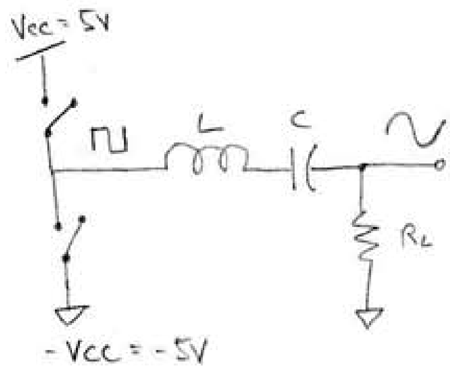
ECE Problem Set 2: Solutions

Problem 1



→ switches produce 1 GHz square wave
→ LC filter passes only 1GHz fundamental freq.

a) find AC voltage at the load: square wave can be deconstructed into a fourier series with fundamental frequency: $n=1$

$$f(t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4A}{n\pi} \sin(2\pi f t) \quad \left(\text{table 13.4 T\&R}\right)$$

Using $A = 5$ volts, $f = 1$ GHz, $n = 1$

$$\boxed{V_{out}(t) = \frac{20}{\pi} \sin(2\pi \times 10^9 \text{Hz}\, t) \text{ Volts}}$$

b) Find the power delivered to the load if $R_L = 50\Omega$.

$$P_{rms} = \frac{(V_{out\,rms})^2}{R} = \frac{\left(\frac{20}{\sqrt{2}\pi}\right)^2}{50} = \boxed{\frac{4}{\pi^2} \text{ watts}}$$

c) Filter is now changed to pass only 3GHz. This is the third Harmonic only, $n=3$

new AC voltage at the load = $\boxed{V_{out}(t) = \frac{20}{3\pi} \sin(6\pi \times 10^9 \text{Hz}\, t) \text{ volts}}$

new Pwr delivered to the load = $P_{rms} = \frac{\left(\frac{20}{3\sqrt{2}\pi}\right)^2}{50} = \boxed{\frac{4}{9\pi^2} \text{ watts}}$

```
%Problem 2

clear
clc
close all

%Variables
f=100;                  %Frequency
ts=1e-6;                %Sampling Rate
t=0:ts:2/f;             %Defining a time scale
A=1;                    %Amplitude
V=0;                    %Initialize V

%Equations
V1=A*cos(2*pi*f*t);
V3=V1+A/3*cos(3*2*pi*f*t);
V5=V3+A/5*cos(5*2*pi*f*t);
V7=V5+A/7*cos(7*2*pi*f*t);
V9=V7+A/9*cos(9*2*pi*f*t);

%Calculate Vn as n --> infinity
for q=1:2:1000
    V=V+(A/q)*cos(q*2*pi*f*t);
end

%Ploting functions
figure ('Name','Voltage VS Time for n=1,3,5,7,9')
subplot(5,1,1), plot(t,V1), ylabel('V1 (V)','FontSize',12)
subplot(5,1,2), plot(t,V3), ylabel('V2 (V)','FontSize',12)
subplot(5,1,3), plot(t,V5), ylabel('V3 (V)','FontSize',12)
subplot(5,1,4), plot(t,V7), ylabel('V4 (V)','FontSize',12)
subplot(5,1,5), plot(t,V9), ylabel('V5 (V)','FontSize',12)

figure ('Name','Voltage VS Time as n Approaches Infinity')
plot(t,V), title('Voltage VS Time as n Approaches Infinity','FontSize',14),
ylabel('Voltage (V)','FontSize',12), xlabel('Time (s)','FontSize',12)
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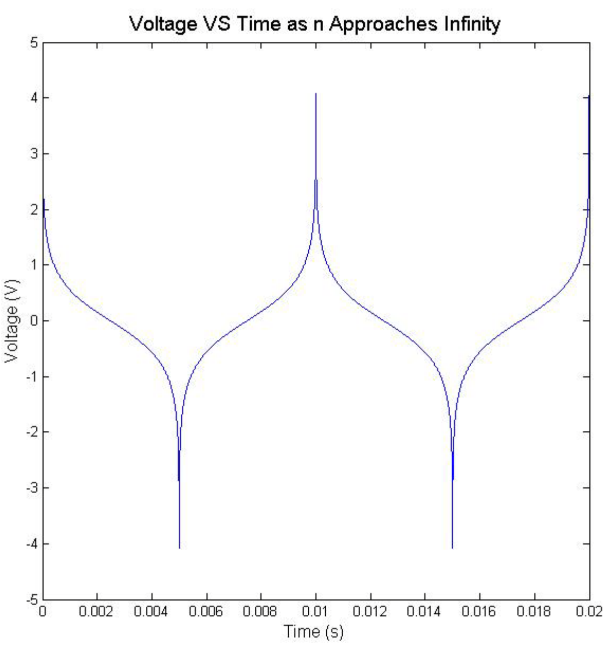

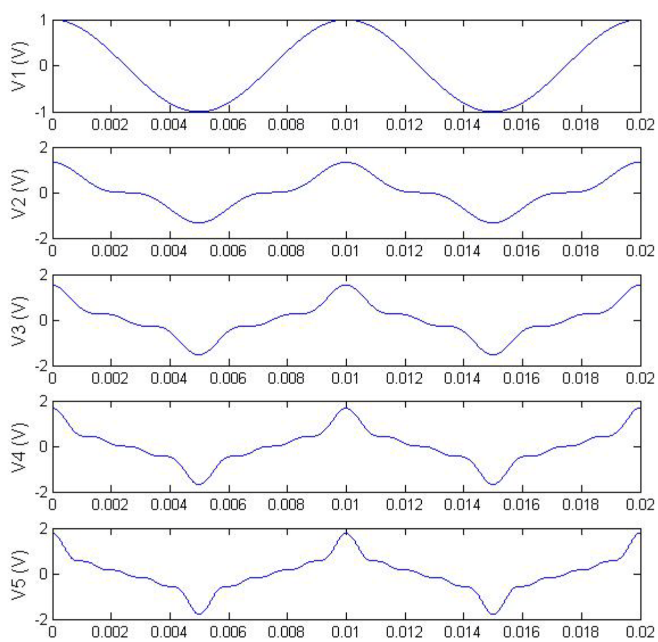

