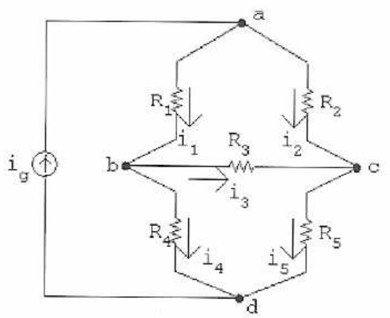
Homework 3 Solutions

1.

[a] There are six circuit components, five resistors and the current source. Since the current is known only in the current source, it is unknown in the five resistors. Therefore there are **five** unknown currents.

[b] There are four essential nodes in this circuit, identified by the dark black dots in Fig. P4.4. At three of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the fourth node would be dependent on the first three. Therefore there are **three** independent KCL equations.

[c]



Sum the currents at any three of the four essential nodes a, b, c, and d. Using nodes a, b, and c we get

$$-i_g + i_1 + i_2 = 0$$

$$-i_1 + i_4 + i_3 = 0$$

$$i_5 - i_2 - i_3 = 0$$

[d] There are three meshes in this circuit: one on the left with the components i_g, R_1, and R_4; one on the top right with components R_1, R_2, and R_3; and one on the bottom right with components R_3, R_4, and R_5. We cannot write a KVL equation for the left mesh because we don't know the voltage drop across the current source. Therefore, we can write KVL equations for the two meshes on the right, giving a total of **two** independent KVL equations.

[e] Sum the voltages around two independent closed paths, avoiding a path that contains the independent current source since the voltage across the current source is not known. Using the upper and lower meshes formed by the five resistors gives

$$R_1 i_1 + R_3 i_3 - R_2 i_2 = 0$$

$$R_3 i_3 + R_5 i_5 - R_4 i_4 = 0$$

1 R2 2

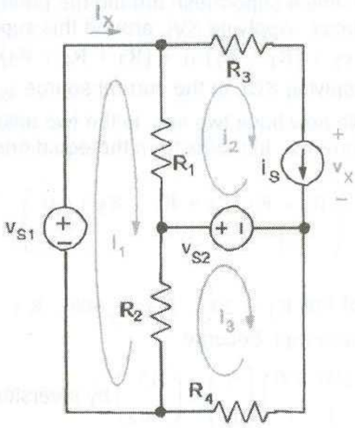
(a) Assign mesh currents as shown at right. Applying KCL to the current source $i_2 = i_S$. Hence, i_1 and i_2 are the only unknown mesh currents. Applying KVL to

Mesh 1 $-v_{S1} + R_1 \cdot (i_1 - i_S) + R_2 \cdot (i_1 - i_3) = 0$

Mesh 3 $v_{S2} + R_4 \cdot i_3 + R_2 \cdot (i_3 - i_1) = 0$

In matrix form these equations are

$$\begin{pmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_4 \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ i_3 \end{pmatrix} = \begin{pmatrix} v_{S1} + R_1 \cdot i_S \\ -v_{S2} \end{pmatrix}$$



(b) For $R_1 = 10^4$, $R_2 = 10^4$, $R_3 = 2 \cdot 10^3$, $R_4 = 10^3$, $i_S = 0.0025$, $v_{S1} = 12$ V, and $v_{S2} = 0.5$ V these equations become

$$\begin{pmatrix} 2 \cdot 10^4 & -10^4 \\ -10^4 & 11 \cdot 10^3 \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ i_3 \end{pmatrix} = \begin{pmatrix} 37 \\ -0.5 \end{pmatrix}$$ Using matrix inversion

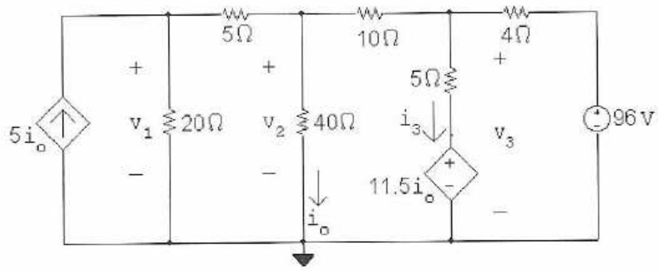
$$\begin{pmatrix} i_1 \\ i_3 \end{pmatrix} := \begin{pmatrix} 2 \cdot 10^4 & -10^4 \\ -10^4 & 11 \cdot 10^3 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 37 \\ -0.5 \end{pmatrix}$$ yielding $$\begin{pmatrix} i_1 \\ i_3 \end{pmatrix} = \begin{pmatrix} 3.35 \times 10^{-3} \\ 3 \times 10^{-3} \end{pmatrix}$$

