Homework 6 Solutions

1)

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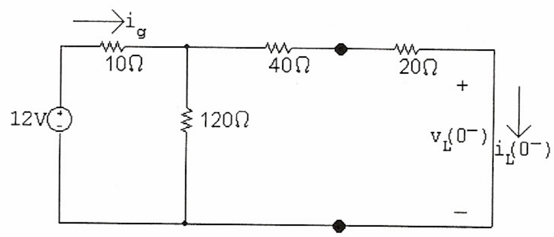
[a] $i_o(0^-) = 0$ since the switch is open for $t < 0$.

[b] For $t = 0^-$ the circuit is:

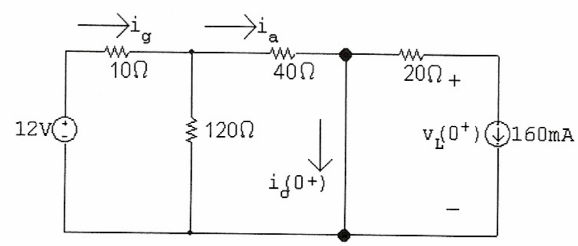


$$120\,\Omega \| 60\,\Omega = 40\,\Omega$$

$$\therefore\ i_g = \frac{12}{10+40} = 0.24\text{ A} = 240\text{ mA}$$

$$i_L(0^-) = \left(\frac{120}{180}\right) i_g = 160\text{ mA}$$

[c] For $t = 0^+$ the circuit is:



$$120\,\Omega \| 40\,\Omega = 30\,\Omega$$

$$\therefore\ i_g = \frac{12}{10+30} = 0.30\text{ A} = 300\text{ mA}$$

$$i_a = \left(\frac{120}{160}\right) 300 = 225\text{ mA}$$

$$\therefore\ i_o(0^+) = 225 - 160 = 65\text{ mA}$$

[d] $i_L(0^+) = i_L(0^-) = 160\text{ mA}$

[e] $i_o(\infty) = i_a = 225\text{ mA}$

[f] $i_L(\infty) = 0$, since the switch short circuits the branch containing the 20 Ω resistor and the 100 mH inductor.

[g] $\tau = \dfrac{L}{R} = \dfrac{100 \times 10^{-3}}{20} = 5\text{ ms}; \qquad \dfrac{1}{\tau} = 200$

$$\therefore\ i_L = 0 + (160 - 0)e^{-200t} = 160e^{-200t}\text{ mA}, \qquad t \geq 0$$

[h] $v_L(0^-) = 0$ since for $t < 0$ the current in the inductor is constant

[i] Refer to the circuit at $t = 0^+$ and note:

$$20(0.16) + v_L(0^+) = 0; \qquad \therefore\ v_L(0^+) = -3.2\text{ V}$$

[j] $v_L(\infty) = 0$, since the current in the inductor is a constant at $t = \infty$.

[k] $v_L(t) = 0 + (-3.2 - 0)e^{-200t} = -3.2e^{-200t}\text{ V}, \qquad t \geq 0^+$

[l] $i_o = i_a - i_L = 225 - 160e^{-200t}\text{ mA}, \qquad t \geq 0^+$

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2)

$$w(0) = \frac{1}{2}(20 \times 10^{-3})(11.2)^2 = 1254.4\,\text{mJ}$$

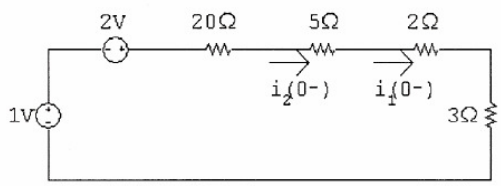
$$p_{30i_\Delta} = -30 i_\Delta i_L = -30(-4.9e^{-2625t})(11.2e^{-2625t}) = 1646.4e^{-5250t}\,\text{W}$$

$$w_{30i_\Delta} = \int_0^\infty 1646.4e^{-5250t}\,dt = 1646.4\frac{e^{-5250t}}{-5250}\Big|_0^\infty = 313.6\text{mJ}$$

$$\%\text{ dissipated } = \frac{313.6}{1254.4}(100) = 25\%$$

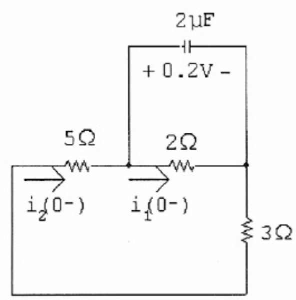
3)

