Home work 7 solution

Problem 1

$i_C(0) = 0; \qquad v_o(0) = 50\,\text{V}$

$$\alpha = \frac{R}{2L} = \frac{8000}{2(160 \times 10^{-3})} = 25{,}000\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(160 \times 10^{-3})(10 \times 10^{-9})} = 625 \times 10^6$$

$\therefore \quad \alpha^2 = \omega_o^2; \qquad$ critical damping

$$v_o(t) = V_f + D_1' te^{-25{,}000t} + D_2' e^{-25{,}000t}$$

$V_f = 250\,\text{V}$

$v_o(0) = 250 + D_2' = 50; \qquad D_2' = -200\,V$

$$\frac{dv_o}{dt}(0) = -25{,}000 D_2' + D_1' = 0$$

$D_1' = 25{,}000 D_2' - 5 \times 10^6\ \text{V/s}$

$$v_o = 250 - 5 \times 10^6 te^{-25{,}000t} - 200e^{-25{,}000t}\,\text{V}, \quad t \geq 0$$

Problem 2

[a] $v_c = V_f + [B_1' \cos\omega_d t + B_2' \sin\omega_d t]\, e^{-\alpha t}$

$$\frac{dv_c}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]e^{-\alpha t}$$

Since the initial stored energy is zero,

$$v_c(0^+) = 0 \quad \text{and} \quad \frac{dv_c(0^+)}{dt} = 0$$

It follows that $B_1' = -V_f$ and $B_2' = \dfrac{\alpha B_1'}{\omega_d}$

When these values are substituted into the expression for $[dv_c/dt]$, w

$$\frac{dv_c}{dt} = \left(\frac{\alpha^2}{\omega_d} + \omega_d\right) V_f e^{-\alpha t} \sin\omega_d t$$

But $V_f = V$ and $\dfrac{\alpha^2}{\omega_d} + \omega_d = \dfrac{\alpha^2 + \omega_d^2}{\omega_d} = \dfrac{\omega_o^2}{\omega_d}$

Therefore $\dfrac{dv_c}{dt} = \left(\dfrac{\omega_o^2}{\omega_d}\right) V e^{-\alpha t} \sin\omega_d t$

[b] $\dfrac{dv_c}{dt} = 0$ when $\sin\omega_d t = 0,$ or $\omega_d t = n\pi$

where $n = 0, 1, 2, 3, \ldots$

Therefore $t = \dfrac{n\pi}{\omega_d}$

[c] When $t_n = \frac{n\pi}{\omega_d}$, $\cos\omega_d t_n = \cos n\pi = (-1)^n$

and $\sin\omega_d t = \sin n\pi = 0$

Therefore $v_c(t_n) = V[1 - (-1)^n e^{-\alpha n\pi/\omega_d}]$

[d] It follows from [c] that

$$v(t_1) = V + Ve^{-(\alpha\pi/\omega_d)} \quad \text{and} \quad v_c(t_3) = V + Ve^{-(3\alpha\pi/\omega_d)}$$

Therefore $$\frac{v_c(t_1) - V}{v_c(t_3) - V} = \frac{e^{-(\alpha\pi/\omega_d)}}{e^{-(3\alpha\pi/\omega_d)}} = e^{(2\alpha\pi/\omega_d)}$$

But $\frac{2\pi}{\omega_d} = t_3 - t_1 = T_d$, thus $\alpha = \frac{1}{T_d}\ln\frac{[v_c(t_1) - V]}{[v_c(t_3) - V]}$

Problem 3

[a] $$\frac{d^2v_o}{dt^2} = \frac{1}{R_1C_1R_2C_2}v_g$$

$$\frac{1}{R_1C_1R_2C_2} = \frac{10^{-6}}{(100)(400)(0.5)(0.2)\times 10^{-6}\times 10^{-6}} = 250$$

$$\therefore \quad \frac{d^2v_o}{dt^2} = 250v_g$$

$0 \le t \le 0.5^-$:

