ECE PS 1 SOLUTIONS

1a) Given:

$V_T = 0.25\ V$

$v_{sat} C_{ox} W_g = 1.0\ mA/V$

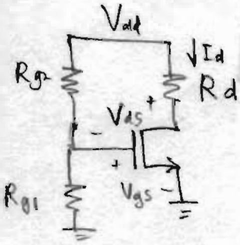
$\lambda = 0\ V^{-1}$

$R_{g1} = 1M\Omega$

$V_{dd} = 1.0V$

$I_d = 0.1mA$

$V_{ds} = 0.6V$



$$I_d = v_{sat} C_{ox} W_g (V_{gs} - V_T)(1 + \lambda V_{ds})$$

$$\Rightarrow V_{gs} = V_T + \frac{I_d}{v_{sat} C_{ox} W_g (1 + \lambda V_{ds})}$$

$$= 0.25\ V + \frac{10^{-4} A}{(10^{-3} A/V)(1)}$$

$$= 0.35V$$

$$V_s = 0V \Rightarrow V_g = V_{gs} = 0.35V$$

$$I_{Rg1} = \frac{V_g}{R_{g1}} = \frac{0.35V}{1M\Omega} = 0.35\mu A$$

$$I_{Rg2} = \frac{V_{dd} - V_g}{R_{g2}} = I_{Rg1}$$

$$\Rightarrow R_{g2} = \frac{V_{dd} - V_g}{I_{Rg1}} = \frac{1.0 - 0.35}{0.35 \times 10^{-6}} = \boxed{1.86M\Omega = R_{g2}}$$

$$R_d = \frac{V_{dd} - V_d}{I_d} = \frac{1 - 0.6}{1mA} = \boxed{4k\Omega = R_d}$$

1b) $v_{sat} C_{ox} W_g$ increased by 10%:

$v_{sat} C_{ox} W_g = 1.1\ mA/V$

V_{gs} remains unchanged

$$I_d = v_{sat} C_{ox} W_g (V_{gs} - V_T)(1 + \lambda V_{ds})$$

$$= (0.0011\ A/V)(0.35\ V - 0.25\ V)(1)$$

$$\boxed{I_d = 0.11\ mA}$$

$$V_{dd} - V_d = I_d R_d = (0.00011\ A)(4000\ \Omega) = \boxed{0.56\ V = V_{ds}}$$

power supply increased by 10%:

$V_{dd} = 1.1\ V$

$$V_{gs} = \frac{R_{g1}}{R_{g1} + R_{g2}} V_{dd} = \frac{1M\Omega}{1M\Omega + 1.86M\Omega} \times 1.1V = 0.385\ V$$

$$I_d = v_{sat} C_{ox} W_g (V_{gs} - V_T)(1 + \lambda V_{ds})$$

$$= (10^{-3}\ A/V)(0.385 - 0.25)(1) = \boxed{0.135\,mA = I_d}$$

$$V_{dd} - V_d = I_d R_d = (0.135)(4) = \boxed{0.56\ V = V_{ds}}$$

1c) Given:

$V_{dd} = 2V$

$V_T = 0.25V$

$\mu C_{ox} W_g / 2L_g = 1 mA/V^2$

$\lambda = 0 \ V^{-1}$

$I_{Rg1} = I_{Rg2} = 10 \mu A$

$I_d = 1mA$

$V_{ds} = 0.7V$

$I_d = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_T)^2 (1 + \lambda V_{ds})$

$\Rightarrow V_{gs} = V_T + \left(\frac{I_d}{(\mu C_{ox} W_g / 2L_g)(1 + \lambda V_{ds})} \right)^{1/2}$

$= 0.25 + \left(\frac{10^{-3}}{(10^{-3})(1)} \right)^{1/2} = 1.25V \Rightarrow V_{gs} - V_t > V_{ds} \Rightarrow$ linear mode.

use following eqn. to solve for V_{gs}:

$I_d = \left(\frac{\mu C_{ox} W_g}{L_g} \right) \left[(V_{gs} - V_T) V_{ds} - \frac{V_{ds}^2}{2} \right]$

$1mA = 2(1mA/V^2)\left[(V_{gs} - 0.25)\, 0.7 - 0.7^2/2 \right] \Rightarrow V_{gs} = 1.31V$

$R_{g1} = \frac{V_{gs}}{I_{Rg1}} = \frac{1.31 V}{10 \mu A} = \boxed{130 k\Omega = R_{g1}}$

$R_d = \frac{V_{dd} - V_{ds}}{I_d} = \frac{2 - 0.7}{1mA} = \boxed{1.3k\Omega = R_d}$

$R_{g2} = \frac{V_{dd} - V_{gs}}{I_{Rg2}} = \frac{2 - 1.31 V}{10 \mu A} = \boxed{70 k\Omega = R_{g2}}$

1d) Given:

$V_T = -0.25V$

$v_{sat} C_{ox} W_g = 0.3 \ mA/V$

$V_{ss} = 1V$

$I_{Rg1} = I_{Rg2} = 2 \mu A$

$I_d = 100 \mu A$

$V_s = 0.9V$

$V_d = 0.1V$

$I_d = v_{sat} C_{ox} W_g (-V_{gs} + V_T)(1 + \lambda V_{ds})$

$\Rightarrow V_{gs} = V_T - \frac{I_d}{(v_{sat} C_{ox} W_g)(1 - \lambda V_{ds})}$

$= -0.25 - \frac{100 \times 10^{-6}}{3 \times 10^{-4}} = -0.58V$

$V_g = V_{gs} + V_s = -0.58 + 0.9 = 0.32 \ V$

$R_{g1} = \frac{V_g}{I_{Rg1}} = \frac{0.32}{2 \times 10^{-6}} = \boxed{160 \ k\Omega = R_{g1}}$

$R_{g2} = \frac{V_{ss} - V_g}{I_{Rg2}} = \frac{1 - 0.32}{2 \times 10^{-6}} = \boxed{340 k\Omega = R_{g2}}$

$R_s = \frac{V_{ss} - V_s}{I_d} = \frac{1 - 0.9}{10^{-4}} = \boxed{1k\Omega = R_s}$

$R_d = \frac{V_d}{I_d} = \frac{0.1}{10^{-4}} = \boxed{1k\Omega = R_d}$

2a) Given:

$\beta = 100$

$V_{cc} = 3.3V$

$V_c = 2.5V$

$V_E = 1V$

$I_{Rb2} = 100 \mu A$

$I_c = 1mA$

$$R_c = \frac{V_{cc}-V_c}{I_c} = \frac{3.3-2.5}{10^{-3}} = \boxed{800\Omega = R_c}$$

$$I_B = \frac{I_c}{\beta} = \frac{10^{-3}A}{100} = 0.01mA$$

$$I_E = (\beta+1) I_B = 1.01mA$$

$$R_{EE} = \frac{V_E}{I_E} = \frac{1}{1.01\times10^{-3}} = \boxed{990\Omega = R_{EE}}$$

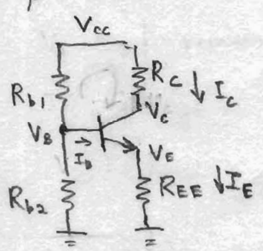
$$V_B = V_{BE} + V_E = 0.7 + 1 = 1.7V$$

$$R_{b2} = \frac{V_B}{I_{Rb2}} = \frac{1.7}{10^{-4}} = \boxed{17k\Omega = R_{b2}}$$

$$I_{Rb1} = I_B + I_{Rb2} = 100\mu A + 0.01mA = 110\mu A$$

$$R_{b1} = \frac{V_{cc}-V_B}{I_{Rb1}} = \frac{3.3-1.7}{110\times10^{-6}} = \boxed{14.5k\Omega = R_{b1}}$$

