ECE 2C Mid-Term Exam B Solutions

May 8, 2012

Do not open exam until instructed to.

Closed book: Crib sheet and 1 page personal notes permitted

There are 2 problems on this exam, and you have 75 minutes.

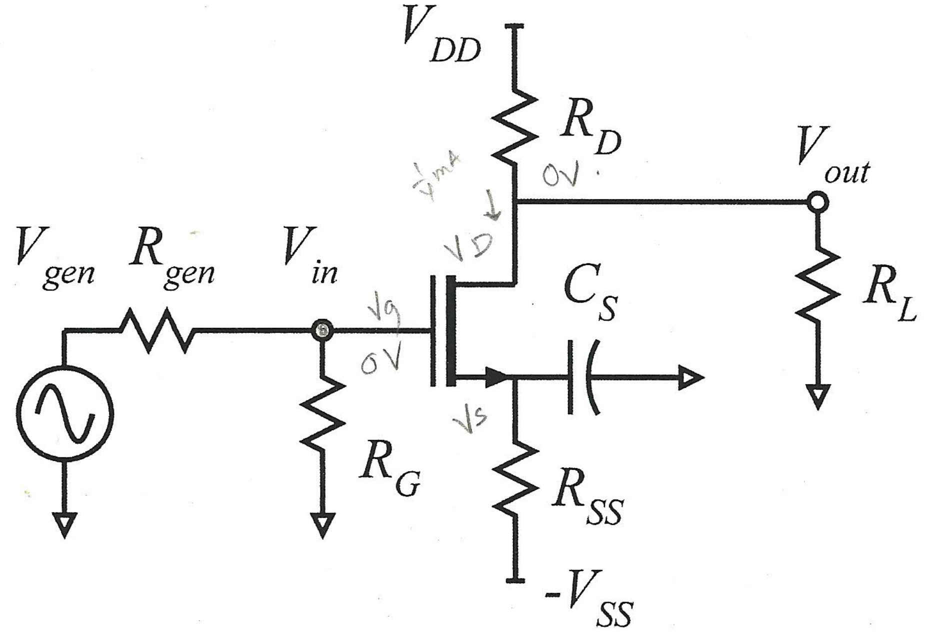
Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING and approximately Justifying them.***

Name: ______________________

Problem	Points Received	Points Possible
1a		10
1b		4
1c		10
1d		15
1e		5
1f		10
1g		10
2a		15
2b		15
2c		5
total		100

Problem 1, 65 points
Transistor circuit analysis

You will be working on the circuit below:



Q1 is a mobility-limited FET, ie. $I_d = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$ where $(\mu C_{ox} W_g / 2L_g)$=4 mA/V^2, $\lambda = 0.1\,\mathrm{V}^{-1}$, and V_{th}=0.5 V.

V_{DD} = +4.0 Volts, $-V_{SS}$ = −4 Volts
C_S is very big and has negligible AC impedance.
R_L=10 kOhm, R_{gen}=100 kOhm, R_G = 900 kOhm

Part a, 10 points

DC bias.

Q1 is to be biased with 1/4 mA drain current.
The drain of Q1 is to be biased at 0.0 Volts
Ignore λ while solving this part*

Find: R_{SS}= 13 kΩ R_D= 16 kΩ.

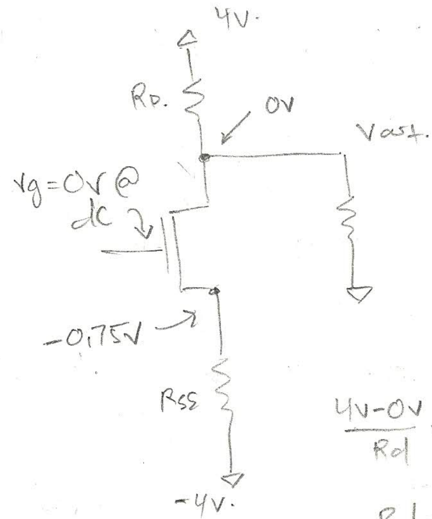
$$I_d = \underbrace{\frac{\mu C_{ox} W_g}{2 L_g}}_{4\,mA/V^2} (V_{gs} - V_{th})^2 (1 + \cancel{\lambda V_{ds}})^{1} = \frac{1}{4}\,mA.$$