$v_g = 80\,\text{mV}$

$$\frac{d^2v_o}{dt^2} = 20$$

Let $g(t) = \frac{dv_o}{dt}$, then $\frac{dg}{dt} = 20$ or $dg = 20\,dt$

$$\int_{g(0)}^{g(t)} dx = 20\int_0^t dy$$

$$g(t) - g(0) = 20t, \quad g(0) = \frac{dv_o}{dt}(0) = 0$$

$$g(t) = \frac{dv_o}{dt} = 20t$$

$$dv_o = 20t\,dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = 20\int_0^t x\,dx; \qquad v_o(t) - v_o(0) = 10t^2, \quad v_o(0) = 0$$

$$v_o(t) = 10t^2\,\text{V}, \quad 0 \le t \le 0.5^-$$

$$\frac{dv_{o1}}{dt} = -\frac{1}{R_1C_1}v_g = -20v_g = -1.6$$

$$dv_{o1} = -1.6\,dt$$

$$\int_{v_{o1}(0)}^{v_{o1}(t)} dx = -1.6\int_0^t dy$$

$$v_{o1}(t) - v_{o1}(0) = -1.6t, \qquad v_{o1}(0) = 0$$

$$v_{o1}(t) = -1.6t\,\text{V}, \qquad 0 \le t \le 0.5^-$$

$0.5^+ \le t \le t_{\text{sat}}$:

$$\frac{d^2v_o}{dt^2} = -10, \qquad \text{let} \quad g(t) = \frac{dv_o}{dt}$$

$$\frac{dg(t)}{dt} = -10; \qquad dg(t) = -10\,dt$$

$$\int_{g(0.5^+)}^{g(t)} dx = -10\int_{0.5}^t dy$$

$$g(t) - g(0.5^+) = -10(t - 0.5) = -10t + 5$$

$$g(0.5^+) = \frac{dv_o(0.5^+)}{dt}$$

$$C\frac{dv_o}{dt}(0.5^+) = \frac{0 - v_{o1}(0.5^+)}{400 \times 10^3}$$

$$v_{o1}(0.5^+) = v_o(0.5^-) = -1.6(0.5) = -0.80\,\text{V}$$

$$\therefore \quad C\frac{dv_o(0.5^+)}{dt} = \frac{0.80}{0.4 \times 10^6} = 2\,\mu\text{A}$$

$$\frac{dv_o}{dt}(0.5^+) = \frac{2 \times 10^{-6}}{0.2 \times 10^{-6}} = 10\,\text{V/s}$$

$$\therefore \quad g(t) = -10t + 5 + 10 = -10t + 15 = \frac{dv_o}{dt}$$

$$\therefore \quad dv_o = -10t\,dt + 15\,dt$$

$$\int_{v_o(0.5^+)}^{v_o(t)} dx = \int_{0.5^+}^t -10y\,dy + \int_{0.5^+}^t 15\,dy$$

$$v_o(t) - v_o(0.5^+) = -5y^2 \Big|_{0.5}^{t} + 15y \Big|_{0.5}^{t}$$

$$v_o(t) = v_o(0.5^+) - 5t^2 + 1.25 + 15t - 7.5$$

$$v_o(0.5^+) = v_o(0.5^-) = 2.5\,\text{V}$$

$$\therefore \quad v_o(t) = -5t^2 + 15t - 3.75\,\text{V}, \qquad 0.5^+ \leq t \leq t_{\text{sat}}$$

$$\frac{dv_{o1}}{dt} = -20(-0.04) = 0.8, \qquad 0.5^+ \leq t \leq t_{\text{sat}}$$

$$dv_{o1} = 0.8\,dt; \qquad \int_{v_{o1}(0.5^+)}^{v_{o1}(t)} dx = 0.8\int_{0.5^+}^{t} dy$$

$$v_{o1}(t) - v_{o1}(0.5^+) = 0.8t - 0.4; \qquad v_{o1}(0.5^+) = v_{o1}(0.5^-) = -0.8\,\text{V}$$

$$\therefore \quad v_{o1}(t) = 0.8t - 1.2\,\text{V}, \qquad 0.5^+ \leq t \leq t_{\text{sat}}$$

Summary:

$0 \leq t \leq 0.5^-\,\text{s}:$ $\qquad v_{o1} = -1.6t\,\text{V}, \quad v_o = 10t^2\,\text{V}$

$0.5^+\,\text{s} \leq t \leq t_{\text{sat}}:$ $\qquad v_{o1} = 0.8t - 1.2\,\text{V}, \quad v_o = -5t^2 + 15t - 3.75\,\text{V}$

