problem 1

given: $\upsilon_{sat} C_{ox} W_g = 1.0 \text{mA/V}$

$V_{th} = 0.25\,\text{V}$

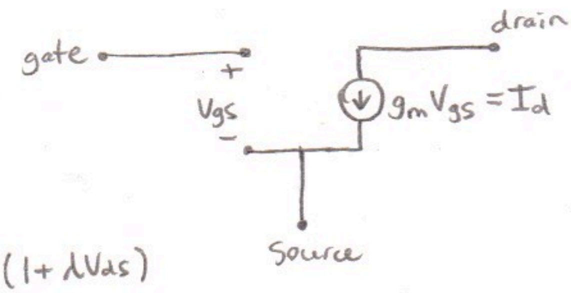
$\lambda = 0.25\,\text{V}^{-1}$

$V_{ds} = 1\text{V}$

$V_{gs} = 0.5\,\text{V}$

$I_d = \upsilon_{sat} C_{ox} W_g (V_{gs} - V_{th})(1 + \lambda V_{ds})$

small signal model:



• calculate transconductance:

$$g_m \triangleq \frac{\partial I_d}{\partial V_{gs}} = \upsilon_{sat} C_{ox} W_g (1 + \lambda V_{ds}) = (1.0\,\text{mA/V})(1.25\,\text{V/V}) = \boxed{1.25\,\text{mA/V}}$$

• calculate output conductance:

$$G_{ds} \triangleq \frac{\partial I_d}{\partial V_{ds}} = \upsilon_{sat} C_{ox} W_g (V_{gs} - V_{th})\lambda = (1.0\,\text{mA/V})(0.25\,\text{V})(0.25\,\text{V}^{-1})$$
$$= \boxed{62.5\,\mu\text{A/V}}$$

problem 2

given: $\mu C_{ox} W_g / 2L_g = 0.8\,\text{mA/V}^2$

$\lambda = 0.25\,\text{V}^{-1}$

$V_{ds} = 1.5\text{V}$

$I_d = 0.25\text{mA}$

$I_d = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$

small signal model:

(same as Problem 1)

• calculate gate overdrive:

$$V_{gs} - V_{th} = \sqrt{\frac{I_d}{(\mu C_{ox} W_g/2L_g)(1+\lambda V_{ds})}} = \sqrt{\frac{0.25\text{mA}}{(0.8\,\text{mA/V}^2)(1.375\,\text{V/V})}} = 0.4767\,\text{V}$$

• calculate transconductance and output conductance:

$$g_m \triangleq \frac{\partial I_d}{\partial V_{gs}} = 2(\mu C_{ox} W_g / 2L_g)(V_{gs} - V_{th})(1 + \lambda V_{ds})$$
$$= 2(0.8\,\text{mA/V})(0.4767\,\text{V})(1.375\,\text{V/V}) \approx \boxed{1.05\,\text{mA/V}}$$

$$G_{ds} \triangleq \frac{\partial I_d}{\partial V_{ds}} = \frac{\lambda I_0}{1 + \lambda V_{DS}} = \frac{(0.25\,\text{V}^{-1})(0.25\text{mA})}{1.375\,\text{V/V}} = \boxed{45.4\,\mu\text{A/V}}$$

problem 3

given: same as problem 2, except $V_{ds} = 1V$

- gate overdrive:

$$V_{gs} - V_{th} = \sqrt{\frac{I_d}{(\mu C_{ox} W_g/2L_g)(1+\lambda V_{ds})}} = \sqrt{\frac{0.25\text{ mA}}{(0.8\text{mA}/V^2)(1.25\,V/V)}} = 0.5\,V$$

- calculate transconductance & output conductance:

$$g_m \triangleq \frac{\partial I_d}{\partial V_{gs}} = 2(\mu C_{ox} W_g/2L_g)(V_{gs}-V_{th})(1+\lambda V_{ds})$$

$$= 2(0.8\text{mA}/V)(0.5\,V)(1.25\,V/V) = \boxed{1\text{mA}/V}$$

$$G_{ds} \triangleq \frac{\partial I_d}{\partial V_{ds}} = \lambda(\mu C_{ox} W_g/2L_g)(V_{gs}-V_{th})^2$$

$$= (0.25\,V^{-1})(0.8\text{mA}/V)(0.25\,V^2) = \boxed{50\,\mu A/V}$$

problem 4

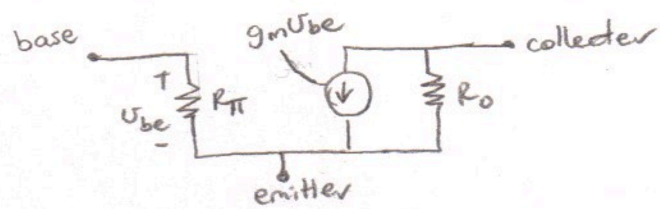
given: $\beta = 100$

$V_A = 50V$

$V_{CE} = 1.5V$

$I_C = 0.2mA$

small signal model:



Assuming room temperature operation, $V_T = \frac{kT}{q} = 25.8\,mV$.

$$g_m = \left.\frac{\partial I_c}{\partial V_{be}}\right|_{V_{ce}\text{ constant}} = \frac{I_c}{V_T} = \frac{0.2mA}{25.8mV} = \boxed{7.75\text{ mA}/V = g_m}$$

$$G_{ce} = \left.\frac{\partial I_c}{\partial V_{ce}}\right|_{V_{be}\text{ const}} = \frac{I_c}{V_{CE}+V_A} = \frac{0.2mA}{51.5V} = \boxed{3.88\,\mu A/V = G_{CE}}$$

To compute the input impedance we start with

$$g_m = \frac{\alpha}{r_e} = \frac{\beta}{(\beta+1)r_e} = \frac{\beta}{R_\pi} \Rightarrow R_\pi = \frac{\beta}{g_m} = \boxed{12.9k\Omega = R_\pi}$$

problem 5

a. given: $V_t = 0.25\,V$ need: $I_d = 0.5\,mA$

$\lambda = 0.1\,V^{-1}$ $V_{gs} = 0.4\,mA$

$L_g = 130\,nm$ $W_g = ?$

$\mu = 300\,cm^2/Vs$

$T_{ox} = 1.2\,nm$

Assuming SiO_2 we have $\varepsilon_r = 3.8$. So,

$$C_{ox} = \frac{\varepsilon_r \varepsilon_0}{T_{ox}} = \frac{3.8(8.854\times10^{-12}\,F\cdot m^{-1})}{1.2\times10^{-9}\,m} \approx 28\,mF/m^2$$

For mobility limited case we need $V_{ds} > V_{knee} = V_{gs} - V_{th}$. Therefore,

$$I_d = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_t)^2(1 + \lambda V_{ds})$$

$$> (\mu C_{ox}/2L_g)(V_{gs} - V_t)^2(1 + \lambda V_{gs} - \lambda V_t)\,W_g$$

Or, ~~under the~~ approximation (see lecture notes) that $1 + \lambda V_{ds} \approx 1$,

$$W_g = \frac{2(0.5\,mA)(130\times10^{-9}\,m)}{(300\,cm^2/Vs)\left(\frac{1m}{100cm}\right)^2(28\,mF/m^2)(0.15\,V)^2} = \boxed{6.88\,\mu m}$$

b. Here, $V_d = V_{dd} - I_d R_d = 2V - (0.5\,mA)R_D = 1V$

$$\Rightarrow R_D = \frac{1V}{0.5\,mA} = \boxed{2k\Omega}$$

Now we determine R_{g_1} and R_{g_2} so that constraints are met:

- $V_{gs} = 0.4V$, $V_{DD} = 2V \Rightarrow \frac{R_{g_2}}{R_{g_1} + R_{g_2}} = \frac{0.4}{2} = \frac{1}{5} \Rightarrow R_{g_1} = 4R_{g_2}$
- $R_{in}^{\star} = R_{g_1} \| R_{g_2} = 1\,M\Omega$

$$\Rightarrow \frac{1}{R_{g_1}} + \frac{4}{R_{g_1}} = \frac{5}{R_{g_1}} = \frac{1}{1\,M\Omega}$$

★ Note:
R_{in} does not include R_{gen}; is just impedance seen by gate of NMOS.

$$\Rightarrow R_{g_1} = 5(1\,M\Omega) = \boxed{5.0\,M\Omega = R_{g_1}}$$

$$R_{g_2} = \frac{1}{4}R_{g_1} = \boxed{1.25\,M\Omega = R_{g_2}}$$

thanks Justin Liang!

c.

