKS)}$$

If $v_3 = -100$ V, then $v_1 = -120$ V and $v_2 = -200$ V. Then

$$-120 - 400 + 300 = -220 \text{ (CHECKS)}$$

2.)



[a] Write a KVL equation clockwise around the right loop:

$$-v_{60} + v_{30} + v_{90} = 0$$

From Ohm's law,

$$v_{60} = (60\,\Omega)(4\,\text{A}) = 240\,\text{V}, \qquad v_{30} = 30i_o, \qquad v_{90} = 90i_o$$

Substituting,

$$-240\,\text{V} + 30i_o + 90i_o = 0 \qquad \text{so} \qquad 120i_o = 240\,\text{V}$$

Thus $\quad i_o = \dfrac{240\,\text{V}}{120} = 2\,\text{A}$

Now write a KCL equatiohn at the top middle node, summing the currents leaving:

$$-i_g + 4\,\text{A} + i_o = 0 \qquad \text{so} \qquad i_g = 4\,\text{A} + 2\,\text{A} = 6\,\text{A}$$

[b] Write a KVL equation clockwise around the left loop:

$$-v_o + v_{60} = 0 \qquad \text{so} \qquad v_o = v_{60} = 240\,\text{V}$$

[c] Calculate power using $p = vi$ for the source and $p = Ri^2$ for the resistors:

$$p_{\text{source}} = -v_o i_g = -(240\,\text{V})(6\,\text{A}) = -1440\,\text{W}$$

$$p_{60\Omega} = 4^2(60) = 960\,\text{W}$$

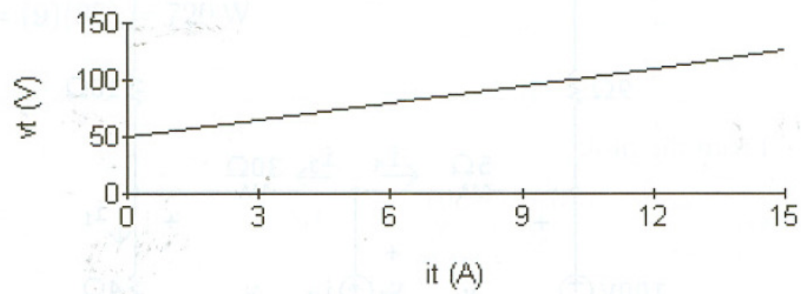
$$p_{30\Omega} = 30i_o^2 = (30)2^2 = 120\,\text{W}$$

$$p_{90\Omega} = 90i_o^2 = (90)2^2 = 360\,\text{W}$$

$$\sum P_{\text{dev}} = 1440\,\text{W} \qquad \sum P_{\text{abs}} = 960 + 120 + 360 = 1440\,\text{W}$$

3.)

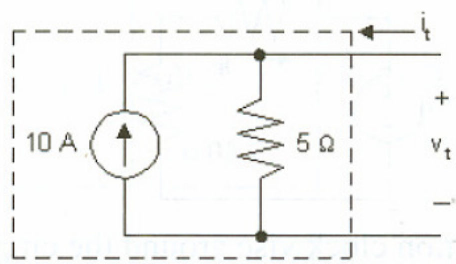
[a] Plot tl



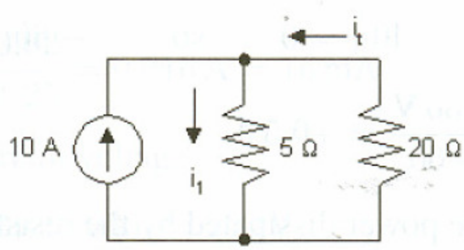
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(125 - 50)}{(15 - 0)} = 5\,\Omega$$

When $i_t = 0$, $v_t = 50$ V; therefore the ideal current source has a current of 10 A



[b]



$10 + i_t = i_1$ and $5i_1 = -20i_t$

Therefore, $10 + i_t = -4i_t$ so $i_t = -2$ A

4.)

[a] Start with the $22.5\,\Omega$ resistor. Since the voltage drop across this resistor is 90 V, we can use Ohm's law to calculate the current:

$$i_{22.5\,\Omega} = \frac{90\text{ V}}{22.5\,\Omega} = 4\text{ A}$$

Next we can calculate the voltage drop across the $15\,\Omega$ resistor by writing a KVL equation around the outer loop of the circuit:

$$-240\text{ V} + 90\text{ V} + v_{15\,\Omega} = 0 \qquad \text{so} \qquad v_{15\,\Omega} = 240 - 90 = 150\text{ V}$$