[b] $12.5 = -5t_{\text{sat}}^2 + 15t_{\text{sat}} - 3.75$

$$\therefore \quad 5t_{\text{sat}}^2 - 15t_{\text{sat}} - 8.75 = 0$$

Solving, $\qquad t_{\text{sat}} = 3.5\,\text{sec}$

$$v_{o1}(t_{\text{sat}}) = 0.8(3.5) - 1.2 = 1.6\,\text{V}$$

Problem 4

[a] $f(t) =$ inertial force + frictional force + spring force

$$= M[d^2x/dt^2] + D[dx/dt] + Kx$$

[b] $$\frac{d^2x}{dt^2} = \frac{f}{M} - \left(\frac{D}{M}\right)\left(\frac{dx}{dt}\right) - \left(\frac{K}{M}\right)x$$

Given $v_A = \dfrac{d^2x}{dt^2}$, then

$$v_B = -\frac{1}{R_1C_1}\int_0^t \left(\frac{d^2x}{dy^2}\right) dy = -\frac{1}{R_1C_1}\frac{dx}{dt}$$

$$v_C = -\frac{1}{R_2C_2}\int_0^t v_B\, dy = \frac{1}{R_1R_2C_1C_2}x$$

$$v_D = -\frac{R_3}{R_4}\cdot v_B = \frac{R_3}{R_4R_1C_1}\frac{dx}{dt}$$

$$v_E = \left[\frac{R_5 + R_6}{R_6}\right] v_C = \left[\frac{R_5 + R_6}{R_6}\right] \cdot \frac{1}{R_1 R_2 C_1 C_2} \cdot x$$

$$v_F = \left[\frac{-R_8}{R_7}\right] f(t), \qquad v_A = -(v_D + v_E + v_F)$$

Therefore $\dfrac{d^2x}{dt^2} = \left[\dfrac{R_8}{R_7}\right] f(t) - \left[\dfrac{R_3}{R_4 R_1 C_1}\right] \dfrac{dx}{dt} - \left[\dfrac{R_5 + R_6}{R_6 R_1 R_2 C_1 C_2}\right] x$

Therefore $M = \dfrac{R_7}{R_8}$, $\quad D = \dfrac{R_3 R_7}{R_8 R_4 R_1 C_1}\quad$ and $\quad K = \dfrac{R_7(R_5 + R_6)}{R_8 R_6 R_1 R_2 C_1 C_2}$

Box Number	Function
1	inverting and scaling
2	inverting and scaling
3	integrating and scaling
4	integrating and scaling
5	inverting and scaling
6	noninverting and scaling

Problem 5

[a] $\omega = 2\pi f = 314{,}159.27$ rad/s

[b] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{10 \times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(10 \times 10^{-3})\underline{/0^\circ} = 10 \times 10^{-3}\omega C\underline{/90^\circ}$

$$\therefore \quad \theta_i = 90^\circ$$

[c] $628.32 \times 10^{-6} = 10 \times 10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{10 \times 10^{-3}}{628.32 \times 10^{-6}} = 15.92\,\Omega, \quad \therefore \quad X_C = -15.92\,\Omega$$

[d] $C = \dfrac{1}{15.92(\omega)} = \dfrac{1}{(15.92)(100\pi \times 10^3)}$

$$C = 0.2\,\mu\text{F}$$

[e] $Z_c = j\left(\dfrac{-1}{\omega C}\right) = -j15.92\,\Omega$

Problem 6

[a] $\mathbf{V}_g = 300\angle 78^\circ; \qquad \mathbf{I}_g = 6\angle 33^\circ$

$$\therefore \quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = \frac{300\angle 78^\circ}{6\angle 33^\circ} = 50\angle 45^\circ\,\Omega$$

[b] i_g lags v_g by 45°:

$$2\pi f = 5000\pi; \qquad f = 2500\,\text{Hz}; \qquad T = 1/f = 400\,\mu\text{s}$$

$$\therefore \quad i_g \text{ lags } v_g \text{ by } \frac{45^\circ}{360^\circ}(400\,\mu\text{s}) = 50\,\mu\text{s}$$

