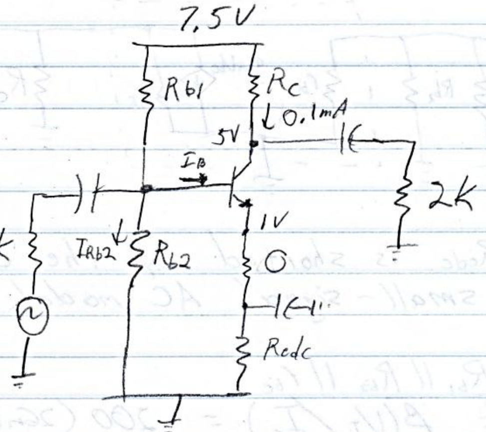
Homework

#5

Problem 1



$\beta = 200$

$V_a = 100V$

$I_{Rb2} = 20 \cdot I_b \qquad I_c = 0.1mA$

$V_e = 1V \qquad V_b \approx 1.7V \qquad V_{cb} > 0 \Rightarrow$ linear active

$\beta I_B = I_c$

$200\, I_B = 0.1mA$

$I_B = 0.5\,\mu A$

$I_{Rb2} = 20 \cdot 0.5\mu A = 10\,\mu A$

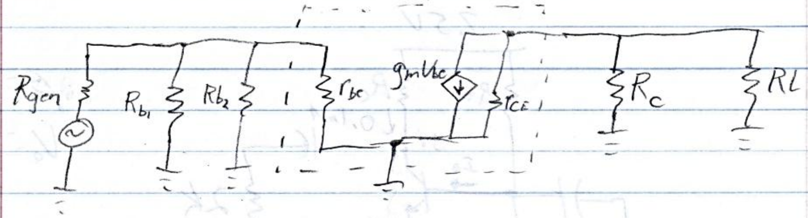
$I_{Rb1} = 10.5\,\mu A$

a.) $R_{b2} = \dfrac{1.7V}{10\mu A} = 170\,k\Omega$

$R_{b1} = \dfrac{7.5V - 1.7V}{10.5\mu A} = 552\,k\Omega$

$R_c = \dfrac{7.5V - 5V}{0.1mA} = 25\,k\Omega$

$R_{edc} = \dfrac{1V}{0.1mA + 0.5\mu A} = 9.95\,k\Omega$



Note: R_{edc} is shorted by the capacitor in the small-signal AC model.

$R_{in} = R_{b1} \,//\, R_{b2} \,//\, r_{be}$

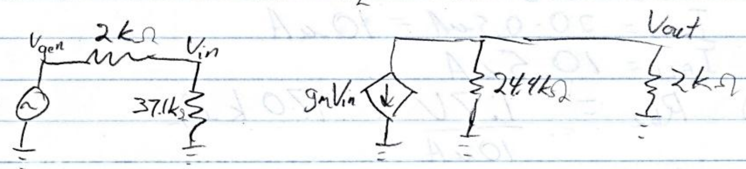
$r_\pi = r_{be} = \beta(V_T / I_c) = 200\,(26mV / 0.1mA) = 52k\Omega$

$R_{in} = 552k \,//\, 170k \,//\, 52k = 37.1k\,\Omega$

$R_{out} = r_{ce} \,//\, R_c$

$r_{ce} = r_o = |V_A| / I_c = 100V / 0.1mA = 1M\Omega$

$R_{out} = 1M\Omega \,//\, 25k\Omega = 24.4\,k\Omega$



$g_M = I_c / V_T = 0.1mA / 26mV = 3.8\ m\Omega^{-1}$

b.) $\dfrac{V_{out}}{V_{in}} = \dfrac{-g_M V_{in}\ R_{out} // R_L}{V_{in}} = -3.8m\Omega^{-1} \cdot 24.4k // 2k = -7.0$

$\dfrac{V_{in}}{V_{gen}} = \dfrac{R_{in}}{R_{in} + R_{gen}} = \dfrac{37.1k}{37.1k + 2k} = 0.949$