Using these mesh currents to find the desired signal variables. First $i_x := i_1$. Apply KVL around the perimeter of the circuit $-v_{S1} + R_3 \cdot i_S + v_x + R_4 \cdot i_3 = 0$. Inserting numbers and solving for v_x yields

$v_x := 12 - 2000 \cdot 0.0025 - 1000 \cdot 0.003$ or $v_x = 4$ V and $i_x = 3.35 \times 10^{-3}$ = 3.35 mA

(c) The power delivered by v_{S1} is $v_{S1} \cdot (-i_1) = 12 \cdot (-3.35 \cdot 10^{-3}) = -4.02 \cdot 10^{-2}$ = -40.2 mW

3:



The node voltage equations are:

$$-5i_o + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{40} + \frac{v_2 - v_3}{10} = 0$$

$$\frac{v_3 - v_2}{10} + \frac{v_3 - 11.5i_o}{5} + \frac{v_3 - 96}{4} = 0$$

The dependent source constraint equation is:

$i_o = v_2/40$

Place these equations in standard form:

$$v_1\left(\frac{1}{20} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) + v_3(0) + i_o(-5) = 0$$

$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{5} + \frac{1}{40} + \frac{1}{10}\right) + v_3\left(-\frac{1}{10}\right) + i_o(0) = 0$$

$$v_1(0) + v_2\left(-\frac{1}{10}\right) + v_3\left(\frac{1}{10} + \frac{1}{5} + \frac{1}{4}\right) + i_o\left(-\frac{11.5}{5}\right) = \frac{96}{4}$$

$$v_1(0) + v_2\left(-\frac{1}{40}\right) + v_3(0) + i_o(1) = 0$$

Solving, $v_1 = 156$ V; $v_2 = 120$ V; $v_3 = 78$ V; $i_o = 3$ A

[b] Calculate the power:

$$
\begin{aligned}
p_{\text{cccs}} &= -[5(3)](156) = -2340 \text{ W} \\
p_{20\Omega} &= (156)^2/20 = 1216.8 \text{ W} \\
p_{5\Omega} &= (156-120)^2/5 = 259.2 \text{ W} \\
p_{40\Omega} &= (120)^2/40 = 360 \text{ W} \\
p_{10\Omega} &= (120-78)^2/10 = 176.4 \text{ W} \\
p_{5\Omega} &= (78-11.5\cdot 3)^2/5 = 378.45 \text{ W} \\
p_{4\Omega} &= (78-96)^2/4 = 81 \text{ W} \\
p_{96\text{V}} &= [(78-96)/4](96) = -432 \text{ W} \\
p_{\text{ccvs}} &= [(78-3\cdot 11.5)/5](11.5\cdot 3) = 300.15 \text{ W}
\end{aligned}
$$

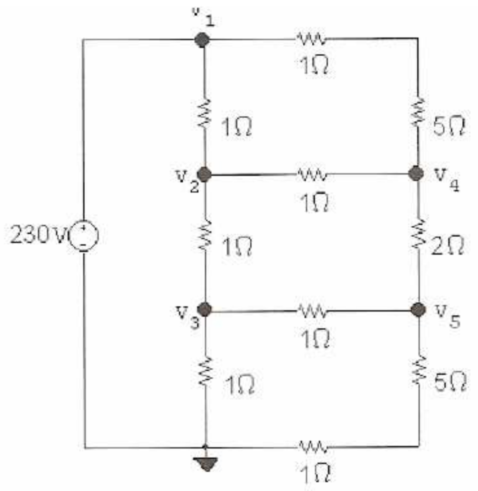
$\sum p_{\text{dev}} = 2340 + 432 = 2772$ W

