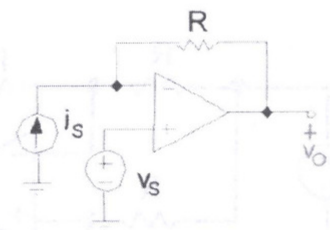
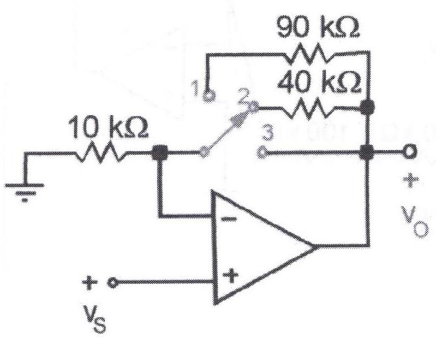
1.) Using superposition first let $i_S = 0$ and the circuit becomes a voltage follower with $v_{O1} = v_S$. Next let $v_S = 0$. This grounds the noninverting input so $v_P = 0$ *The sum of currents at the inverting input is* $i_S + \frac{v_{O2} - v_N}{R} + i_N = 0$. But $i_N = 0$ and $v_N = v_P = 0$ Hence $v_{O2} = -R \cdot i_S$. Using superposition $v_O = v_{O1} + v_{O2} = v_S - R \cdot i_S$



Therefore, $K_1 = 1$ and $K_2 = -R$.

2.)

The circuit is a noninverting amplifier with $R_1 = 10^4$ and R_2 determined by the switch position



Position 1 $R_2 = 90 \cdot 10^3$ and $K_V = 1 + \frac{R_2}{R_1} = 10$

Position 2 $R_2 = 40 \cdot 10^3$ and $K_V = 1 + \frac{R_2}{R_1} = 5$

Position 3 $R_2 = 0$ and $K_V = 1 + \frac{R_2}{R_1} = 1$

3.) Circuit is a subtractor with

$R_1 = 30 \cdot 10^3$, $R_2 = 60 \cdot 10^3$, $R_3 = 10 \cdot 10^3$ and

$R_4 = 20 \cdot 10^3$

Using superposition with $v_{S2} = 0$

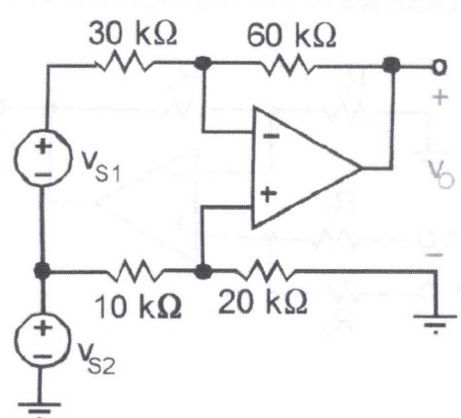
$$v_{O1} = \frac{-R_2}{R_1} \cdot v_{S1} = -2 \cdot v_{S1}$$

Using superposition with $v_{S1} = 0$

$$v_{O2} = \left(\frac{R_4}{R_3 + R_4}\right) \cdot \left(\frac{R_1 + R_2}{R_1}\right) \cdot v_{S2} + \frac{-R_2}{R_1} \cdot v_{S2} \quad \text{or}$$

$$v_{O2} = \left(\frac{2}{3}\right) \cdot (3) \cdot v_{S2} - 2 \cdot v_{S2} = 0$$ So finally

$$v_O = v_{O1} + v_{O2} = -2 \cdot v_{S1}$$

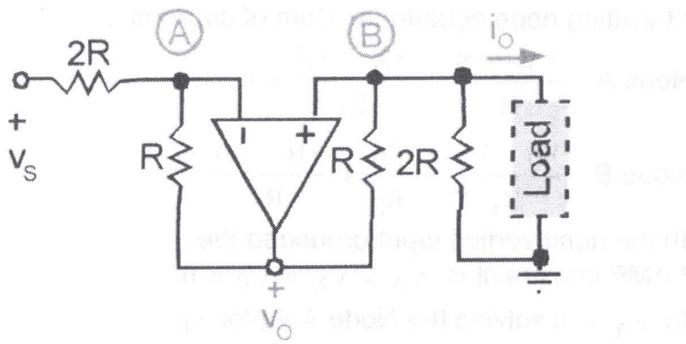


4.) Writing node equations. Sum of currents

at Node A: $\frac{v_A - v_S}{2\cdot R} + \frac{v_A - v_O}{R} = 0$

at Node B: $\frac{v_B - v_O}{R} + \frac{v_B}{2\cdot R} + i_O = 0$

The OP AMP voltage constraint is $v_A = v_B$. Inserting this into the Node A eq. and solving for v_B gives $v_B = \frac{1}{3}\cdot v_S + \frac{2}{3}\cdot v_O$. Inserting this into the Node B equation gives