$\dfrac{V_{out}}{V_{gen}} = \dfrac{V_{out}}{V_{in}} \cdot \dfrac{V_{in}}{V_{gen}} = -7.0 \cdot 0.949 = -6.7$

c.) As found above

$R_{in} = 37.1\,k\Omega \quad R_{out} = 24.4\,k\Omega$

d.) $R_{Leq} = 25k\Omega \| 2k\Omega = 1.85k\Omega$

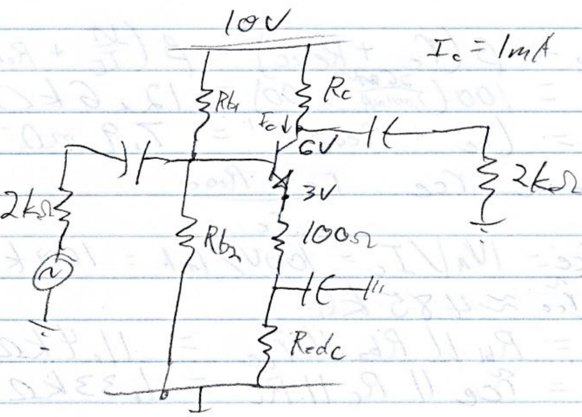
$\Delta V_{out} \uparrow_{max} = I_{CQ} \cdot R_{Leq} = 0.1mA \cdot 1.85k\Omega = 0.185V$

$V_{CEQ} = 5V - 1V = 4V$

$V_{CEmin} = V_{CE sat} \approx 0.5V$

$\Delta V_{CE} \downarrow max = 4V - 0.5V = 3.5V$

Problem # 2



$\beta = 100$

$V_a = 100$

$I_{Rb2} = 20 I_B \qquad I_C = 1mA$

$V_E = 3V \quad V_B \approx 3.7V \qquad V_{CB} > 0 \rightarrow$ linear active

$\beta I_B = I_C$

$100 I_B = 1mA$

$I_B = 1\mu A$

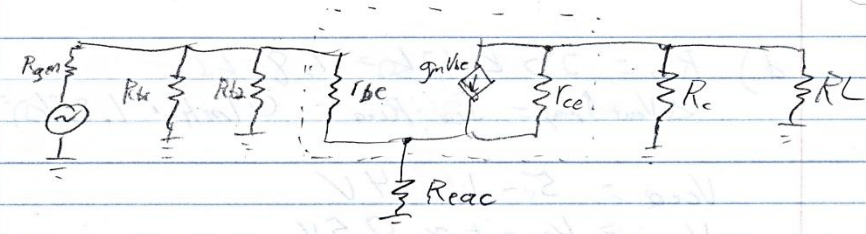
$I_{Rb2} = 20\mu A \qquad I_{Rb1} = 21\mu A$

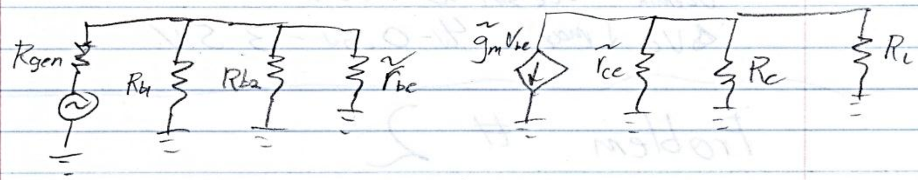
a.) $R_{b2} = 3.7V / 20\mu A = 185k\Omega$

$R_{b1} = (10V - 3.7V) / 21\mu A = 300k\Omega$

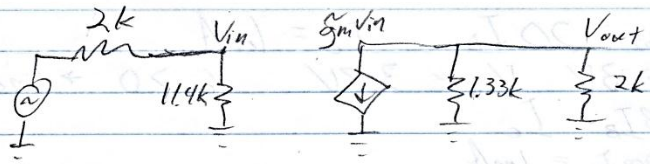
$R_C = (10V - 6V) / 1mA = 4k\Omega$

$R_{edc} = \dfrac{3 - (1mA + 1\mu A) \cdot 100\Omega}{1mA + 1\mu A} = 2.9k\Omega$