$$\frac{1}{4}\,mA = \frac{4\,mA}{V^2} (V_{gs} - 0.5)^2$$

$$V_{gs} = 0.75\,V.$$

$$R_{SS}\left(\frac{1}{4}\,mA\right) = 3.25\,V.$$

$$\underline{R_{SS} = 13\,k\Omega}$$

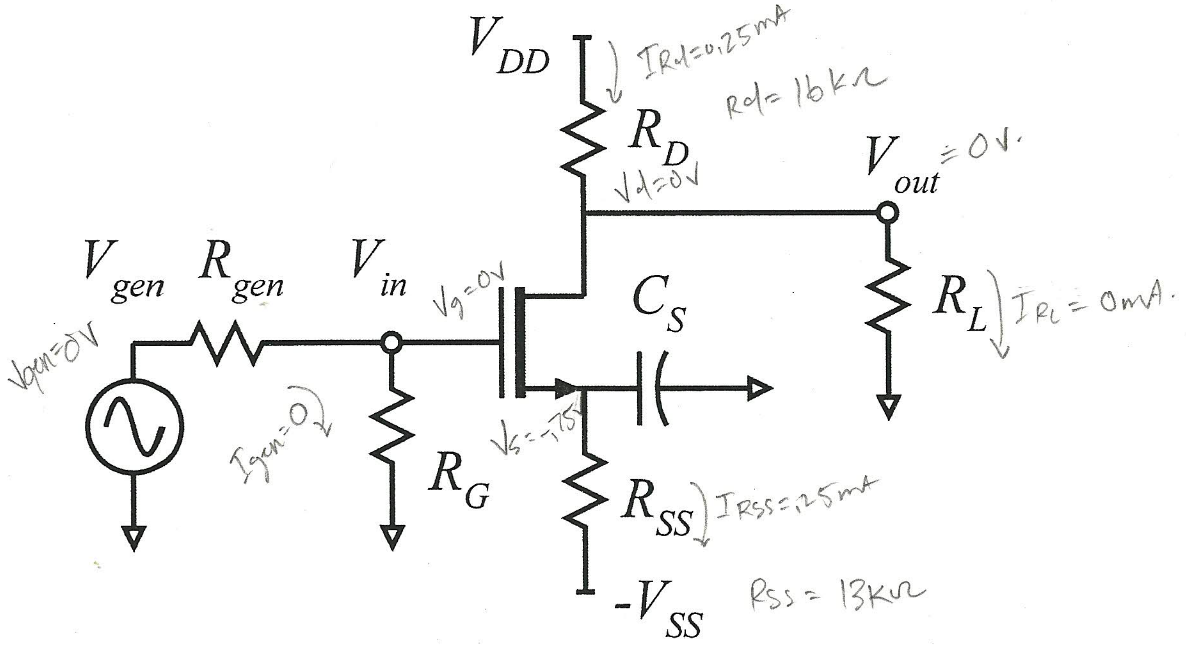


$$\frac{4V - 0V}{R_d} = \frac{1}{4}\,mA.$$

$$\underline{R_d = 16\,k\Omega}$$

Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes** and the DC currents through **ALL resistors**

Part c, 10 points

Find the small signal parameters of Q1. Use the mobility-limited model.

$g_m =$ 2.15 mS

$R_{DS} =$ 43 kΩ

$$I_d = \left(\frac{\mu \cdot C_{ox} \cdot W_g}{2 L_g}\right)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds}).$$

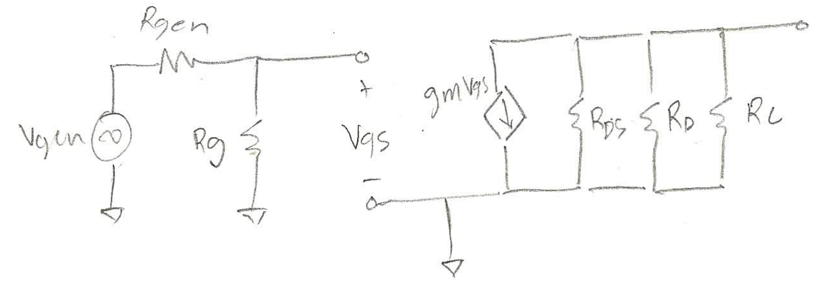
$$g_m = \frac{\partial I_d}{\partial V_{gs}} = 2\left(\frac{\mu C_{ox} W_g}{2 L_g}\right)(V_{gs} - V_{th})(1 + \lambda V_{ds})$$

$$= 2(4\,mA/V^2)(0.25\,V)(1 + (0.1\,V^{-1})(.75\,V)) = 2.15\,mS$$

$$R_{ds} = \frac{\partial V_{ds}}{\partial I_d} = \frac{V_{ds} + \frac{1}{\lambda}}{I_d} = \frac{.75\,V + \frac{1}{.1\,V^{-1}}}{.00025\,A} = \underline{43\,k\Omega}.$$

Part d, 15 points

Replacing the transistor with its small-signal model, draw a small-signal equivalent circuit diagram for the amplifier. Give values for all elements on the diagram.



