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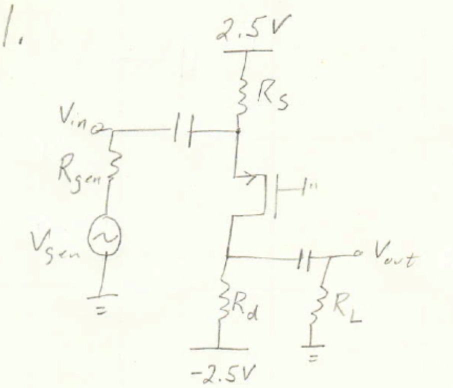
1.



$K_p = \frac{1}{2}\mu_p C_{ox} W/L =$ 0.345 (W/L) mA/V^2

$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.9\varepsilon_o}{t_{ox}} =$ 3.45*10^-6 F/cm^2

$\lambda = 0.05V^{-1}$; $L =$ 130 nm

$R_L =$ 10 $k\Omega$; $R_{gen} =$ 200 Ω

a) DC Bias

$i_D = 1mA$; $V_{GS} = -0.7V$; $V_{DS} = -1.7V$

$i_D = K_p(|V_{GS}| - |V_{tn}|)^2(1 + \lambda|V_{DS}|)$

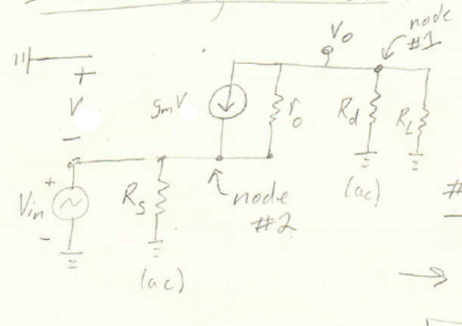
$\rightarrow K_p = \frac{i_D}{(|V_{GS}| - |V_{tn}|)^2(1+\lambda|V_{DS}|)} =$ 10.24 $mA/V^2 = (0.345)(W/L)\ mA/V^2$

$\boxed{\Rightarrow W = 3.86\ \mu m}$

$V_s = 2.5V - i_D R_s = 0.7V \Rightarrow \boxed{R_s = 1.8\ k\Omega}$

$V_D = -2.5V + i_D R_d = -1.0V \Rightarrow \boxed{R_d = 1.5k\Omega}$

b) Small Signal Model



$\Rightarrow V_{in} = -V$; $r_o \approx \frac{1}{\lambda i_D} = 20k\Omega$

$g_m = \frac{2 i_D}{|V_{GS}| - |V_{tn}|} = 5mS$ (positive by convention)

#1: $g_m V + \frac{V_o - V_{in}}{r_o} + \frac{V_o}{R_d} + \frac{V_o}{R_L} = 0$

$\rightarrow V_o\left(\frac{1}{r_o} + \frac{1}{R_d} + \frac{1}{R_L}\right) = V_{in}\left(g_m + \frac{1}{r_o}\right)$

$\Rightarrow V_o/V_{in} = \frac{R_d R_L(g_m r_o + 1)}{R_d R_L + R_L r_o + R_d r_o} = \boxed{6.18 = V_o/V_i}$

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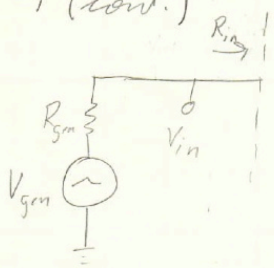
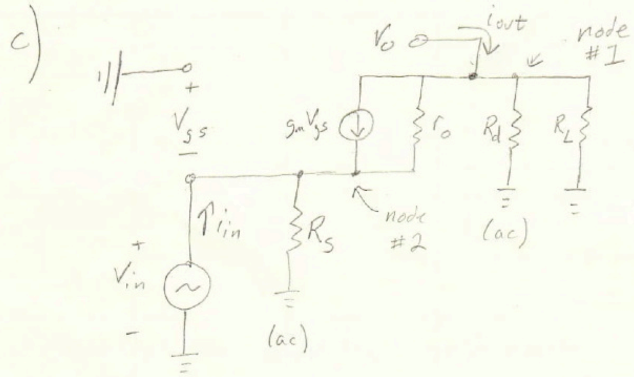
1. b) (cont.)



$$\Rightarrow \boxed{V_{in}/V_{gen} = \left(\frac{R_{in}}{R_{gen}+R_{in}}\right) = 0.486}$$

$$\boxed{V_{out}/V_{gen} = \left(V_{out}/V_{in}\right)\left(V_{in}/V_{gen}\right) = 3.00}$$

c)



Node #2 : $i_{in} = V_{in}/R_s - g_m V_{gs} + \frac{V_{in}-V_o}{r_o} = V_{in}\left(\frac{1}{R_s} + g_m + \frac{1}{r_o}\right) - \frac{V_o}{r_o}$

$$i_{in} = V_{in}\left[\frac{1}{R_s} + g_m + \frac{1}{r_o} - \frac{(V_{out}/V_{in})}{r_o}\right] \Rightarrow \boxed{R_{in} = V_{in}/i_{in} = 189\ \Omega}$$

Node #1

$$i_{out} = \frac{V_o}{R_d} + \frac{V_o - V_{in}}{r_o} + g_m V$$

$$i_{out} \approx V_o\left(\frac{1}{R_d} + \frac{1}{r_o}\right) \Rightarrow \boxed{R_{out} = V_{out}/i_{out} = 1.4\ k\Omega} ;$$

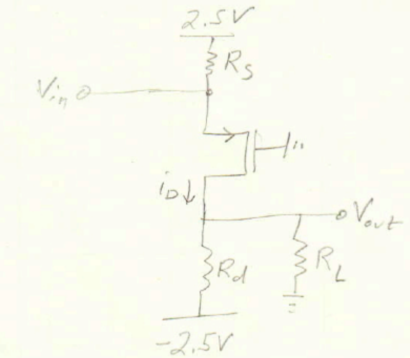
Alternatly $\boxed{R_{out} \approx R_d // r_{ds}(1+g_m R_s) = 1.5\ k\Omega}$

1. (cont.)

d) Seek: Maximum positive- and negative-going outputs.

