ECE 2C Mid-Term Exam A Solutions

May 8, 2012

Do not open exam until instructed to.

Closed book: Crib sheet and 1 page personal notes permitted

There are 2 problems on this exam, and you have 75 minutes.

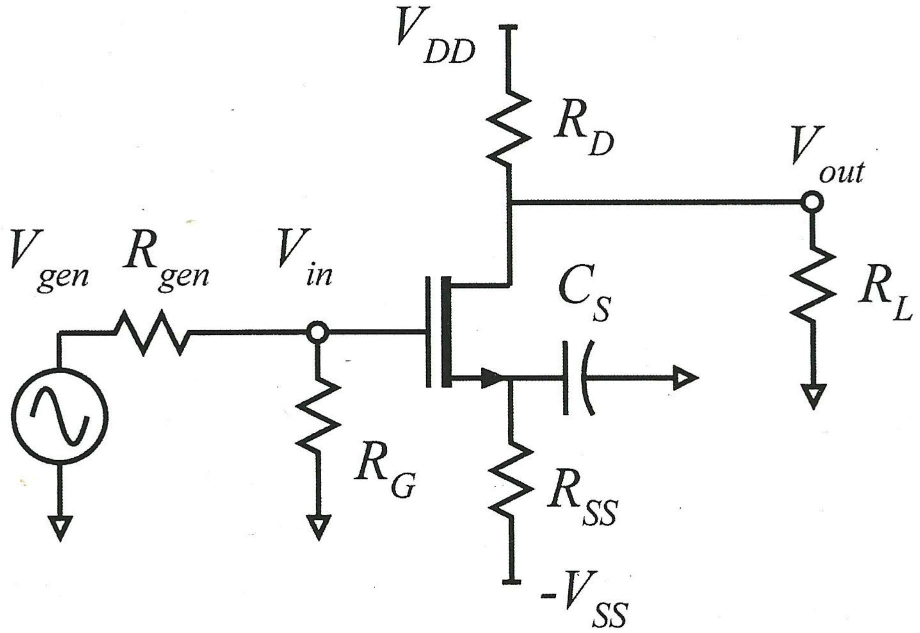
Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING and approximately Justifying them.***

Name: ______________________

Problem	Points Received	Points Possible
1a		10
1b		4
1c		10
1d		15
1e		5
1f		10
1g		10
2a		15
2b		15
2c		5
total		100

Problem 1, 65 points
Transistor circuit analysis

You will be working on the circuit below:



Q1 is a mobility-limited FET, ie. $I_d = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2(1+\lambda V_{ds})$ where $(\mu C_{ox} W_g / 2L_g)$=4 mA/V^2, $\lambda = 0.1\,\text{V}^{-1}$, and V_{th}=0.5 V.

V_{DD} = +2.0 Volts, $-V_{SS}$ = −2 Volts
C_S is very big and has negligible AC impedance.
R_L=5 kOhm, R_{gen}=100 kOhm, R_G = 1 MOhm

Part a, 10 points

DC bias.

Q1 is to be biased with 1 mA drain current.

The drain of Q1 is to be biased at 0.0 Volts

Ignore λ while solving this part*

Find: R_{SS}= 1kΩ R_D= 2kΩ

We are given that Q1 is a mobility-limited FET with current equation

$$I_d = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$$

We plug in $\mu C_{ox} W_g / 2L_g = 4 mA \cdot V^{-2}$, $V_{th} = 0.5V$, and assume $\lambda = 0$ to get

(2) —— $$V_{gs} = \left(\frac{I_D}{\mu C_{ox} W_g / 2L_g}\right)^{1/2} + V_{th} = \left(\frac{10^{-3}\,A}{4 mA \cdot V^{-2}}\right)^{1/2} + 0.5V = 1V$$

And now we can solve for R_{SS}, R_D:

Since V_{gen} is a sinusoidal signal, we assume that its average DC value is zero, and hence $V_g = 0$.