$R_{gen} = 100k\Omega$

$R_g = 900k\Omega$

$R_L = 10k\Omega$

$g_m = .00215\,S$

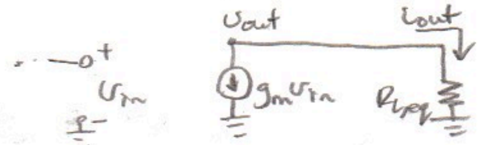
$R_{ds} = 43k\Omega$

$R_d = 16k.$

B-EXAM

problem 1

e. From the small signal model we have:



$$v_{out} = -i_{out} R_{L,eq} = -g_m v_{in} (R_D \| R_L \| R_{DS})$$

$$= (-2.105\ mA/V)(43k\Omega \| 16k\Omega \| 10k\Omega)\, v_{in}$$

$$= -11.33\, v_{in} \Rightarrow \boxed{\frac{v_{out}}{v_{in}} = -11.33\ V/V}$$

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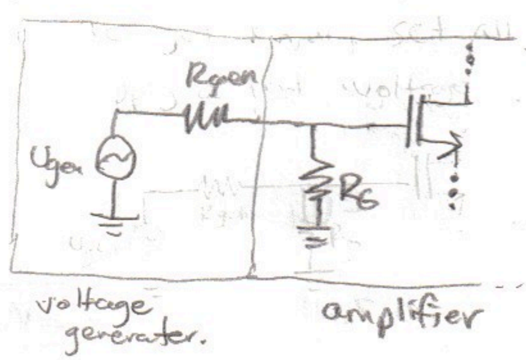
f.



To get R_{in}, pretend the input of the "amplifier" is disconnected from the "voltage generator" circuit, and calculate the resistance seen by the gate.

$$R_{in} = R_G \| \cancel{R_{gate}} \quad \underset{\infty \text{ for MOSFETs}}{}$$

$$= R_G$$

$$\therefore \boxed{R_{in,amp} = 900k\Omega}$$

To find $\frac{v_{in}}{v_{gen}}$ note that R_{gen}, R_G form a voltage divider:

$$\Rightarrow v_{in} = \frac{R_G}{R_G + R_{gen}} v_{gen} \Rightarrow \boxed{\frac{v_{in}}{v_{gen}} = \frac{900k\Omega}{1M\Omega} = 0.9\ V/V}$$

And finally

$$\frac{v_{out}}{v_{gen}} = \frac{v_{out}}{v_{in}} \cdot \frac{v_{in}}{v_{gen}} = (-11.33)(0.9) = \boxed{-10.2\ V/V = \frac{v_{out}}{v_{gen}}}$$

B - EXAM

problem 1

g. cutoff $\Rightarrow V_{gs} - V_{th} = 0 \Rightarrow I_d = 0$

As per the lecture notes (BJT common emitter / MOSFET common source);

$$\Delta V_{out} = -R_{L,eq}\,\Delta I_d = -R_{L,eq}(0 - I_d)$$

$$= (R_L \| R_O \| R_{OS})\, I_0 = (0.25mA)(5.383k\Omega)$$

$$\boxed{\Delta V_{out}\uparrow = 1.346\ V}$$

knee voltage $\Rightarrow V_{ds} = V_{gs} - V_{th}$

As per the same lecture notes:

$$\Delta V_{out} = V_d = V_{ds} - V_s = V_{gs} - V_{th} - V_s$$

$$= 0.75V - 0.5V - 0.75V = \boxed{-0.5V = \Delta V_{out}}$$

$$\boxed{\Delta V_{out}\downarrow = 0.5V}$$

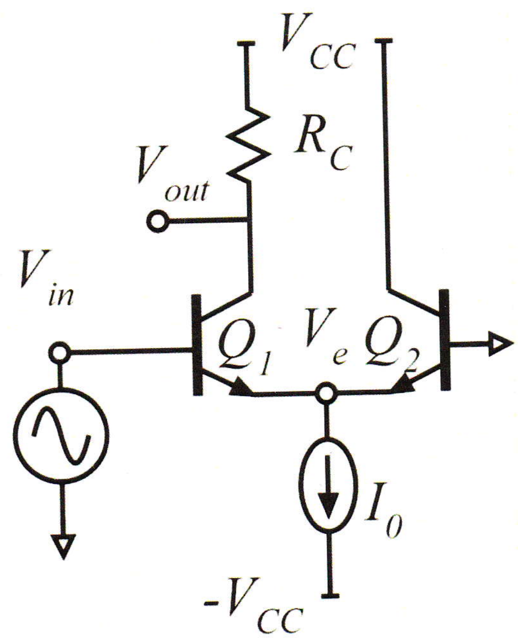
Problem 2, 35 points
Small signal analysis, Nodal analysis

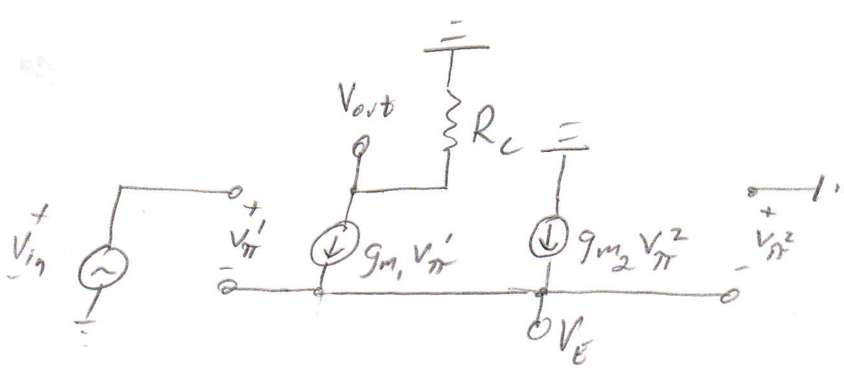
Part a, 15 points

In the circuit at right, the transitors both have $g_{m1} = g_{m2} = g_m$ and $R_{CE1} = R_{CE2} = \infty$. Further, $\beta = \infty$, so $R_{BE1} = R_{BE2} = \infty$

V_{CC} and $-V_{EE}$ are DC bias voltages, while I_0 is a DC bias current source.

Replacing the two transistors with their small-signal models, draw below a small-signal diagram of the circuit. Be sure to label the nodes V_{in}, V_{out}, and V_e in the diagram.





Part b, 15 points

Using nodal analysis, determine an algebaic expression for V_{out}/V_{in}

Nodal equation at V_e: $V_E = g_{m_1} V_{in} / (g_{m_1} + g_{m_2})$

Nodal equation at V_{out}: $V_{out}/R_C + g_{m_1}(V_{in} - V_E) = 0$

V_{out}/V_{in} $V_{out}/V_{in} = -R_c\left(\frac{g_{m_1} g_{m_2}}{g_{m_1}+g_{m_2}}\right) = -R_c\left(\frac{g_m^2}{2g_m}\right)$ $= -\frac{R_c g_m}{2}$

$-g_{m_2} V_{\pi_2} - g_{m_1} V_{\pi_1} = 0$; $V_{\pi_2} = -V_E$; $V_{\pi_1} = V_{in} - V_E$

$+g_{m_2} V_E + g_{m_1}(V_E - V_{in}) = 0$

$\boxed{V_E(g_{m_1} + g_{m_2}) = g_{m_1} V_{in}}$ (5 pts)

$V_{out}/R_C + g_{m_1} V_{\pi_1} = 0$

$\boxed{V_{out}/R_C + g_{m_1}(V_{in} - V_E) = 0}$ (5 pts)

$= V_{out}/R_c + g_{m_1} V_{in} - g_{m_1}\left[g_{m_1} V_{in} / (g_{m_1} + g_{m_2})\right] = 0$

$\boxed{V_{out}/V_{in} = R_C\left[g_{m_1}^2 / (g_{m_1} + g_{m_2}) - g_{m_1}\right] = -R_c\left(\frac{g_{m_1} g_{m_2}}{g_{m_1}+g_{m_2}}\right)}$ (5 pts)

Part C, 5 points

$R_C = 1$ kOhm, $g_m = 100$ mS.

Find the numerical value of V_{out}/V_{in}:

V_{out}/V_{in} ≈ -50

b