Now that we know the voltage drop across the $15\,\Omega$ resistor, we can use Ohm's law to find the current in this resistor:

$$i_{15\,\Omega} = \frac{150\text{ V}}{15\,\Omega} = 10\text{ A}$$

Write a KCL equation at the middle right node to find the current through the $5\,\Omega$ resistor. Sum the currents entering:

$$4\text{ A} - 10\text{ A} + i_{5\,\Omega} = 0 \qquad \text{so} \qquad i_{5\,\Omega} = 10\text{ A} - 4\text{ A} = 6\text{ A}$$

Write a KVL equation clockwise around the upper right loop, starting below the $4\,\Omega$ resistor. Use Ohm's law to express the voltage drop across the resistors in terms of the current through the resistors:

$$-v_{4\,\Omega} + 90\text{ V} + (5\,\Omega)(-6\text{ A}) = 0 \qquad \text{so} \qquad v_{4\,\Omega} = 90\text{ V} - 30\text{ V} = 60\text{ V}$$

Using Ohm's law we can find the current through the $4\,\Omega$ resistor:

$$i_{4\,\Omega} = \frac{60\text{ V}}{4\,\Omega} = 15\text{ A}$$

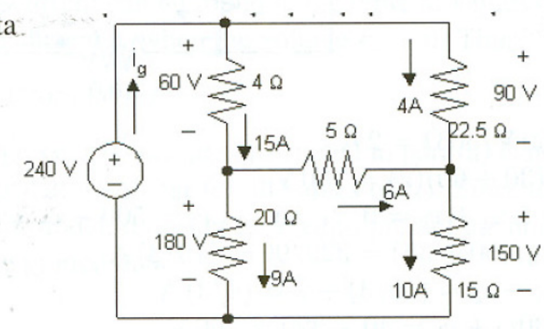
Write a KCL equation at the middle node. Sum the currents entering:

$$15\text{ A} - 6\text{ A} - i_{20\,\Omega} = 0 \qquad \text{so} \qquad i_{20\,\Omega} = 15\text{ A} - 6\text{ A} = 9\text{ A}$$

Use Ohm's law to calculate the voltage drop across the $20\,\Omega$ resistor:

$$v_{20\,\Omega} = (20\,\Omega)(9\text{ A}) = 180\text{ V}$$

All of the volta ... n in the figure below:



Calculate the power dissipated by the resistors using the equation $p_R = Ri_R^2$:

$$p_{4\Omega} = (4)(15)^2 = 900\text{ W} \qquad p_{20\Omega} = (20)(9)^2 = 1620\text{ W}$$

$$p_{5\Omega} = (5)(6)^2 = 180\text{ W} \qquad p_{22.5\Omega} = (22.5)(4)^2 = 360\text{ W}$$

$$p_{15\Omega} = (15)(10)^2 = 1500\text{ W}$$

[b] We can calculate the current in the voltage source, i_g by writing a KCL equation at the top middle node:

$i_g = 15\,\text{A} + 4\,\text{A} = 19\,\text{A}$

Now that we have both the voltage and the current for the source, we can calculate the power supplied by the source:

$p_g = -240(19) = -4560\,\text{W}$ thus $p_g\,(\text{supplied}) = 4560\,\text{W}$

[c] $\sum P_{\text{dis}} = 900 + 1620 + 180 + 360 + 1500 = 4560\,\text{W}$

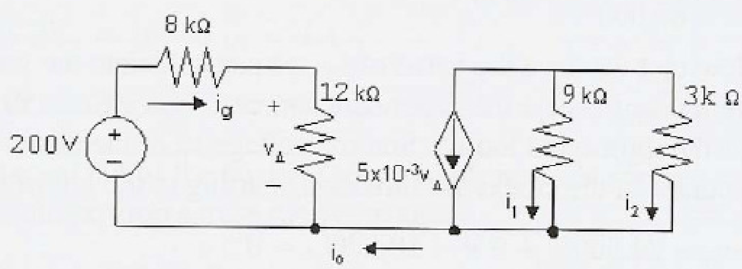
Therefore,

$\sum P_{\text{supp}} = \sum P_{\text{dis}}$

5.)

[a] $i_o = 0$ because no current can exist in a single conductor connecting two parts of a circuit.

[b]



$-200 + 8000i_g + 12{,}000i_g = 0$ so $i_g = 200/20{,}000 = 10\,\text{mA}$

$v_\Delta = (12\times10^3)(10\times10^{-3}) = 120\,\text{V}$

$5\times10^{-3}v_\Delta = 0.6\,\text{A}$

$9000i_1 = 3000i_2$ so $i_2 = 3i_1$

$0.6 + i_1 + i_2 = 0$ so $0.6 + i_1 + 3i_1 = 0$ thus $i_1 = -0.15\,\text{A}$

[c] $i_2 = 3i_1 = -0.45\text{A}$

6.)