$\sum p_{\text{dis}} = 1216.8 + 259.2 + 360 + 176.4 + 378.45 + 81 + 300.15 = 2772$ W

(checks)

Thus, the circuit dissipates 2772 W

4:



The node voltage equations are:

$$\begin{aligned}
\frac{v_2 - 230}{1} + \frac{v_2 - v_4}{1} + \frac{v_2 - v_3}{1} &= 0 \\
\frac{v_3 - v_2}{1} + \frac{v_3 - v_5}{1} + \frac{v_3}{1} &= 0 \\
\frac{v_4 - 230}{5+1} + \frac{v_4 - v_2}{1} + \frac{v_4 - v_5}{2} &= 0 \\
\frac{v_5 - v_4}{2} + \frac{v_5 - v_3}{1} + \frac{v_5}{5+1} &= 0
\end{aligned}$$

Place these equations in standard form:

$$\begin{aligned}
v_2(1+1+1) + v_3(-1) + v_4(-1) + v_5(0) &= 230 \\
v_2(-1) + v_3(1+1+1) + v_4(0) + v_5(-1) &= 0 \\
v_2(-1) + v_3(0) + v_4\left(\frac{1}{6} + 1 + \frac{1}{2}\right) + v_5\left(-\frac{1}{2}\right) &= \frac{230}{6} \\
v_2(0) + v_3(-1) + v_4\left(-\frac{1}{2}\right) + v_5\left(\frac{1}{2} + 1 + \frac{1}{6}\right) &= 0
\end{aligned}$$

Solving, $v_2 = 150$ V; $v_3 = 80$ V; $v_4 = 140$ V; $v_5 = 90$ V

Find the power dissipated by the $2\,\Omega$ resistor:

$i_{2\Omega} = \frac{v_4 - v_5}{2} = \frac{140 - 90}{2} = 25$ A

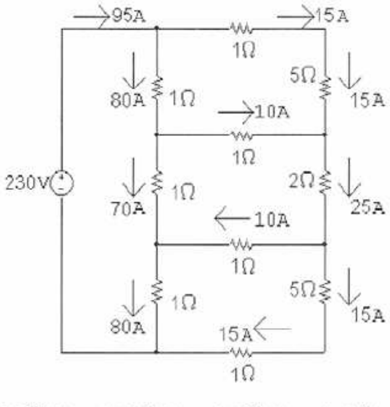
$p_{2\Omega} = (25)^2(2) = 1250$ W

[b] Find the power developed by the 230 V source:

$i_{230\text{V}} = \frac{v_2 - 230}{1} + \frac{v_4 - 230}{6} = -80 - 15 = -95$ A

$p_{230\text{V}} = (230)(-95) = -21{,}850$ W, so the source supplies 21,850 W

Check:



$$\begin{aligned}
\sum P_{\text{dis}} &= (80)^2(1) + (15)^2(1) + (15)^2(5) + (70)^2(1) + (10)^2(1) \\
&\quad + (25)^2(2) + (10)^2(1) + (80)^2(1) + (15)^2(5) + (15)^2(1) \\
&= 21{,}850 \text{ W(checks)}
\end{aligned}$$

5. :

Calculate currents and voltages needed to calculate the power for the various components:

$$i_\phi = \frac{v_4 - v_3}{8} = \frac{81.6 - 108}{8} = -3.3 \text{ A}$$

$$\frac{40}{3} i_\phi = \frac{40}{3}(-3.3) = -44 \text{ V}$$

$$v_1 = v_4 + \frac{40}{3} i_\phi = 81.6 - 44 = 37.6 \text{ V}$$

$$v_3 + v_\Delta = 120 \qquad \therefore \qquad v_\Delta = 120 - 108 = 12 \text{ V}$$

$$1.75 v_\Delta = (1.75)(12) = 21 \text{ A}$$

$$i_{120\text{V}} = \frac{v_1 - 120}{4} + \frac{v_3 - 120}{2} = \frac{37.6 - 120}{4} + \frac{108 - 120}{2} = -26.6 \text{ A}$$

$$i_{\text{ccvs}} = \frac{0 - v_1}{20} + \frac{v_2 - v_1}{4} = \frac{-37.6}{20} + \frac{120 - 37.6}{4} = 18.72 \text{ A}$$