```matlab
%Problem 3

clear
clc
close all

RC=1e-9;             % RC Time Constant
f=1e8;               % Frequency (Hz)
w=2*pi*f;            % Frequency (Rad/s)
ts=1e-11;            % Sample rate
t=0:ts:2/f;          % Define time scale
A=1;                 % Amplitude
Vout_Total=0;        % Initialize V

% Using phasor analysis, the transfer function is found to be:
%
%                    H(s) = 1/(1+RC*j*w*i)
%
% Where i is an odd integer between 1 and 9 representing the
% various frequencies of Vin_i.
%
% The for loop preceding this comment calculates the magnitude and phase
% of each Vout_i and finds the time domain expression for each.
% VinS is proportional to the magnitude and also varies inversly with i.
%
% Finally all Vout_i values are added together into Vout_Total as follows:
%
%       Vout_Total=Vout_1+Vout_3+Vout_5+Vout_7+Vout_9 where
%          Vout_i=Magnitude*Cos(w*i*t+Phase(use radians))
%              Vout_1=0.8467*Cos(w*i*t-32.1419deg)
%              Vout_3=0.1562*Cos(w*i*t-62.0533deg)
%              Vout_5=0.0607*Cos(w*i*t-72.3432deg)
%              Vout_7=0.0317*Cos(w*i*t-77.1908deg)
%              Vout_9=0.0193*Cos(w*i*t-79.9716deg)

for i=1:2:9

    VinS(:,i)=A/i;
    VoutS(:,i)=1/(1+RC*j*w*i)*VinS(:,i);

    MagVout(:,i)=abs(VoutS(:,i));
    PhaseVout(:,i)=phase(VoutS(:,i));