2b) $\beta = 200$:



Nodal analysis at V_B:

$$\frac{V_B - V_{cc}}{R_{b1}} + I_B + \frac{V_B}{R_{b2}} = 0 \quad \text{—— ①}$$

Assuming $V_{BE}(on) \approx 0.7V$

$$V_B = V_{BE} + V_E = V_{BE}(on) + (\beta+1) I_B R_{EE} \quad \text{—— ②}$$

Plugging ② into ① gives:

$$\left[V_{BE}(on) + (\beta+1) I_B R_{EE}\right]\left(\frac{1}{R_{b1}} + \frac{1}{R_{b2}}\right) + I_B - \frac{V_{cc}}{R_{b1}} = 0$$

Solving for I_B gives $I_B = 5.23\mu A$

$$V_c = V_{cc} - \beta I_B R_c = 3.3 - (200)(5.23\times10^{-6})(800) = \boxed{2.46V = V_c}$$

$$V_E = (\beta+1) I_B R_{EE} = (201)(5.23\times10^{-6})(990) = 1.04\ V$$

$$V_B = V_{BE}(on) + (\beta+1) I_B R_{EE} = 0.7 + (201)(5.23\times10^{-6})(990) = 1.74V$$

$V_{BE} > 0$

$V_{BC} < 0$ $\Rightarrow$ forward active mode

$$I_c = \beta I_B = (200)(5.23\times10^{-6}) = \boxed{1.046mA = I_c}$$

2b) V_{cc} increased by 10%:

$V_{cc} = 3.63$ V.

Using similar analysis as in previous part, we get

$$\left[V_{BE}(on) + (\beta+1) I_B R_{EE}\right]\left(\frac{1}{R_{b1}} + \frac{1}{R_{b2}}\right) + I_B - \frac{V_{cc}}{R_{b2}} = 0$$

$\Rightarrow I_B = 11.68\ \mu A$

$V_c = V_{cc} - \beta I_B R_c = 3.63 - (100)(11.68\times10^{-6})(800) = \boxed{2.42\text{ V} = V_c}$

$V_E = (\beta+1) I_B R_{EE} = (101)(11.68\times10^{-6})(990) = 1.17$ V

$V_B = V_{BE}(on) + (\beta+1) I_B R_{EE} = 0.7 + (101)(11.68\times10^{-6})(990) = 1.89$ V

$V_{BE} > 0$

$V_{BC} < 0$ $\Rightarrow$ fwd active mode

$I_c = \beta I_B = \boxed{1.168\text{ mA} = I_c}$

2c) Given:

$\beta = 300$

$V_{EE} = 5$ V

$V_c = 2$ V

$V_E = 3$ V

$I_c = 0.1$ mA

$I_{Rb2} = 10\ \mu A$

$I_B = \frac{I_c}{\beta} = \frac{10^{-4}\text{ A}}{300} = 0.33\ \mu A$

$R_{EE} = \frac{V_{EE} - V_E}{(\beta+1) I_B} = \frac{5-3}{(301)(.33\times10^{-6})} = \boxed{20\text{ k}\Omega = R_{EE}}$

$R_c = \frac{V_c}{I_c} = \frac{2}{10^{-4}} = \boxed{20\text{ k}\Omega = R_c}$

$V_B = V_E - V_{EB} = 3 - 0.7 = 2.3$ V

$R_{b2} = \frac{V_{EE} - V_B}{I_{Rb2}} = \frac{5-2.3}{10^{-5}} = \boxed{270\text{ k}\Omega = R_{b2}}$

$I_{Rb1} = I_{Rb2} - I_B = 10\ \mu A - 0.33\ \mu A = 9.67\ \mu A$

$R_{b1} = \frac{V_B}{I_{Rb1}} = \frac{2.3}{9.67\times10^{-6}} = \boxed{240\text{ k}\Omega = R_{b1}}$