Now calculate the power associated with each circuit element:

$$p_{20\Omega} = (37.6)^2/20 = 70.688 \text{ W}$$

$$p_{4\Omega} = (37.6 - 120)^2/4 = 1697.44 \text{ W}$$

$$p_{120\text{V}} = (120)(-26.6) = -3192 \text{ W}$$

$$p_{2\Omega} = (12)^2/2 = 72 \text{ W}$$

$$p_{40\Omega} = (108)^2/40 = 291.6 \text{ W}$$

$$p_{8\Omega} = (108 - 81.6)^2/8 = 87.12 \text{ W}$$

$$p_{80\Omega} = (81.6)^2/80 = 83.232 \text{ W}$$

$$p_{\text{vccs}} = (81.6)[1.75(12)] = 1713.6 \text{ W} \qquad \sum p_{\text{abs}} = \sum p_{\text{del}} = 4015.6 \text{ W}$$

$$p_{\text{ccvs}} = (18.72)(-44) = -823.68 \text{ W}$$

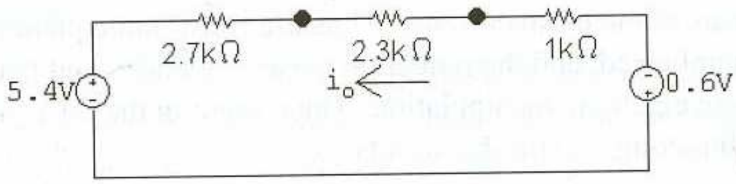
Now sum the powers:

$$\sum p_{\text{total}} = 70.688 + 1697.44 - 3192 + 72 + 291.6 + 87.12$$
$$+83.232 + 1712.6 - 823.68 = 0 \text{ W}$$

Thus, the power balances and the staff analyst has correctly calculated the voltage values

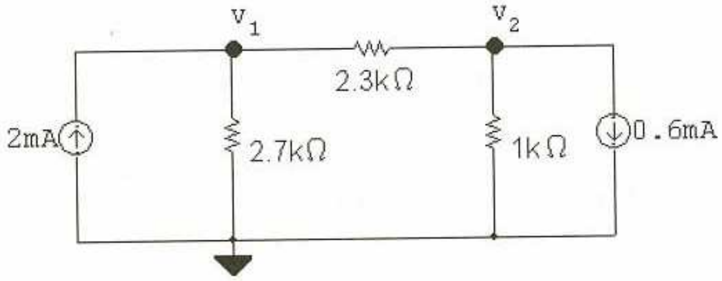
6.

[a] Apply source transformations to both current sources to get



$$i_o = \frac{-(5.4 + 0.6)}{2700 + 2300 + 1000} = -1 \text{ mA}$$

[b]



The node voltage equations:

$$-2 \times 10^{-3} + \frac{v_1}{2700} + \frac{v_1 - v_2}{2300} = 0$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{2300} + 0.6 \times 10^{-3} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{2700} + \frac{1}{2300}\right) + v_2\left(-\frac{1}{2300}\right) = 2 \times 10^{-3}$$

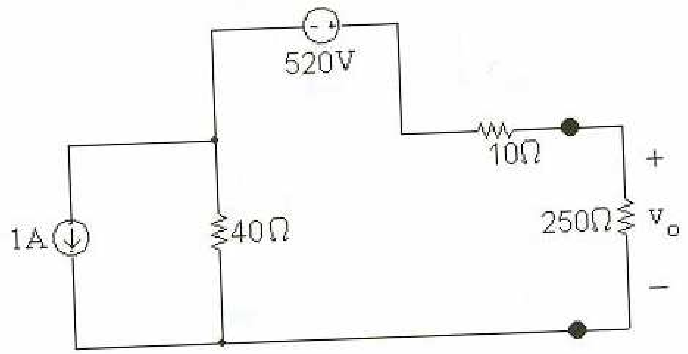
$$v_1\left(-\frac{1}{2300}\right) + v_2\left(\frac{1}{1000} + \frac{1}{2300}\right) = -0.6 \times 10^{-3}$$