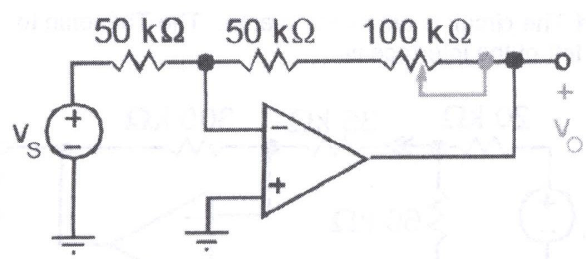


$$\frac{\frac{1}{3}\cdot v_S + \frac{2}{3}\cdot v_O - v_O}{R} + \frac{\frac{1}{3}\cdot v_S + \frac{2}{3}\cdot v_O}{2\cdot R} + i_O = 0$$ which reduces to $\frac{2}{3}\cdot v_S - \frac{2}{3}\cdot v_O + \frac{1}{3}\cdot v_S + \frac{2}{3}\cdot v_O + 2\cdot R\cdot i_O = 0$

Hence $i_O = \frac{-v_S}{2\cdot R}$ QED

5.) The circuit is an inverting amplifier with $R_1 := 50\cdot 10^3\ \Omega$ and $50\cdot 10^3\ \Omega < R_2 < 150\cdot 10^3\ \Omega$

Since $K_V = \frac{-R_2}{R_1}$ the gain falls in the range $-3 < K_V < -1$



6.)

[a] Assume the op-amp is operating within its linear range, then

$$i_L = \frac{8}{4000} = 2\,\text{mA}$$

For $R_L = 4\,\text{k}\Omega$ $\qquad v_o = (4+4)(2) = 16\ \text{V}$

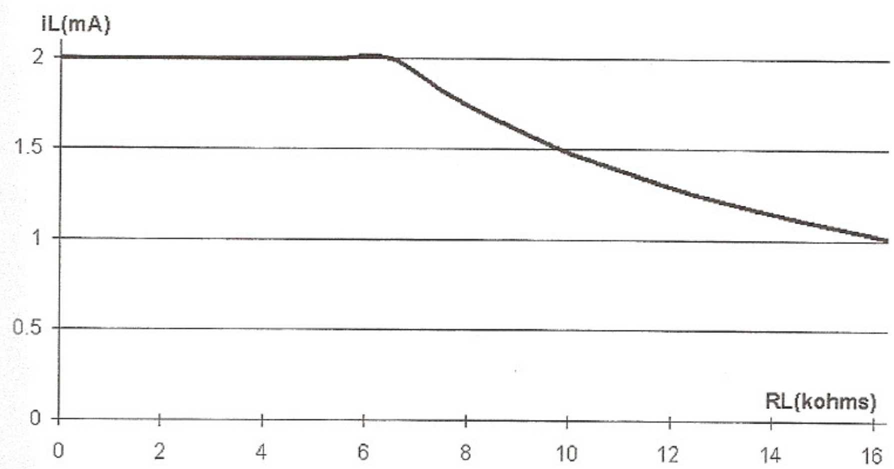
Now since $v_o < 20\ \text{V}$ our assumption of linear operation is correct, therefore

$i_L = 2\,\text{mA}$

[b] $20 = 2(4 + R_L); \qquad R_L = 6\,\text{k}\Omega$

[c] As long as the op-amp is operating in its linear region i_L is independent of R_L. From (b) we found the op-amp is operating in its linear region as long as $R_L \leq 6\,\text{k}\Omega$. Therefore when $R_L = 16\,\text{k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 20/(4{,}0000 + 16{,}000) = 1\,\text{mA}$. To justify neglecting the current into the op-amp assume the drop across the 50 kΩ resistor is negligible, and the input resistance to the op-amp is at least 500 kΩ. Then $i_p = i_n = (8-4)/(500 \times 10^3) = 8\,\mu\text{A}$. But $8\,\mu\text{A} \ll 1\,\text{mA}$, hence our assumption is reasonable.

[d]



7.)