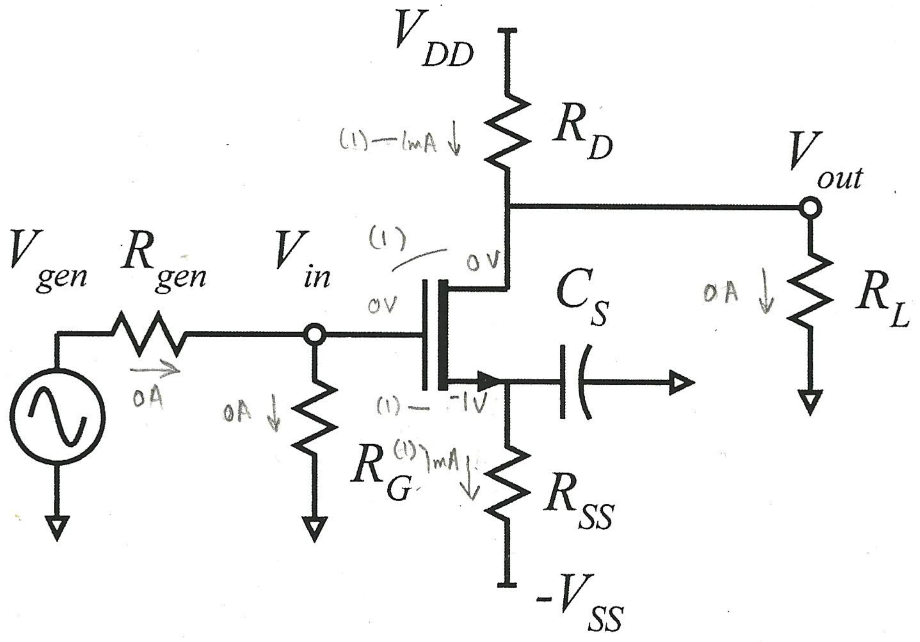
(2) —— Thus $V_s = -V_{gs} = -1\ V.$

(3) —— $$R_{SS} = \frac{V_S - V_{SS}}{I_D} = \frac{(-1+2)V}{10^{-3}A} = 1k\Omega$$

(3) —— $$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{(2-0)V}{10^{-3}A} = 2k\Omega$$

Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes** and the DC currents through **ALL resistors**

Part c, 10 points

Find the small signal parameters of Q1. Use the mobility-limited model.

g_m = 4.4mS

R_{DS} = 11 kΩ

$$(3) - g_m = \frac{\partial I_D}{\partial V_{gs}} = (\mu C_{ox} W_g / L_g)(V_{gs} - V_{th})(1 + \lambda V_{ds})$$

$$= 2\,(4\,mA \cdot V^{-2})(1 - 0.5V)(1 + 0.1\,V^{-1} \cdot 1V)$$

$$= 4.4\,mS \; - (2)$$

$$(3) - R_{DS} = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} = \frac{10\,V + 0V}{1mA} = 11\,k\Omega \; - (2)$$

OR

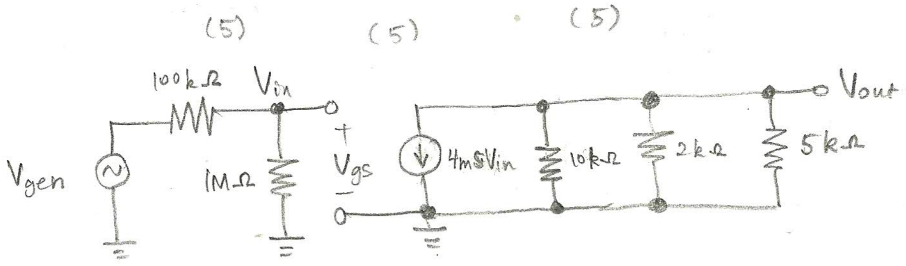
$$(3) - R_{DS} = \left(\frac{\partial I_D}{\partial V_{ds}}\right)^{-1} = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 \lambda$$

$$= (4\,mA \cdot V^{-2})(1 - 0.5V)^2 (0.1\,V^{-1})$$

$$= 10\,k\Omega \; - (2)$$

Part d, 15 points

Replacing the transistor with its small-signal model, draw a small-signal equivalent circuit diagram for the amplifier. Give values for all elements on the diagram.