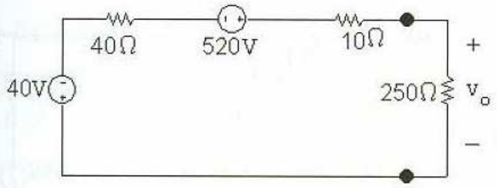
Solving, $v_1 = 2.7$ V; $\quad v_2 = 0.4$ V

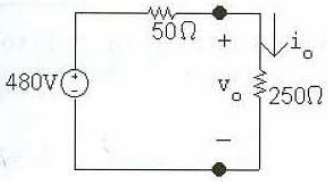
$$\therefore \quad i_o = \frac{v_2 - v_1}{2300} = -1 \text{ mA}$$

7:

[a]

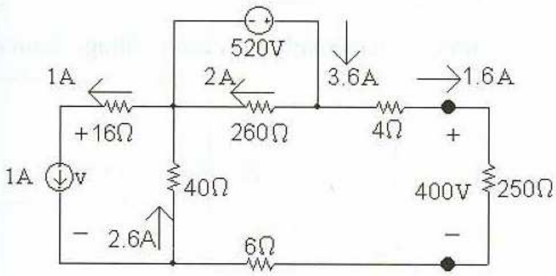






$$\therefore\ v_o = \frac{250}{300}(480) = 400\ \text{V}; \qquad i_o = \frac{400}{250} = 1.6\ \text{A}$$

[b]



$p_{520\text{V}} = -(520)(3.6) = -1872$ W

Therefore, the 520 V source is developing 1872 kW.

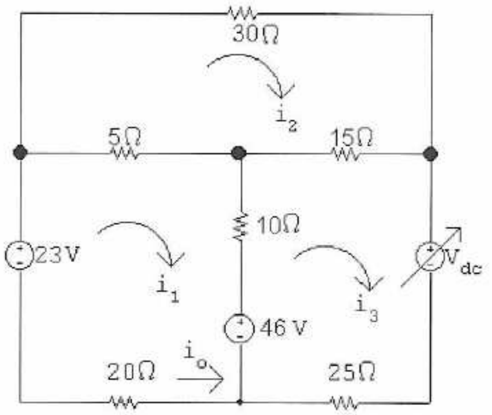
[c] $v = -(16)(1) - 40(2.6) = -120$ V

$p_{1\text{A}} = (-120)(1) = -120$ W

Therefore the 1 A source is developing 120 W.

8.

[a]



Write the mesh current equations. Note that if $i_o = 0$, then $i_1 = 0$:

$$\begin{aligned}
-23 + 5(-i_2) + 10(-i_3) + 46 &= 0 \\
30i_2 + 15(i_2 - i_3) + 5i_2 &= 0 \\
V_{\text{dc}} + 25i_3 - 46 + 10i_3 + 15(i_3 - i_2) &= 0
\end{aligned}$$

Place the equations in standard form:

$$\begin{aligned}
i_2(-5) + i_3(-10) + V_{\text{dc}}(0) &= -23 \\
i_2(30 + 15 + 5) + i_3(-15) + V_{\text{dc}}(0) &= 0 \\
i_2(-15) + i_3(25 + 10 + 15) + V_{\text{dc}}(1) &= 46
\end{aligned}$$

Solving, $\quad i_2 = 0.6$ A; $\quad i_3 = 2$ A; $\quad V_{\text{dc}} = -45$ V

Thus, the value of V_{dc} required to make $i_o = 0$ is -45 V.