    Vout_i(:,i)=MagVout(:,i)*cos(w*i*t+PhaseVout(:,i));
    Vout_Total=Vout_Total+Vout_i(:,i);

end

figure ('Name','Voltage Out Total VS Time')
plot(t,Vout_Total), ylabel('Vout Total (V)','FontSize',12), xlabel('Time
(s)','FontSize',12)

figure ('Name','Voltage Out VS Time for n=1,3,5,7,9')

subplot(5,1,1), plot(t,Vout_i(:,1)), ylabel('Vo n=1 (V)','FontSize',12)
subplot(5,1,2), plot(t,Vout_i(:,3)), ylabel('Vo n=3 (V)','FontSize',12)
subplot(5,1,3), plot(t,Vout_i(:,5)), ylabel('Vo n=5 (V)','FontSize',12)
subplot(5,1,4), plot(t,Vout_i(:,7)), ylabel('Vo n=7 (V)','FontSize',12)
subplot(5,1,5), plot(t,Vout_i(:,9)), ylabel('Vo n=9 (V)','FontSize',12)
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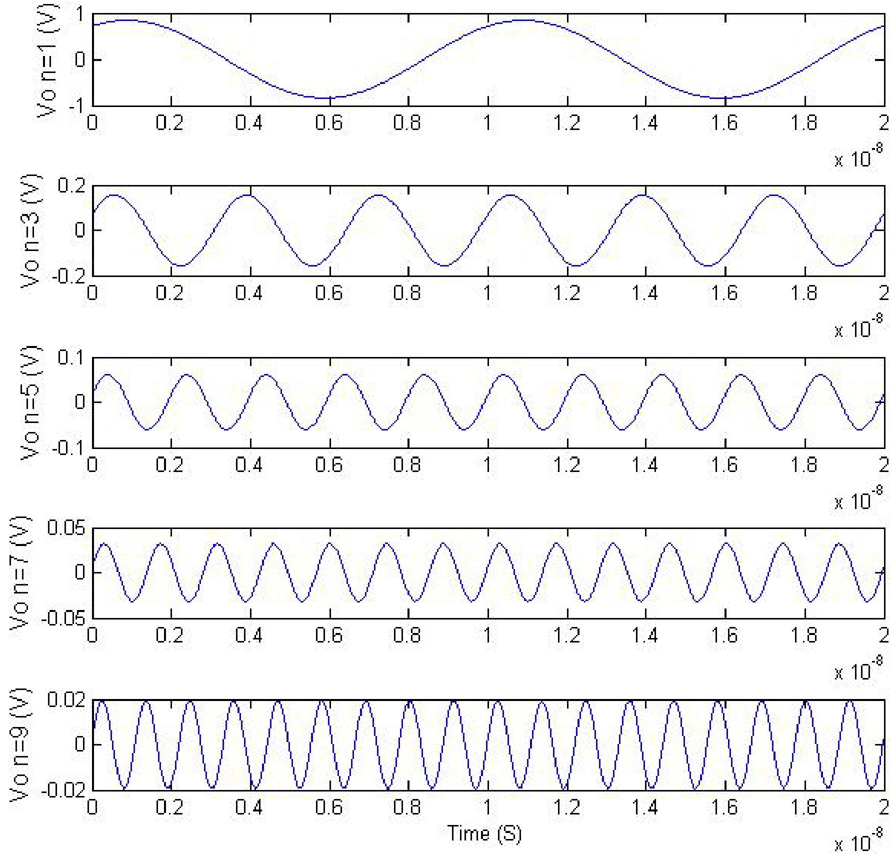
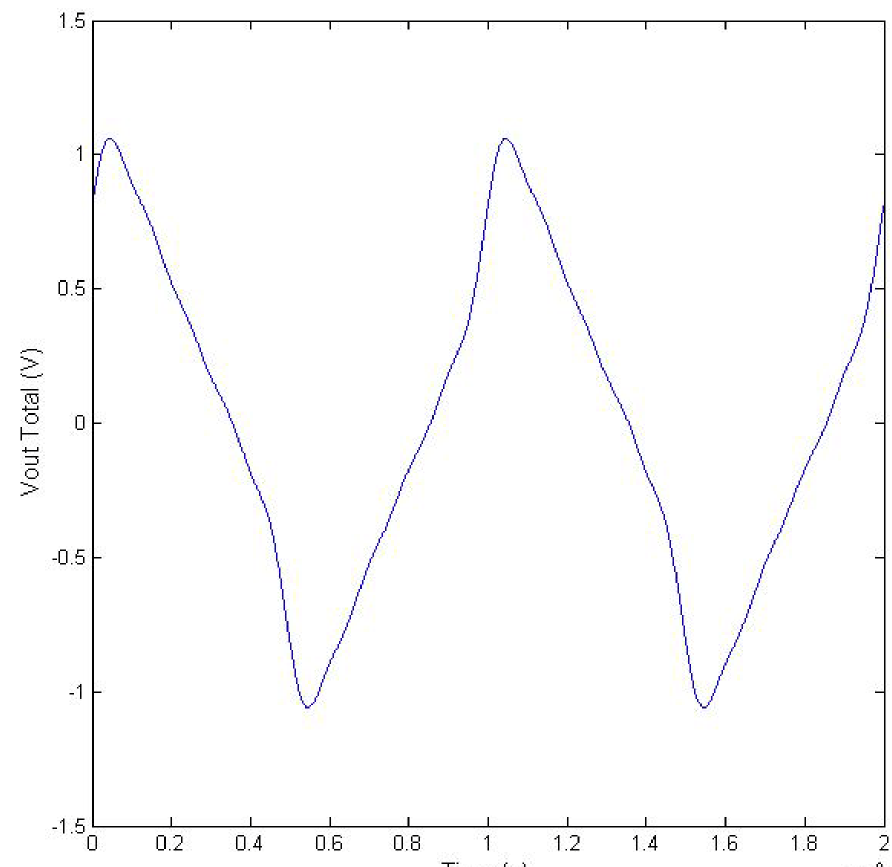