We want the following expression for the output voltage:

$$v_o = -(2v_a + 4v_b + 6v_c + 8v_d)$$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage:

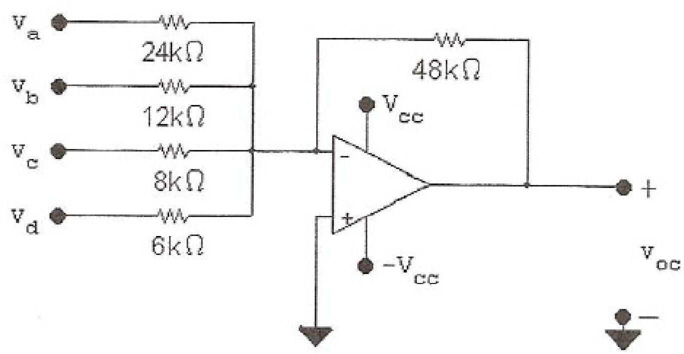
$$v_o = -\left[\frac{48}{R_a}v_a + \frac{48}{R_b}v_b + \frac{48}{R_c}v_c + \frac{48}{R_d}v_d\right]$$

Solve for each input resistance value to yield the desired gain:

$$\therefore \quad R_a = 48{,}000/2 = 24\,\text{k}\Omega \qquad R_c = 48{,}000/6 = 8\,\text{k}\Omega$$

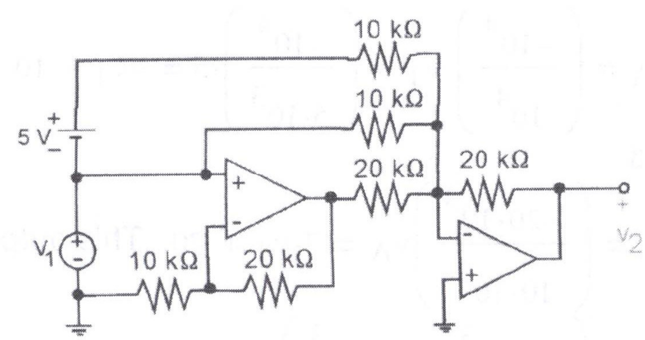
$$R_b = 48{,}000/4 = 12\,\text{k}\Omega \qquad R_d = 48{,}000/8 = 6\,\text{k}\Omega$$

The final circuit is shown here:



8.) The inverting summer has three input resistors. The inputs applied are (from top to bottom)

$v_1 + 5$ and v_1 and $\left(\frac{10+20}{10}\right)\cdot v_1 = 3\cdot v_1$.



Hence, the output of the inverting summer is

$$v_2 = \left(\frac{-20}{10}\right)\cdot(v_1 + 5) + \left(\frac{-20}{10}\right)\cdot v_1 + \left(\frac{-20}{20}\right)\cdot 3\cdot v_1$$

$$v_2 = (-7\cdot v_1 - 10)$$

9.)

$$A_{\mathrm{cm}} = \frac{(20)(50) - (50)R_x}{20(50 + R_x)}$$

$$A_{\mathrm{dm}} = \frac{50(20 + 50) + 50(50 + R_x)}{2(20)(50 + R_x)}$$

$$\frac{A_{\mathrm{dm}}}{A_{\mathrm{cm}}} = \frac{R_x + 120}{2(20 - R_x)}$$

$\therefore \quad \dfrac{R_x + 120}{2(20 - R_x)} = \pm 1000$ for the limits on the value of R_x

If we use $+1000 \quad R_x = 19.93\,\mathrm{k\Omega}$

If we use $-1000 \quad R_x = 20.07\,\mathrm{k\Omega}$

$19.93\,\mathrm{k\Omega} \leq R_x \leq 20.07\,\mathrm{k\Omega}$

10.) The output of the 1st stage is

$$v_A = \left(\frac{-10^4}{10^4}\right)\cdot v_1 + \left(\frac{-10^4}{5\cdot 10^3}\right)\cdot 5 = -v_1 - 10.$$ This output is the input to the 2nd stage

and

$$v_B = \left(\frac{-20\cdot 10^3}{10\cdot 10^3}\right) v_A = 2\cdot v_1 + 20.$$ This output is the input to the 3rd stage

$$v_O = \left(\frac{10\cdot 10^3 + 20\cdot 10^3}{20\cdot 10^3}\right)\cdot v_B = \frac{3}{2}\cdot v_B = 3\cdot v_1 + 30$$