[b] Calculate the power:

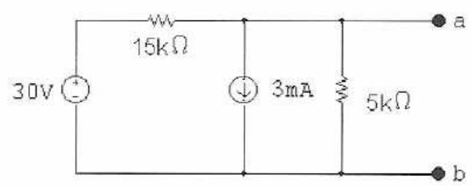
$$\begin{aligned}
p_{23\text{V}} &= -(23)(0) = 0 \text{ W} \\
p_{46\text{V}} &= -(46)(2) = -92 \text{ W} \\
p_{\text{Vdc}} &= (-45)(2) = -90 \text{ W} \\
p_{30\Omega} &= (30)(0.6)^2 = 10.8 \text{ W} \\
p_{5\Omega} &= (5)(0.6)^2 = 1.8 \text{ W} \\
p_{15\Omega} &= (15)(2 - 0.6)^2 = 29.4 \text{ W} \\
p_{10\Omega} &= (10)(2)^2 = 40 \text{ W} \\
p_{20\Omega} &= (20)(0)^2 = 0 \text{ W} \\
p_{25\Omega} &= (25)(2)^2 = 100 \text{ W}
\end{aligned}$$

$$\sum p_{\text{dev}} = 92 + 90 = 182 \text{ W}$$

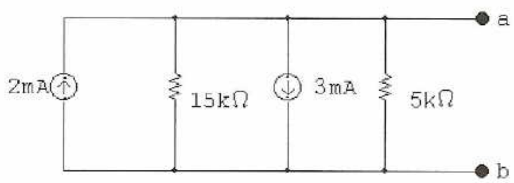
$$\sum p_{\text{dis}} = 10.8 + 1.8 + 29.4 + 40 + 0 + 100 = 182 \text{ W(checks)}$$

9:

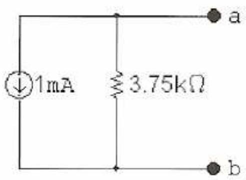
[9] First we make the observation that the 10 mA current source and the 10 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to



or

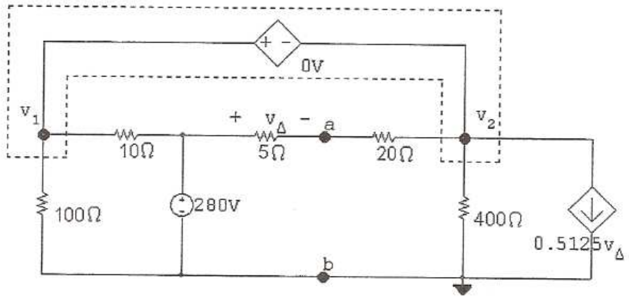


Therefore the Norton equivalent is determined by adding the current sources and combining the resistors in parallel:



10 .

[a] First find the Thévenin equivalent with respect to R_o.
Open circuit voltage: $i_\phi = 0$; $50i_\phi = 0$



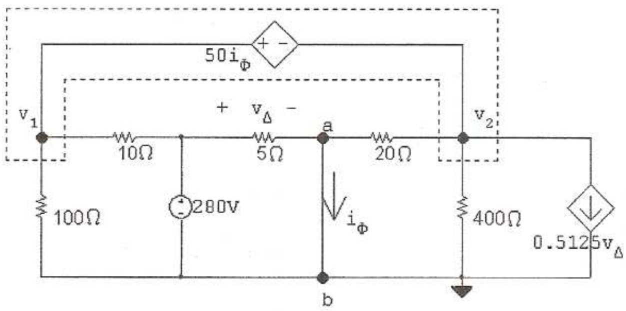
$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_1 - 280}{25} + \frac{v_1}{400} + 0.5125v_\Delta = 0$$

$$v_\Delta = \frac{(280 - v_1)}{25}5 = 56 - 0.2v_1$$

$v_1 = 210$ V; $\qquad v_\Delta = 14$ V

$V_{\text{Th}} = 280 - v_\Delta = 280 - 14 = 266$ V

Short circuit current



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2}{20} + \frac{v_2}{400} + 0.5125(280) = 0$$