Vo n=1 (V)
Vo n=3 (V)
Vo n=5 (V)
Vo n=7 (V)
Vo n=9 (V)
Time (S)
x 10^-8



Vout Total (V)
Time (s)
x 10^-8

```
%Problem 4

clear
clc
close all

%Variables
f=100;                  %Frequency
ts=1e-6;                %Sampling Rate
t=0:ts:2/f;             %Defining t
A=1;                    %Amplitude
V=0;

%Equations
V1=A*sin(2*pi*f*t);
V2=V1+A/2*sin(2*2*pi*f*t);
V3=V2+A/3*sin(3*2*pi*f*t);
V4=V3+A/4*sin(4*2*pi*f*t);

%Calculate Vn as n --> infinity
for q=1:1:100
    V=V+A/q*sin(q*2*pi*f*t);
end

figure ('Name','Voltage VS Time for n=1,2,3,4')
subplot(4,1,1), plot(t,V1), ylabel('V1 (V)','FontSize',12)
subplot(4,1,2), plot(t,V2), ylabel('V2 (V)','FontSize',12)
subplot(4,1,3), plot(t,V3), ylabel('V3 (V)','FontSize',12)
subplot(4,1,4), plot(t,V4), ylabel('V4 (V)','FontSize',12)

figure ('Name','Voltage VS Time as n Approaches Infinity')
plot(t,V), title('Voltage VS Time as n Approaches Infinity','FontSize',14), ylabel('Voltage
(V)','FontSize',12), xlabel('Time (s)','FontSize',12)
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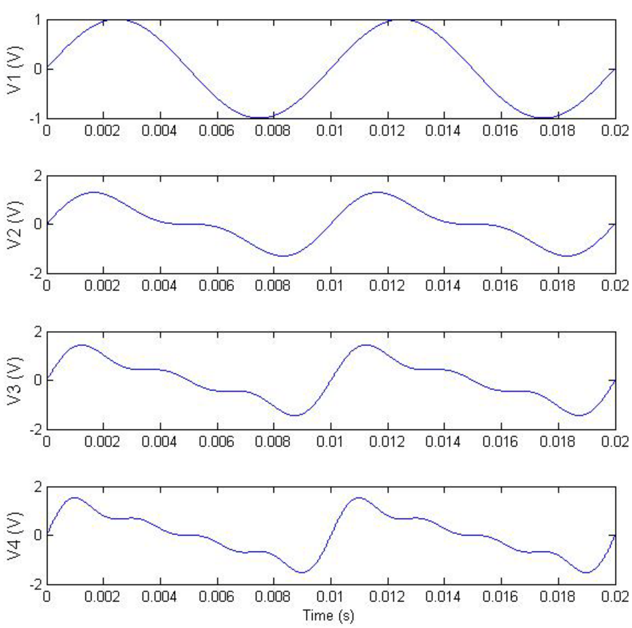
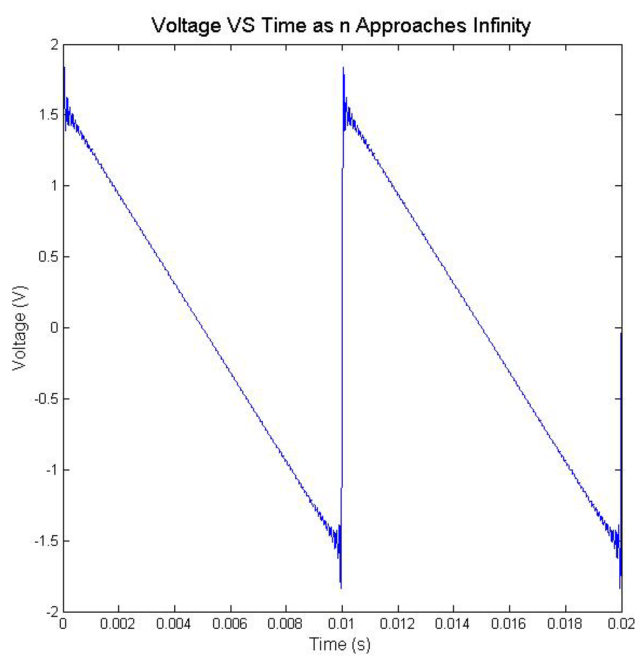
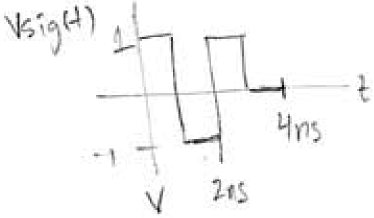