[a] $t < 0$:



$$i_1(0^-) = i_2(0^-) = \frac{3\,\text{V}}{30\,\Omega} = 100\,\text{mA}$$

[b] $t > 0$:



$$i_1(0^+) = \frac{0.2}{2} = 100\,\text{mA}$$

$$i_2(0^+) = \frac{-0.2}{8} = -25\,\text{mA}$$

[c] Capacitor voltage cannot change instantaneously, therefore,

$$i_1(0^-) = i_1(0^+) = 100\,\text{mA}$$

[d] Switching can cause an instantaneous change in the current in a resistive branch. In this circuit

$$i_2(0^-) = 100\,\text{mA} \quad \text{and} \quad i_2(0^+) = -25\,\text{mA}$$

[e] $v_c = 0.2e^{-t/\tau}\,\text{V}, \quad t \geq 0 \qquad R_e = 2||(5+3) = 1.6\,\Omega$

$$\tau = 1.6(2 \times 10^{-6}) = 3.2 \times 10^{-6}\,\text{s}$$

$$v_c = 0.2e^{-312{,}500t}\,\text{V}, \qquad t \geq 0$$

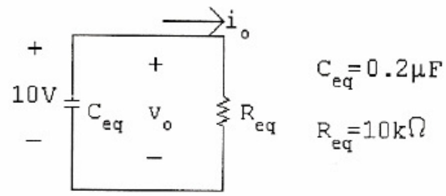
$$i_1 = \frac{v_c}{2} = 0.1e^{-312{,}500t}\,\text{A}, \qquad t \geq 0$$

[f] $i_2 = \dfrac{-v_c}{8} = -25e^{-312{,}500t}\,\text{mA}, \qquad t \geq 0^+$

Homework 6 Solutions

4)

[a] The equivalent circuit for $t > 0$:



$$\tau = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o = 10e^{-500t}\,\text{V}, \qquad t \geq 0$$

$$i_o = e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{24k\Omega} = e^{-500t}\left(\frac{16}{40}\right) = 0.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$p_{24k\Omega} = (0.16 \times 10^{-6}e^{-1000t})(24{,}000) = 3.84e^{-1000t}\,\text{mW}$$

$$w_{24k\Omega} = \int_0^\infty 3.84 \times 10^{-3}e^{-1000t}\,dt = -3.84 \times 10^{-6}(0-1) = 3.84\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.25 \times 10^{-6})(40)^2 + \frac{1}{2}(1 \times 10^{-6})(50)^2 = 1.45\,\text{mJ}$$

$$\%\,\text{diss}\,(24\,\text{k}\Omega) = \frac{3.84 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.26\%$$

[b] $p_{400\Omega} = 400(1 \times 10^{-3}e^{-500t})^2 = 0.4 \times 10^{-3}e^{-1000t}$

$$w_{400\Omega} = \int_0^\infty p_{400}\,dt = 0.40\,\mu\text{J}$$

$$\%\,\text{diss}\,(400\Omega) = \frac{0.4 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.03\%$$

$$i_{16k\Omega} = e^{-500t}\left(\frac{24}{40}\right) = 0.6e^{-500t}\,\text{mA}, \quad t \geq 0^+$$

$$p_{16k\Omega} = (0.6 \times 10^{-3}e^{-500t})^2(16{,}000) = 5.76 \times 10^{-3}e^{-1000t}\,\text{W}$$

$$w_{16k\Omega} = \int_0^\infty 5.76 \times 10^{-3}e^{-1000t}\,dt = 5.76\,\mu\text{J}$$