[a] $5\|20 = 100/25 = 4\,\Omega$

$9\|18 = 162/27 = 6\,\Omega$

$R_{\text{ab}} = 5 + 12 + 3 = 20\,\Omega$

$5\|20 + 9\|18 + 10 = 20\,\Omega$

$20\|30 = 600/50 = 12\,\Omega$

[b] $5 + 15 = 20\,\Omega$

$20\|60 = 1200/80 = 15\,\Omega$

$15 + 10 = 25\,\Omega$

$25\|75 = 1875/100 = 18.75\,\Omega$

$18.75 + 11.25 = 30\,\Omega$

$30\|20 = 600/50 = 12\,\Omega$

$3\|6 = 18/9 = 2\,\Omega$

$3\|6 + 30\|20 = 2 + 12 = 14\,\Omega$

$26\|14 = 364/40 = 9.1\,\Omega$

$R_{\text{ab}} = 2.5 + 9.1 + 3.4 = 15\,\Omega$

[c] $3 + 5 = 8\,\Omega$

$8\|12 = 96/20 = 4.8\,\Omega$

$4.8 + 5.2 = 10\,\Omega$

$45 + 15 = 60\,\Omega$

$60\|40 = 2400/100 = 24\,\Omega$

$24 + 6 = 30\,\Omega$

$30\|10 = 300/40 = 7.5\,\Omega$

$R_{\text{ab}} = 1.5 + 7.5 + 1.0 = 10\,\Omega$

7.)

[a] $v_{9\Omega} = (1)(9) = 9\text{ V}$

$i_{2\Omega} = 9/(2+1) = 3\text{ A}$

$i_{4\Omega} = 1 + 3 = 4\text{ A};$

$v_{25\Omega} = (4)(4) + 9 = 25\text{ V}$

$i_{25\Omega} = 25/25 = 1\text{ A};$

$i_{3\Omega} = i_{25\Omega} + i_{9\Omega} + i_{2\Omega} = 1 + 1 + 3 = 5\text{ A};$

$v_{40\Omega} = v_{25\Omega} + v_{3\Omega} = 25 + (5)(3) = 40\text{ V}$

$i_{40\Omega} = 40/40 = 1\text{ A}$

$i_{5\|20\Omega} = i_{40\Omega} + i_{25\Omega} + i_{4\Omega} = 1 + 1 + 4 = 6\text{ A}$

$v_{5\|20\Omega} = (4)(6) = 24\text{ V}$

$v_{32\Omega} = v_{40\Omega} + v_{5\|20\Omega} = 40 + 24 = 64\text{ V}$

$i_{32\Omega} = 64/32 = 2\text{ A};$

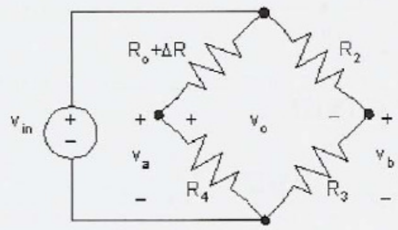
$i_{10\Omega} = i_{32\Omega} + i_{5\|20\Omega} = 2 + 6 = 8\text{ A}$

$v_g = 10(8) + v_{32\Omega} = 80 + 64 = 144\text{ V}.$

[b] $P_{20\Omega} = \dfrac{(v_{5\|20\Omega})^2}{20} = \dfrac{24^2}{20} = 28.8\text{ W}$

8.)

[a]



$$v_a = \frac{v_{in} R_4}{R_o + R_4 + \Delta R}$$

$$v_b = \frac{R_3}{R_2 + R_3} v_{in}$$

$$v_o = v_a - v_b = \frac{R_4 v_{in}}{R_o + R_4 + \Delta R} - \frac{R_3}{R_2 + R_3} v_{in}$$

When the bridge is balanced,

$$\frac{R_4}{R_o + R_4} v_{in} = \frac{R_3}{R_2 + R_3} v_{in}$$

$$\therefore \quad \frac{R_4}{R_o + R_4} = \frac{R_3}{R_2 + R_3}$$

Thus,

$$\begin{aligned} v_o &= \frac{R_4 v_{in}}{R_o + R_4 + \Delta R} - \frac{R_4 v_{in}}{R_o + R_4} \\ &= R_4 v_{in} \left[\frac{1}{R_o + R_4 + \Delta R} - \frac{1}{R_o + R_4} \right] \\ &= \frac{R_4 v_{in}(-\Delta R)}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &\approx \frac{-(\Delta R) R_4 v_{in}}{(R_o + R_4)^2} \end{aligned}$$