$$\tilde{r}_{be} = \beta\left(r_e + R_{eac}\right) = \beta\left(\frac{V_T}{I_E} + R_{eac}\right)$$
$$= 100\left(\frac{26\,mV}{1.001\,mA} + 100\right) = 12.6\,k\Omega$$
$$\tilde{g}_m = \left(r_e + R_{eac}\right)^{-1} = 7.9\,m\Omega^{-1}$$
$$\tilde{r}_{ce} \approx r_{ce}\,\frac{r_e + R_{eac}}{r_e}$$
$$r_{ce} = |V_A|/I_c = 100V/1mA = 100\,k\Omega$$
$$\tilde{r}_{ce} \approx 485\,k\Omega$$
$$R_{in} = R_{b1} \,\|\, R_{b2} \,\|\, \tilde{r}_{be} = 11.4\,k\Omega$$
$$R_{out} = \tilde{r}_{ce} \,\|\, R_C \,\|\, R_L = 1.33\,k\Omega$$



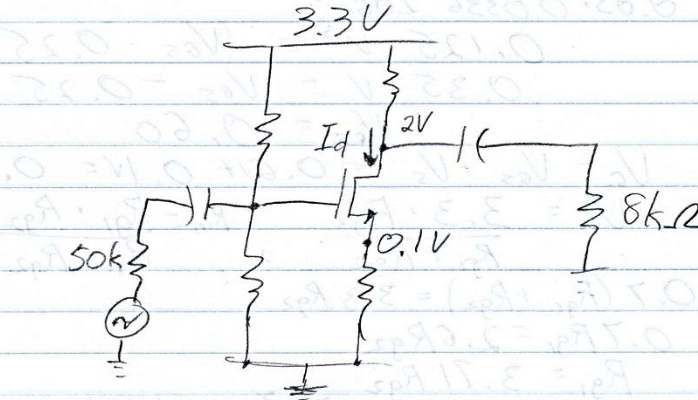
b.) $$\frac{V_{out}}{V_{in}} = \frac{-g_m \cancel{V_{in}}\, R_{out}\|R_L}{\cancel{V_{in}}} = -7.9\,m\Omega^{-1} \cdot 1.33\,k\Omega \| 2\,k\Omega$$
$$V_{out}/V_{in} = -6.3$$
$$\frac{V_{in}}{V_{gen}} = \frac{R_{in}}{R_{in}+R_{gen}} = \frac{11.4\,k\Omega}{12.73\,k\Omega} = 0.896$$
$$\frac{V_{out}}{V_{gen}} = \frac{V_{out}}{V_{in}} \cdot \frac{V_{in}}{V_{gen}} = -6.3 \cdot 0.896 = -5.64$$

c.) As found above
$R_{in} = 11.4k\Omega \quad R_{out} = 1.33k\Omega$

d.) $R_{Leq} = R_c \| R_L = 4k \| 2k = 1.33k\Omega$
$\Delta V_{out} \uparrow max = I_{cq} R_{Leq} = 1mA \cdot 1.33k = 1.33V$

$V_{ceq} = 6V - 3V = 3V$
$V_{cemin} = V_{ce\,sat} \approx 0.5V$
$\Delta V_{ce} \downarrow max = 3 - 0.5V = 2.5V$

Problem 3



$T_{ox} = 1nm, \quad L_g = 130nm \quad V_T = 0.25V$
$\mu = 300\, cm^2/(V \cdot s) \quad \lambda = 0$

a.) $I_D = \frac{1}{2} \mu C_{ox} \frac{W_g}{L_g} (V_{gs} - V_T)^2 (1 + \cancel{\lambda V_{DS}})$
For $I_D = 0.5mA$, $V_{gs} = 0.5V$
$C_{ox} = \frac{\varepsilon_r \varepsilon_0}{T_{ox}} = \frac{3.8\varepsilon_0}{1nm} = 0.0336\ F/m^2$
(Assuming SiO_2)
$0.5mA = \frac{1}{2} \frac{300cm^2}{V \cdot s} \cdot \frac{0.0336F}{m^2} \cdot \frac{W_g}{130nm} \cdot (0.5 - 0.25)^2 V$