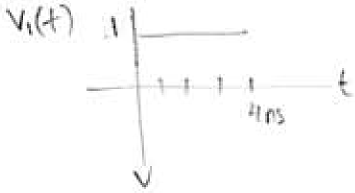
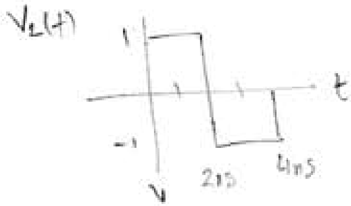


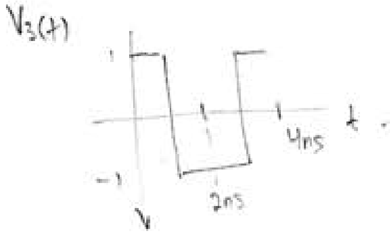


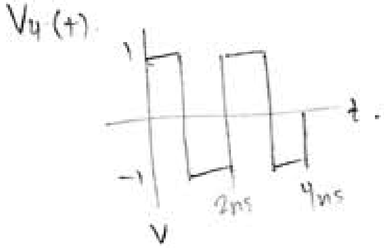
Problem 5











a) $V_{sig}(t) = a_1(V_1(t)) + a_2(V_2(t)) + a_3(V_3(t)) + a_4(V_4(t))$.

Separating Vsig into 1 ns segments.

$$V_{sig} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

we can do the same for $V_1(t) - V_4(t)$.

$$\begin{aligned} \underbrace{a_1}_{V_1} + \underbrace{a_2}_{V_2} + \underbrace{a_3}_{V_3} + \underbrace{a_4}_{V_4} &= \underbrace{1}_{V_{sig}} \\ a_1 + a_2 - a_3 - a_4 &= -1 \\ a_1 - a_2 - a_3 + a_4 &= 1 \\ a_1 - a_2 + a_3 - a_4 &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} .25 \\ -.25 \\ .25 \\ .75 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}}$$

b) check $\bar{V}_3 \cdot \bar{V}_4 = \langle V_3 | V_4 \rangle = \frac{1}{T}\int_0^T V_3(t) V_4(t)\,dt$

$$= \frac{1}{4ns}\left[\int_{0ns}^{1ns} (1V)(1V)\,dt + \int_{1ns}^{2ns} (-1V)(-1V)\,dt + \int_{2ns}^{3ns} (1V)(-1V)\,dt + \int_{3ns}^{4ns} (1V)(-1V)\,dt\right]$$

$$= \frac{1}{4ns}\left(1 + 1 - 1 - 1\right) V^2 ns = \boxed{0 = \langle V_3 | V_4 \rangle}$$