$v_\Delta = 280$ V

$v_2 + 50i_\phi = v_1$

$$i_\phi = \frac{280}{5} + \frac{v_2}{20} = 56 + 0.05v_2$$

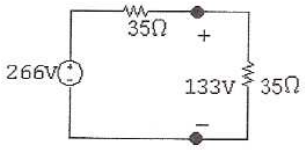
$v_2 = -968$ V; $\qquad v_1 = -588$ V

$i_\phi = i_{\text{sc}} = 56 + 0.05(-968) = 7.6$ A

$R_{\text{Th}} = V_{\text{Th}}/i_{\text{sc}} = 266/7.6 = 35\,\Omega$

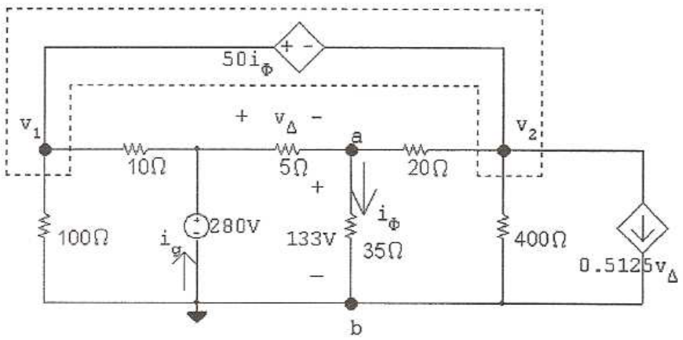
$\therefore \quad R_o = 35\,\Omega$

[b]



$$p_{\max} = (133)^2/35 = 505.4 \text{ W}$$

[c]



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2 - 133}{20} + \frac{v_2}{400} + 0.5125(280 - 133) = 0$$

$v_2 + 50i_\phi = v_1$; $\quad i_\phi = 133/35 = 3.8$ A

Therefore, $v_1 = -189$ V and $v_2 = -379$ V; thus,

$$i_g = \frac{280 - 133}{5} + \frac{280 + 189}{10} = 76.30 \text{ A}$$

p_{280V} (dev) $= (280)(76.3) = 21{,}364$ W

11: **[a]** Data sheet for LM 741 can simply be googled. But for future references, some of the good websites for searching datasheet of IC's are :

www.datasheetcatalog.com
www.alldatasheet.com
www.datasheetarchive.com

[b] Input offset voltage : the voltage required across the op-amp's input terminals to drive the output voltage to zero
Bandwidth : The range of frequency for which Op- amp maintains it gain (gain does not drop below 3 db) is called band width.
Input resistance: It's the resistance between two input terminals of the opamp seen by the incoming signal.

Ideally, input offset voltage should be "Zero" , bandwidth and input resistance should be "Infinite".

12:

Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the $2.2\,\mathrm{M}\Omega$ resistor is $(2.2 \times 10^6)(3.5 \times 10^{-6})$ or 7.7 V. Therefore the voltmeter reads 7.7 V.

13:

[a] Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0 - v_g}{2000} + \frac{0 - v_\Delta}{50{,}000} = 0$$

$$-25v_g - v_\Delta = 0, \qquad \therefore\ v_\Delta = -25(40 \times 10^{-3}) = -1\ \mathrm{V}$$

$$\frac{v_\Delta}{\alpha R_\Delta} + \frac{v_\Delta - 0}{50{,}000} + \frac{v_\Delta - v_o}{(1-\alpha)R_\Delta} = 0$$

$$\frac{v_\Delta}{\alpha} + 2v_\Delta + \frac{v_\Delta - v_o}{1-\alpha} = 0$$

$$v_\Delta\left(\frac{1}{\alpha} + 2 + \frac{1}{1-\alpha}\right) = \frac{v_o}{1-\alpha}$$

$$\therefore\ v_o = -1\left[1 + 2(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right]$$

When $\alpha = 0.2, \qquad v_o = -1(1 + 1.6 + 4) = -6.6\ \mathrm{V}$

When $\alpha = 1, \qquad v_o = -1(1 + 0 + 0) = -1\ \mathrm{V}$

$$\therefore\ -6.6\ \mathrm{V} \le v_o \le -1\ \mathrm{V}$$

[b] $-1\left[1 + 2(1-\alpha) + \dfrac{(1-\alpha)}{\alpha}\right] = -7$

$$\alpha + 2\alpha(1-\alpha) + (1-\alpha) = 7\alpha$$

$$\alpha + 2\alpha - 2\alpha^2 + 1 - \alpha = 7\alpha$$

$$\therefore\ 2\alpha^2 + 5\alpha - 1 = 0 \quad \text{so} \quad \alpha \cong 0.186$$