Problem 7

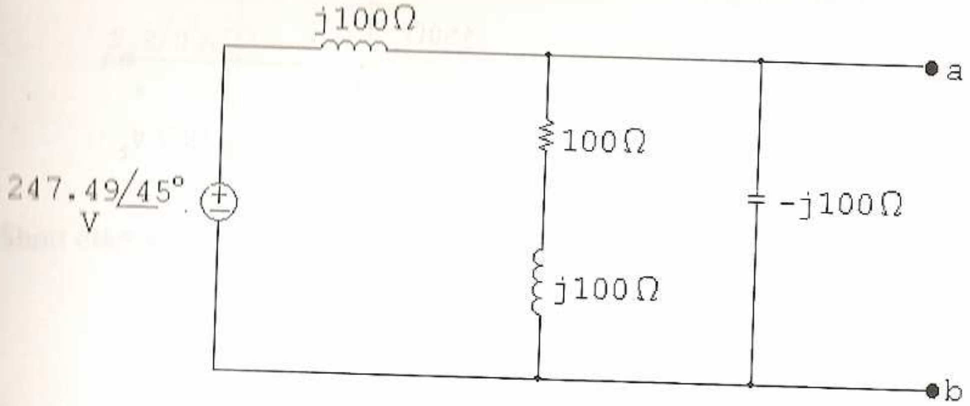
$$Z_{ab} = 1 - j8 + (2 + j4)\|(10 - j20) + (40\|j20)$$

$$= 1 - j8 + 3 + j4 + 8 + j16 = 12 + j12\,\Omega = 16.971\angle 45^\circ\,\Omega$$

Problem 8

[a] $j\omega L = j(1000)(100)\times 10^{-3} = j100\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(1000)(10)} = -j100\,\Omega$$



Using voltage division,

$$\mathbf{V}_{ab} = \frac{(100+j100)\|(-j100)}{j100+(100+j100)\|(-j100)}(247.49\underline{/45^\circ}) = 350\underline{/0^\circ}$$

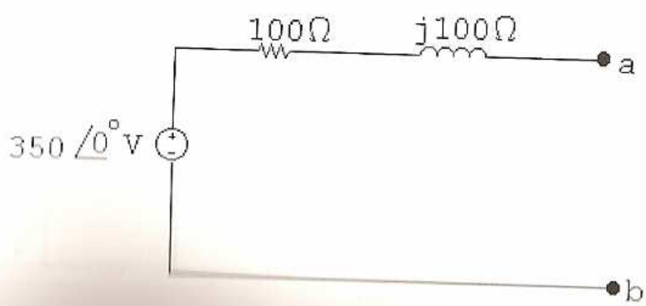
$$\mathbf{V}_{Th} = \mathbf{V}_{ab} = 350\underline{/0^\circ}\,\text{V}$$

[b] Remove the voltage source and combine impedances in parallel to find $Z_{Th} = Z_{ab}$:

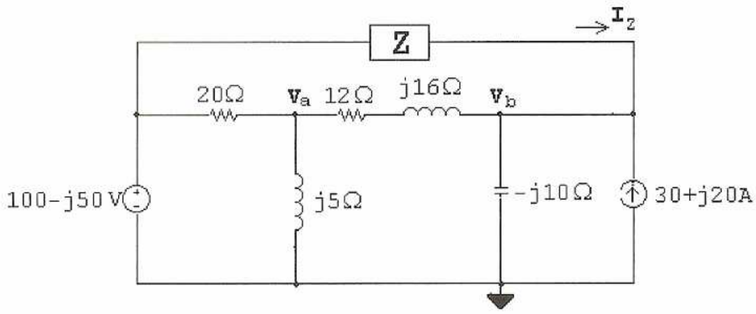
$$Y_{ab} = \frac{1}{j100} + \frac{1}{100+j100} + \frac{1}{-j100} = 5 - j5\text{ mS}$$

$$Z_{Th} = Z_{ab} = \frac{1}{Y_{ab}} = 100 + j100\,\Omega$$

c]



Problem 9



$$\frac{\mathbf{V}_a - (100 - j50)}{20} + \frac{\mathbf{V}_a}{j5} + \frac{\mathbf{V}_a - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{V}_a = 40 + j30\,\mathrm{V}$$

$$\mathbf{I}_Z + (30 + j20) - \frac{140 + j30}{-j10} + \frac{(40 + j30) - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{I}_Z = -30 - j10\,\mathrm{A}$$

$$Z = \frac{(100 - j50) - (140 + j30)}{-30 - j10} = 2 + j2\,\Omega$$