Part e, 5 points.

Find the small signal voltage gain (Vout/Vin) of Q1.

Vout/Vin= -5.54 V/V

$$R_{LEQ} = R_{DS} \,||\, R_L \,||\, R_D = \frac{2k\Omega \cdot 5k\Omega \cdot 11k\Omega}{2k\Omega \cdot 5k\Omega + 2k\Omega \cdot 11k\Omega + 5k\Omega \cdot 11k\Omega}$$

$$R_{LEQ} \approx 1.26\,k\Omega$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_m V_{gs} R_{LEQ}}{V_{in}} \quad , \quad V_{in} = V_{gs}$$

$$\frac{V_{out}}{V_{in}} = -g_m R_{LEQ} = -4.4mS \cdot 1.26k\Omega \approx -5.54$$

Part f, 10 points

Find the *** amplifier *** input resistance, Vin/Vgen, and Vout/Vgen

Rin,amplifier = $1\ M\Omega$

Vin/Vgen= $0.\overline{90}$ V/V

(Vout/Vgen) = -5.04 V/V

$$R_{in,\,amplifier} = \frac{V_{in}}{I_{in}} = R_g$$ No current into gate

$$= 1\ M\Omega$$

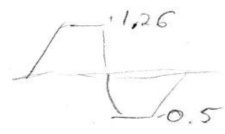
$$\frac{V_{in}}{V_{gen}} = \frac{\frac{\cancel{V_{gen}}\ R_g}{R_g + R_{gen}}}{\cancel{V_{gen}}} = \frac{R_g}{R_g + R_{gen}} = \frac{1\ M\Omega}{1\ M\Omega + 100\ k\Omega}$$

$$\frac{V_{in}}{V_{gen}} = 0.\overline{90}$$

$$\frac{V_{out}}{V_{gen}} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_{gen}} = -5.54 \times 0.\overline{90} \approx -5.04$$

Part g, 10 points

Now you must find the maximum signal swings. Find the output voltage due to the knee voltage and due to cutoff in Q1.



Cutoff of Q1; Maximum ΔVout resulting = +1.26 V

Knee voltage of Q1; Maximum ΔVout resulting = - 0.5V

$\Delta V_{out} \uparrow_{max} = R_{LEQ} \cdot \Delta I_D \downarrow_{max} = 1.26k \times 1mA = 1.26V$

$\Delta I_D \downarrow_{max}$ limited by the 1mA DC bias current

$V_{dg} = -V_t$ at the knee

$V_{DS_{min}} = V_{gs} + V_{dg} = 1 - 0.5 = 0.5V$

$\Delta V_{out} \downarrow_{max} = V_{DS,bias} - V_{DS,min} = 1V - 0.5V = 0.5V$

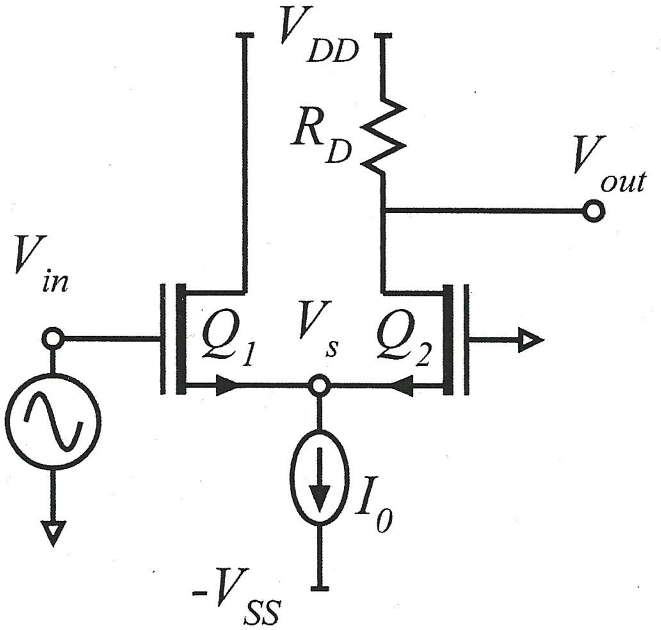
Problem 2, 35 points
Small signal analysis, Nodal analysis