$$0.5e^{-3}A = \frac{1}{32} V^2 \cdot \frac{0.03 m^2}{V \cdot s} \cdot 0.0336 \frac{A \cdot s}{V \cdot m^2} \cdot \frac{W_g}{130e^{-9} m}$$

$$0.5e^{-3}A = W_g \cdot \frac{0.03}{32} \cdot \frac{0.0336}{130e^{-9}} \frac{A}{m}$$

$$W_g = \frac{0.5e^{-3} \cdot 32 \cdot 130e^{-9}}{0.03 \cdot 0.0336} \frac{A \cdot m}{A} = 2.06 \times 10^{-6} m$$

$$W_g = 2.06\ \mu m$$

b.) $1mA = \frac{1}{2} \frac{0.03 m^2}{V \cdot s} \cdot \frac{0.0336 A \cdot s}{V \cdot m^2} \cdot \frac{2.06e^{-6}}{130e^{-9}} (V_{GS} - 0.25)^2$

$$\frac{2e^{-3} \cdot 130e^{-9}}{0.03 \cdot 0.0336 \cdot 2.06e^{-6}} V^2 = (V_{GS} - 0.25)^2$$

$$0.125\ V^2 = (V_{GS} - 0.25)^2$$

$$0.35\ V = V_{GS} - 0.25$$

$$V_{GS} = 0.60$$

$$V_G = V_{GS} + V_S = 0.6V + 0.1V = 0.7\ V$$

$$V_G = \frac{3.3 \cdot R_{g2}}{R_{g1} + R_{g2}}, \quad R_{in} = \frac{R_{g1} \cdot R_{g2}}{R_{g1} + R_{g2}} = 1 M\Omega$$

$$0.7 (R_{g1} + R_{g2}) = 3.3 R_{g2}$$

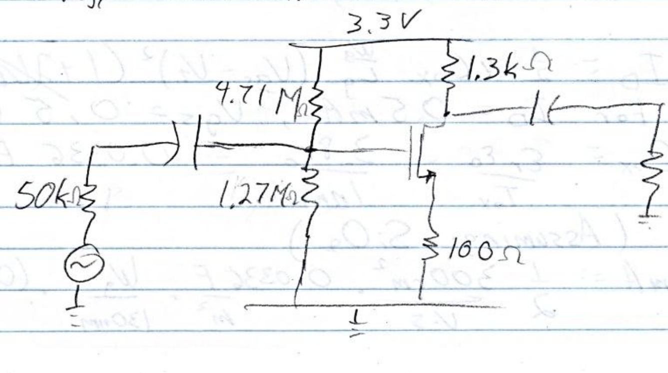
$$0.7 R_{g1} = 2.6 R_{g2}$$

$$R_{g1} = 3.71 R_{g2}$$

$$1 M\Omega = \frac{3.71 R_{g2}^{\cancel{2}}}{4.71 \cancel{R_{g2}}} \qquad 1 M\Omega = 0.788 R_{g2}$$

$$R_{g2} = 1.27 M\Omega$$

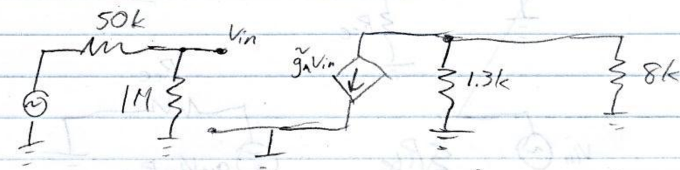
$$R_{g1} = 3.71 \cdot 1.27M = 4.71 M\Omega$$



$$R_d = \frac{3.3 - 2V}{1mA} = 1.3k\Omega$$

$$R_s = \frac{0.1V}{1mA} = 100\Omega$$

c.)