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = (\mu C_{ox} W_g / L_g)(V_{gs} - V_{th})(1 + \lambda V_{DS})$$

here, all the quantities are as before except:

$$W_g = 6.77\mu m \quad \text{and} \quad V_{ds} = V_d = 1V$$

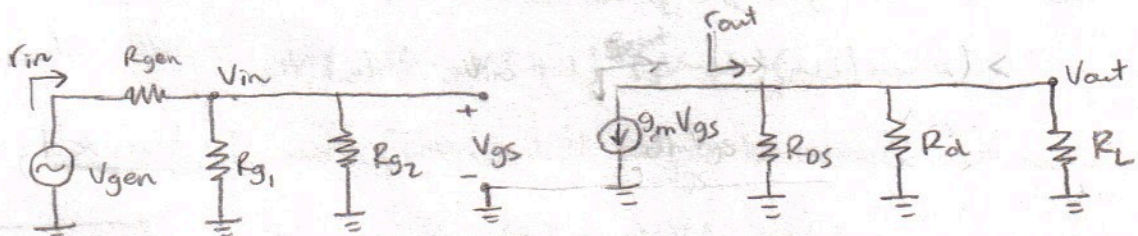
$$\Rightarrow g_m = (0.03\, m^2/Vs)(28\, mF/m^2)(6.77\mu m)(0.15\, V)(1.1\, V/V) / 130\times 10^{-9} m$$

$$= \boxed{7.22\ mA/V}$$

And,

$$G_{DS} \triangleq \frac{\partial I_D}{\partial V_{DS}} = \frac{\lambda I_D}{1 + \lambda V_{DS}} = \frac{(0.1\, V^{-1})(0.5\, mA)}{1.1\, V/V} = \boxed{45.5\ \mu A/V}$$

d.



e. Summary of resistor values:

$R_{gen} = 50k\Omega$ $\qquad R_{DS} = \frac{1}{G_{DS}} = 22k\Omega$

$R_{g_1} = 1.667\, M\Omega$ $\qquad R_D = 2k\Omega$ $\qquad R_{g_1} \| R_{g_2} = 1M\Omega$

$R_{g_2} = 1.111\, M\Omega$ $\qquad R_L = 100\, k\Omega$,

Therefore, with r_{in} and r_{out} as defined in diagram,

$$r_{in} = R_{gen} + R_{g_1} \| R_{g_2} = \boxed{1.05\, M\Omega = r_{in}}$$

$$r_{out} = R_{DS} \| R_d \| R_L = \underline{1.8k\Omega = r_{out}}$$

$$V_{in} = V_{gs} = \frac{R_{g_1} \| R_{g_2}}{R_{gen} + R_{g_1} \| R_{g_2}} V_{gen} = \frac{20}{21} V_{gen} \Rightarrow \boxed{\frac{V_{in}}{V_{gen}} = 0.952}$$

So,

$$\frac{V_{out}}{V_{in}} = \frac{-g_m V_{gs}\, r_{out}}{V_{gs}} = \boxed{-13\, V/V = \frac{V_{out}}{V_{in}}} ,$$

$$\frac{V_{out}}{V_{gen}} = \frac{V_{out}}{V_{in}} \cdot \frac{V_{in}}{V_{gen}} = \boxed{-12.4\, V/V = \frac{V_{out}}{V_{gen}}}$$

problem 6

a. given:

$V_T = \pm 0.25V$ $\quad v_{sat,N} = 10^7$ cm/s $\quad V_{dd} = 1V$

$\lambda = 0.1V^{-1}$ $\quad v_{sat,P} = 5\times10^6$ cm/s $\quad V_{in} = 0.5$

$L_g = 0.45nm$ $\quad W_{g,N} = 5\mu m$

$T_{ox} = 0.8nm$ $\quad W_{g,P} = 10\mu m$

- Determine Cox, assuming $SiO_2 \rightarrow \varepsilon_r = 3.8$

$$C_{ox} = \frac{\varepsilon_r \varepsilon_0}{T_{ox}} = \frac{(3.8)(8.854\times10^{-12}\ F\cdot m^{-1})}{0.8\times10^{-9}\ m} = 42.1\ mF/m^2$$

Now, compare drain currents for NMOS/PMOS

$$I_{D,N} = I_{D,P} \Leftrightarrow C_{ox} W_{g,N} v_{sat,N}(1+\lambda V_{out})(V_{in}-V_{th,N})$$

$$= C_{ox} W_{g,P} v_{sat,P}(1+\lambda(V_{dd}-V_{out}))(V_{DD}-V_{in}+V_{th,P})$$