[b] $\Delta R = 0.03 R_o$

$$R_o = \frac{R_2 R_4}{R_3} = \frac{(1000)(5000)}{500} = 10{,}000\,\Omega$$

$$\Delta R = (0.03)(10^4) = 300\,\Omega$$

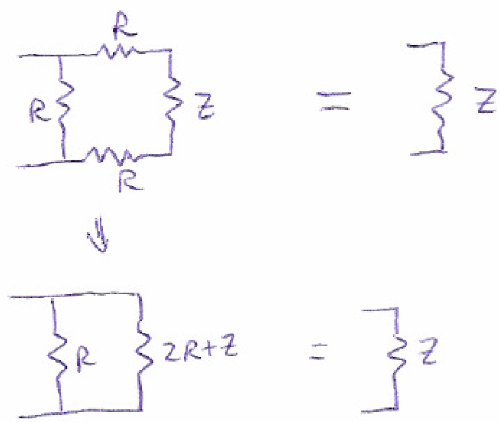
$$\therefore \quad v_o \approx \frac{-300(5000)(6)}{(15{,}000)^2} = -40 \text{ mV}$$

[c]

$$\begin{aligned} v_o &= \frac{-(\Delta R) R_4 v_{in}}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &= \frac{-300(5000)(6)}{(15{,}300)(15{,}000)} \\ &= -39.2157 \text{ mV} \end{aligned}$$

9.)

IF ADDING A STEP TO THE LADDER DOES NOT CHANGE THE EFFECTIVE RESISTANCE, THEN ~~THEN~~ THE EQUIVALENT RESISTANCE OF AN ADDED STEP, INCLUDING THE TERMINATION, Z, MUST BE THE SAMES AS Z ITSELF:



$$\frac{R(2R+Z)}{R+2R+Z} = Z \Rightarrow Z^2 + 2RZ - 2R^2 = 0$$

$$Z = \frac{-2R \pm \sqrt{4R^2 + 8R^2}}{2}$$

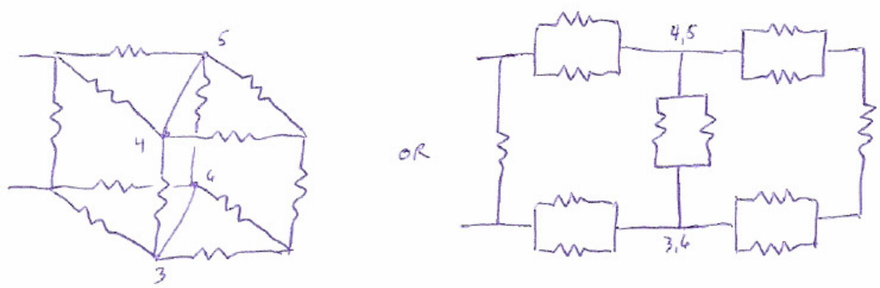
SINCE Z MUST BE POSITIVE,

$$\underline{Z = -R + \sqrt{3}R = (\sqrt{3}-1)R}$$

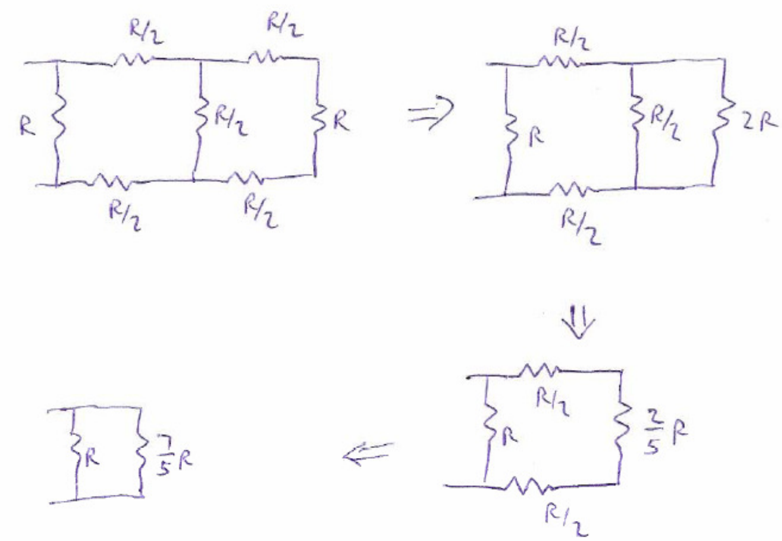
10.)

FROM SYMMETRY NODES 4+5 MUST BE AT THE SAME POTENTIAL; LIKEWISE NODES 3 + 6 MUST BE AT THE SAME POTENTIAL. SINCE NO CURRENT WILL FLOW IN A WIRE CONNECTED BETWEEN NODES AT EQUAL POTENTIAL, WE CAN ADD A WIRE BETWEEN NODES 4+5 WITHOUT CHANGING THE CIRCUIT, AND LIKEWISE BETWEEN 3+6.

SO THE CIRCUIT BECOMES



WITH ALL RESISTORS = R THIS REDUCES TO



=

$\frac{7}{12} R$