$$\%\,\text{diss}\,(16\text{k}\Omega) = 0.4\%$$

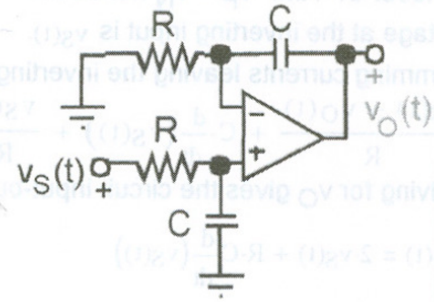
[c] $\sum w_{\text{diss}} = 3.84 + 5.76 + 0.4 = 10\,\mu\text{J}$

$$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 1.45 \times 10^{-3} - 10 \times 10^{-6} = 1.44\,\text{mJ}$$

$$\%\,\text{trapped} = \frac{1.44}{1.45} \times 100 = 99.31\%$$

Check: $0.26 + 0.03 + 0.4 + 99.31 = 100\%$

5)



Summing currents leaving the inverting input

$$\frac{v_N(t)}{R} + C\cdot\frac{d}{dt}\left(v_N(t) - v_O(t)\right) = 0$$

Solving for v_O gives

$$R\cdot C\cdot\frac{d}{dt}v_O(t) = v_N(t) + R\cdot C\cdot\frac{d}{dt}\left(v_N(t)\right)$$

Summing currents leaving the noninverting input

$$\frac{v_P(t) - v_S(t)}{R} + C\cdot\frac{d}{dt}\left(v_P(t)\right) = 0$$

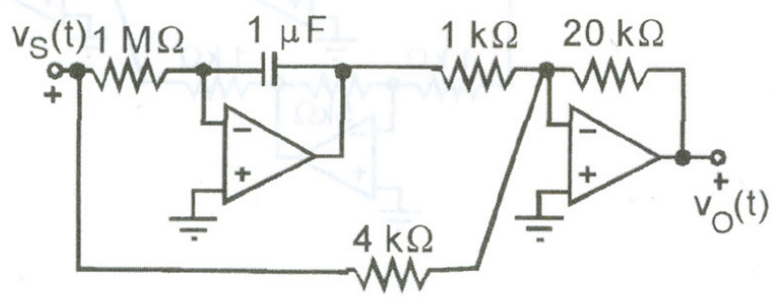
Solving for v_s gives $v_S(t) = v_P(t) + R\cdot C\cdot\frac{d}{dt}\left(v_P(t)\right)$

For an ideal OP AMP $v_p = v_N$ and as a result $v_P(t) + R\cdot C\cdot\frac{d}{dt}\left(v_P(t)\right) = v_N(t) + R\cdot C\cdot\frac{d}{dt}\left(v_N(t)\right)$ which means

that $v_S(t) = R\cdot C\cdot\frac{d}{dt}v_O(t)$ integrating both sides gives $v_O(t) = \frac{1}{R\cdot C}\cdot\int_0^t v_S(x)\,dx + v_O(0)$ QED

6) The required input-output relationship is $v_O(t) = -5\cdot v_S(t) + 20\cdot\int_0^t v_S(x)\,dx$.

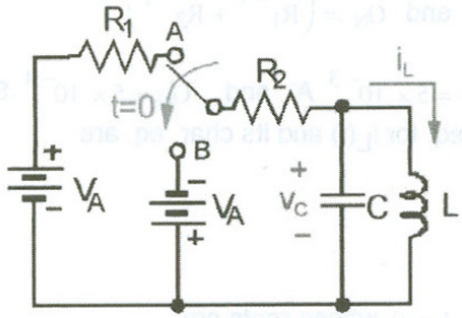
One of many possible designs is shown below



Homework 6 Solutions

7) Parallel RLC circuit with: $R_1 := 20000 = 20\ k\Omega$ $R_2 := 4000 = 4\ k\Omega$ $L := 1.6$ H

$C := 1.25 \cdot 10^{-6} = 1.25\ \mu F$ $V_A := 24$ V



For t < 0 sw in Pos A: $v_C(0) = 0$ and

$i_L(0) = \dfrac{V_A}{R_1 + R_2} = 1 \cdot 10^{-3}$ for t > 0 sw in Pos. B:

The Norton equivalent circuit is $i_N = \dfrac{-V_A}{R_2} = -6 \cdot 10^{-3}$ A

and $G_N := (R_2)^{-1}$ hence $G_N = 2.5 \times 10^{-4}$ S.