Part a, 15 points

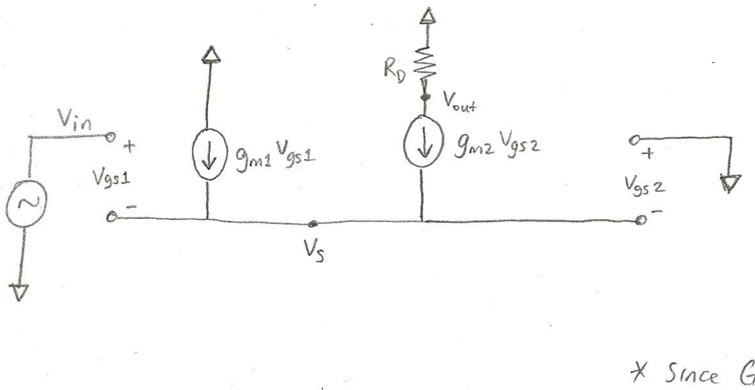
In the circuit at right, the FETs both have $g_{m1} = g_{m2} = g_m$ and $G_{ds1} = G_{ds2} = 0$.

V_{DD} and $-V_{SS}$ are DC bias voltages, while I_0 is a DC bias current source.

Replacing the two FETs with their small-signal models, draw below a small-signal diagram of the circuit. Be sure to label the nodes V_{in}, V_{out}, and V_s in the diagram.



- Replace FETs with small signal models. (5 pts)
- Replace DC voltage sources with short circuits. (3 pts)
- Replace DC current source with open circuit. (3 pts)
- Correct connections and labels. (4 pts)



* Since $G_{ds1} = G_{ds2} = 0$,
$R_{ds1} = R_{ds2} = \infty$.
we can ignore.

Part b, 15 points

Using nodal analysis, determine an algebaic expression for V_{out}/V_{in}

Nodal equation at V_S: $g_m(V_{in}-V_S) - g_m V_S = 0$

Nodal equation at V_{out}: $\frac{V_{out}}{R_D} - g_m V_S = 0$

V_{out}/V_{in} $\frac{1}{2} g_m R_D$

At V_S: $g_{m1} V_{gs1} + g_{m2} V_{gs2} = 0$

$g_m(V_{in}-V_S) + g_m(0-V_S) = 0$

$g_m V_{in} = 2 g_m V_S$

$V_{in} = 2V_S$

Σ currents at node $= 0$

At V_{out}: $(V_{out}-0)\frac{1}{R_D} + g_{m2}V_{gs2} = 0$

$\frac{V_{out}}{R_D} + g_m(0-V_S) = 0$

$V_{out} = g_m R_D V_S$

$$\frac{V_{out}}{V_{in}} = \frac{g_m R_D V_S}{2V_S} = \underline{\frac{1}{2} g_m R_D}$$

- Correct nodal equations for answer from part (a). (8 pts)
- Solving for $\frac{V_{out}}{V_{in}}$. (7 pts)

Part C, 5 points

$R_D = 2$ kOhm, $g_m = 10$ mS.

Find the numerical value of V_{out}/V_{in}:

V_{out}/V_{in} ____10____

- Plug in numbers into answer from part (b) correctly. (5 pts)

$$\frac{V_{out}}{V_{in}} = \frac{1}{2} g_m R_D = \frac{1}{2}(2\ k\Omega)(10\ mS) = \underline{\underline{10}}$$