$\lambda = 0 \rightarrow r_{ce} = \infty = \tilde{r}_{ce}$

$R_{out} = R_c$

$R_{in} = 1M\Omega$

$R_{out} = 1.3k\Omega$

$$\tilde{g}_m = \frac{g_m}{1 + g_m R_s} \qquad g_m = \mu C_{ox}(W_g / L_g) \cdot (V_{GS} - V_{th})$$

$$g_m = \frac{0.03\, m^2}{V \cdot s} \cdot \frac{0.0336\, A \cdot s}{V \cdot m^2} \cdot \frac{2.06e^{-6}}{130e^{-9}} \cdot (0.6 - 0.25)V$$

$$g_m = 5.6\, m\Omega^{-1}$$

$$\tilde{g}_m = \frac{5.6\, m\Omega^{-1}}{1 + 5.6 m\Omega^{-1} \cdot 100\Omega} = 3.6\, m\Omega^{-1}$$

$$R_{Leq} = R_{out} \,||\, R_L = 1.3k \,||\, 8k = 1.12\, k\Omega$$

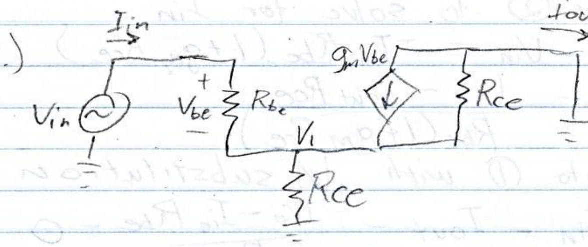
$$\frac{V_{out}}{V_{in}} = \frac{-\tilde{g}_m V_{in} R_{Leq}}{V_{in}} = -3.6m\Omega \cdot 1.12k = -4.0$$

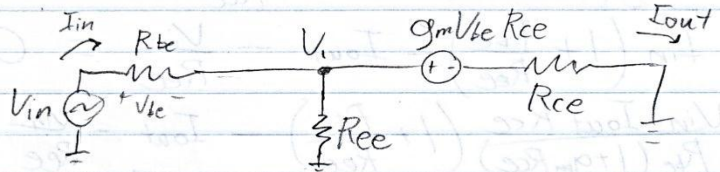
$$\frac{V_{in}}{V_{gen}} = \frac{R_{in}}{R_{in} + R_{gen}} = \frac{1M}{1M + 50k} = 0.952$$

$$\frac{V_{out}}{V_{gen}} = \frac{V_{out}}{V_{in}} \cdot \frac{V_{in}}{V_{gen}} = 0.952 \cdot -4.0 = -3.84$$

Problem 4

a.)





Using KCL at the central node

(1) $I_{in} - I_{out} - V_1/R_{ee} = 0$

$$I_{out} = \frac{V_1 - g_m V_{be} R_{ce}}{R_{ce}}$$

$$V_{be} = I_{in} R_{be}$$

$$I_{out} = \frac{V_1 - g_m I_{in} R_{be} R_{ce}}{R_{ce}}$$

$$V_1 = V_{in} - I_{in} R_{be}$$

(2) $$I_{out} = \frac{V_{in} - I_{in} R_{be}(1 + g_m R_{ce})}{R_{ce}}$$

Plug this and the V_1 substitution back into 1

$$I_{in} - \frac{V_{in} - I_{in} R_{be}(1+g_m R_{ce})}{R_{ce}} - \frac{V_{in} - I_{in} R_{be}}{R_{ee}} = 0$$

$$V_{in}\left(\frac{1}{R_{ce}} + \frac{1}{R_{ee}}\right) = I_{in}\left(1 + \frac{R_{be}}{R_{ce}}(1+g_m R_{ce}) + \frac{R_{be}}{R_{ee}}\right)$$

$$\frac{V_{in}}{I_{in}} = \left[1 + \frac{R_{be}}{R_{ce}}(1+g_m R_{ce}) + \frac{R_{be}}{R_{ee}}\right] \cdot (R_{ce} \| R_{ee})$$