The circuit differential eq. and its characteristic eq. are

$L \cdot C \cdot \dfrac{d^2}{dt^2} i_L(t) + G_N \cdot L \cdot \dfrac{d}{dt} i_L(t) + i_L(t) = i_N$ and

$L \cdot C \cdot s^2 + G_N \cdot L \cdot s + 1 = 0$ whose roots are

$\text{polyroots}\left(\left(1 \quad G_N \cdot L \quad L \cdot C\right)^T\right) = \begin{pmatrix} -100 - 700j \\ -100 + 700j \end{pmatrix}$ **The circuit is underdamped** so the natural response

has the form $i_{LN}(t) = e^{-100 \cdot t} \cdot (K_1 \cdot \cos(700t) + K_2 \cdot \sin(700t))$.

The forced response is $i_{LF} = i_N = -6 \cdot 10^{-3}$.

The total response is $i_L(t) = i_{LN}(t) + i_{LF}(t) = e^{-100 \cdot t} \cdot (K_1 \cdot \cos(700t) + K_2 \cdot \sin(700t)) + \left(-6 \cdot 10^{-3}\right)$.

The initial condition $i_L(0) = 1 \cdot 10^{-3}$ implies $K_1 - 6 \times 10^{-3} = 1 \cdot 10^{-3}$ which means $K_1 = 7 \cdot 10^{-3}$ A.

The initial condition $\dfrac{d}{dt} i_L(0) = \dfrac{1}{L} \cdot v_C(0) = 0$ implies $-100 \cdot K_1 + 700 \cdot K_2 = 0$ which means

$K_2 = \dfrac{K_1}{7} = 10^{-3}$ A.

For t > 0 the inductor current is $i_L(t) = 10^{-3}\left[e^{-100 \cdot t} \cdot (7 \cdot \cos(700t) + \sin(700t)) - 6\right]$ A.

Using KVL and the inductor i-v characteristic leads to

$v_C(t) = v_L(t) = L \cdot \dfrac{d}{dt} i_L(t) = 1.6 \dfrac{d}{dt}\left[10^{-3} \cdot e^{-100 \cdot t} \cdot (7 \cdot \cos(700t) + \sin(700t)) - 6\right]$ which reduces to

$v_C(t) = -8 \cdot e^{-100 \cdot t} \cdot \sin(700t)$ V

8)

When the sw is in position A the capacitor discharges through the short and the inductor current is zero, that is $v_C(0) = 0$ and $i_L(0) = 0$. At t = 0 the sw moves to position B where the capacitor and 10-kΩ resistor form a 1st order RC circuit whose charging time constant is $R \cdot C = 10^4 \cdot 10^{-6} = 0.01$. If left in this position for a long time the final value of the capacitor voltage would be $v_C = 10$ V. Thus, for $0 \le t \le 10$ ms the capacitor voltage is $v_C(t) = 10(1 - \exp(-100t))$ V. The capacitor voltage at t = 10 ms is $v_C(0.01) = 10(1 - \exp(-1)) = 6.32$ V. At t = 10 ms the sw is moved to position C where the capacitor and 1-H inductor form an undamped ($\zeta = 0$) 2nd order system whose undamped natural frequency is $\omega_0 = \dfrac{1}{\sqrt{L \cdot C}} = 1000$ rad/s. Thus, for t > 10 ms the capacitor voltage is an undamped sinusoidal oscillation with a frequency of 1000 rad/s and amplitude of $v_C(0.01) = 6.321$ V.

(a) If R is reduced to 1 kΩ the charging time constant decreases to $R \cdot C = 0.001$, the capacitor voltage at t = 10 ms becomes $v_C(0.01) = 10 \cdot (1 - \exp(-10)) = 10$ V, and $\omega_0 = \dfrac{1}{\sqrt{L \cdot C}}$ is unchanged. Hence the amplitude increases and the frequency remains the same.

(b) If L is reduced to 100 mH the charging time constant is unchanged, but the undamped natural frequency becomes $\omega_0 = \dfrac{1}{\sqrt{L \cdot C}} = 3162$ rad/s. Hence the amplitude is unchanged and the frequency increases.