But in this case (plug in values to verify)

$$W_{g,N} v_{sat,N} = W_{g,P} v_{sat,P} \quad \text{and} \quad V_{in}-V_{th,N} = V_{DD}-V_{in}+V_{th,P}$$

$$\Rightarrow 1+\lambda(V_{DD}-V_{out}) = 1+\lambda V_{out} \Rightarrow \boxed{V_{out} = \frac{V_{DD}}{2} = 0.5V}$$

And so,

$$I_{D,P} = I_{D,N} = C_{ox} W_{g,N} v_{sat,N}(1+\lambda V_{out})(V_{in}-V_{th,N})$$

$$= (42.1mF/m^2)(10^7 cm/s)(5\mu m)(1.05)(0.25V) = \boxed{5.53\ mA = I_D}$$

b.

$$g_{m,N} = \frac{\partial I_{D,N}}{\partial V_{gs}} = C_{ox} W_{g,N} v_{sat,N}(1+\lambda V_{out}) = \boxed{22.1\ mA/V = g_{m,N}}$$

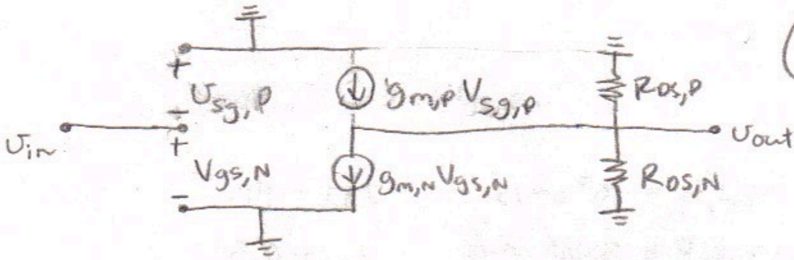
$$g_{m,P} = C_{ox} W_{g,P} v_{sat,P}(1+\lambda V_{out}) = \boxed{22.1\ mA/V = g_{m,P}}$$

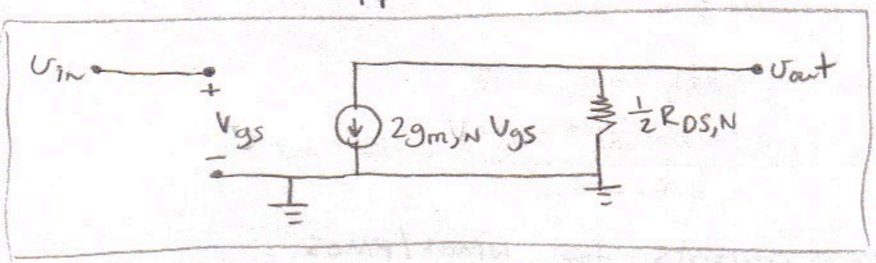
$$G_{ds,N} = \frac{\partial I_{D,N}}{\partial V_{ds}} = \frac{\lambda I_{D,N}}{1+\lambda V_{out}} = \boxed{0.527\ mA/V = G_{ds,N}}$$

$$G_{ds,P} = \frac{\lambda I_{D,P}}{1+\lambda(V_{DD}-V_{out})} = \boxed{0.527\ mA/V = G_{ds,P}}$$

c.

(*)

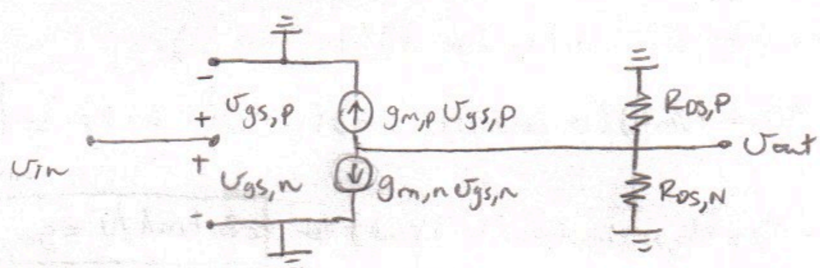




d. using the diagram above,

$$\frac{v_{out}}{v_{in}} = -\frac{2g_{m,N}v_{gs}\cdot\frac{1}{2}R_{DS,N}}{v_{gs}} = -\frac{g_{m,N}}{G_{DS,N}} \approx \boxed{-42\ V/V}$$

(*) Note that this is identical to:



From which we obtain the symplified diagram since

- $v_{gs,P} = v_{gs,n} = v_{in}$
- $g_{m,P} = g_{m,n}$
- $R_{DS,P} = R_{DS,n}$