For extrinsic transconductance

Rearrange ⑤ to solve for I_{in}

$$I_{out} R_{ce} - V_{in} = -I_{in} R_{be}(1 + g_m R_{ce})$$

$$I_{in} = \frac{V_{in} - I_{out} R_{ce}}{R_{be}(1 + g_m R_{ce})}$$

Plug into ① with V_1 substitution

$$I_{in} - I_{out} - \frac{V_{in} - I_{in} R_{be}}{R_{ee}} = 0$$

$$I_{in}\left(1 + \frac{R_{be}}{R_{ee}}\right) - I_{out} - \frac{V_{in}}{R_{ee}} = 0$$

$$\frac{V_{in} - I_{out} R_{ce}}{R_{be}(1 + g_m R_{ce})}\left(1 + \frac{R_{be}}{R_{ee}}\right) - I_{out} - \frac{V_{in}}{R_{ee}} = 0$$

$$I_{out}\left[1 + \frac{R_{ce}}{R_{be}(1 + g_m R_{ce})}\left(1 + \frac{R_{be}}{R_{ee}}\right)\right]$$

Rearranging / simplifying

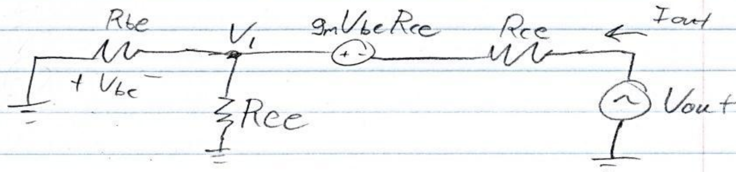
$$I_{out}\left[1 + \frac{R_{be} + R_{ce}}{R_{be}(1 + g_m R_{ce})}\right] = V_{in}\left[\frac{\left(1 + \frac{R_{be}}{R_{ee}}\right)}{R_{be}(1 + g_m R_{ce})} - \frac{1}{R_{ee}}\right]$$

$$\frac{I_{out}}{V_{in}} = \frac{\left[\frac{(1 + R_{be}/R_{ee})}{R_{be}(1 + g_m R_{ce})} - \frac{1}{R_{ee}}\right]}{\left[1 + \frac{R_{be} + R_{ce}}{R_{be}(1 + g_m R_{ce})}\right]}$$

With a bit of work this reduces to

$$\frac{I_{out}}{V_{in}} = \frac{R_{ce} - g_m R_{be} R_{ce}}{R_{ee}(2R_{be} + R_{ce} + g_m R_{be} R_{ce})}$$

b.) The circuit is the same as (a) but with the source on the other side



By KCL

(1) $$-\frac{V_1}{R_{be}} - \frac{V_1}{R_{ee}} + I_{out} = 0$$

Note the redefined <u>direction</u> of I_{out}

$$V_{be} = -V_1$$

$$V_1 = V_{out} - g_m V_1 R_{ce} - I_{out} R_{ce}$$

$$V_1 (1 + g_m R_{ce}) = V_{out} - I_{out} R_{ce}$$

$$V_1 = \frac{V_{out} - I_{out} R_{ce}}{1 + g_m R_{ce}}$$

Plug this into (1)

$$-V_1 \left(\frac{1}{R_{be}} + \frac{1}{R_{ee}}\right) + I_{out} = 0$$

$$-\left(\frac{V_{out} - I_{out} R_{ce}}{1 + g_m R_{ce}}\right)\left(\frac{1}{R_{be}} + \frac{1}{R_{ee}}\right) + I_{out} = 0$$

$$\frac{V_{out}}{1 + g_m R_{ce}}\left(\frac{1}{R_{be}} + \frac{1}{R_{ee}}\right) = I_{out}\left[1 + \frac{R_{ce}}{(1 + g_m R_{ce})} \cdot \left(\frac{1}{R_{be}} + \frac{1}{R_{ee}}\right)\right]$$

$$\frac{V_{out}}{I_{out}} = \frac{\left[1 + \frac{R_{ce}}{1 + g_m R_{ce}}\left(\frac{1}{R_{be}} + \frac{1}{R_{ee}}\right)\right]}{\frac{1}{1 + g_m R_{ce}}\left(\frac{1}{R_{be}} + \frac{1}{R_{ee}}\right)}$$

With a little work this reduces to

$$\frac{V_{out}}{I_{out}} = R_{ce} + (1 + g_m R_{ce}) \cdot (R_{be} \| R_{ee})$$