- Here we continue to assume high frequency so that we can replace —||— with —— .

The amplifier becomes;



- Clearly the amplifier will no longer work when the transistor is off. Evidently this occurs when $V_{in} = 0.3V = |V_{th}|$.

What is V_{out} at this point?

Clearly $i_D \to 0 \Rightarrow V_{out} \to (-2.5V)\left(\frac{R_L}{R_d+R_L}\right) = -2.17\ V$

Since $V_{out} = V_{DS}$ even when $V_{in} \to 0$, obviously the transistor is in saturation at this point. This implies that for $V_{in} > 0.3V$, the amplifier is working in the proper regime.

thus, $\boxed{V_{out_{min}} = -2.17\ V}$

$\Rightarrow$ the max negative voltage swing is:

$$V_{out}\downarrow : = \left| V_{out}(\text{at bias}) - V_{out_{min}} \right|$$

Note that for AC analysis, V_{out}(at bias) is no longer 1V!

Instead, $V_{out} \approx \left(\frac{R_d R_L}{R_d+R_L}\right)\left(\frac{-2.5V}{R_d} + 1mA\right) = -0.87\ V$ (neglecting r_o)

1. d) (cont.)

thus, $\boxed{|V_{out}\downarrow| = |-2.17\,V + 0.87\,V| = 1.3\,V}$

This simple analysis, proves the well-known formula $V_{out}\downarrow = \Delta i_D (R_d // R_L) = 1.3\,V.$

To find the maximum output voltage, we set V_{in} and V_{out} to satisfy the saturation condition: ie $|V_{DS}| = |V_{GS}| - |V_{th}|$

$$|V_{DS}| = |V_{out} - V_{in}| = V_{in} - V_{out} \quad \text{since } V_{in} > V_{out}$$

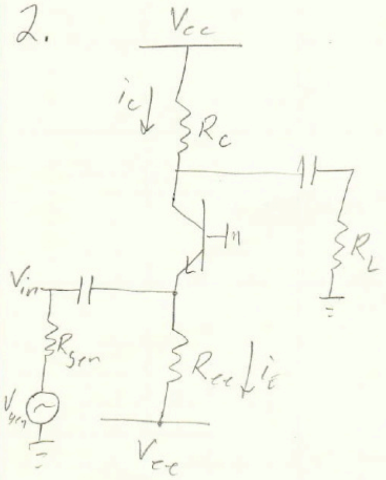
$$|V_{GS}| = |-V_{in}| = V_{in} \quad ; \quad |V_{th}| = 0.3$$

$$\Rightarrow V_{in} - V_{out} = V_{in} - 0.3 \Rightarrow V_{out} = 0.3 \ (V_{out} \text{ maximum})$$

thus, $V\!\!\uparrow := |V_{out}(\text{at bias}) - V_{out}(\text{max})|$

$$\boxed{V\!\!\uparrow = |-0.87\,V - 0.3\,V| = 1.17\,V}$$

2.



a) <u>DC Bias</u>

$$0V - V_{BE} - i_E R_{ee} = V_{ee}$$

$$R_{ee} = \frac{-V_{ee} - V_{BE}}{i_E} = \frac{+3.3V - 0.7V}{0.5\,mA}$$

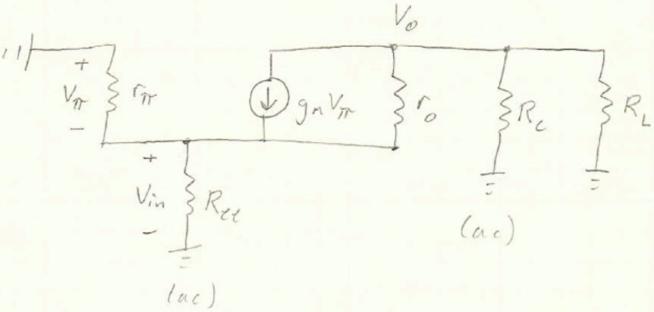
$$\boxed{\Rightarrow R_{ee} = 5.2\ k\Omega}$$

$$V_{cc} - i_c R_c - V_{CB} = 0$$

$$R_c = \frac{V_{cc} - V_{CB}}{i_c} = \frac{3.3\,V - 1.0\,V}{(1+\frac{1}{\beta})^{-1} i_E}$$

$$\Rightarrow \boxed{R_c = 4.65\ k\Omega}$$

b) The small signal model not neglecting the Early effect is:



$\Rightarrow$ <u>$V_{in} = -V_\pi$</u>

$$\Rightarrow V_o/r_o - g_m V_{in} + V_o/R_c + V_o/R_L = 0$$

$$V_o\left(\frac{1}{r_o} + \frac{1}{R_c} + \frac{1}{R_L}\right) = g_m V_{in} \Rightarrow V_o/V_{in} = \frac{g_m r_o R_L R_c}{R_c R_L + r_o(R_c + R_L)}$$

$$g_m \approx \frac{i_c}{V_T} = 19.3\ mS\ ; \quad r_o \approx \frac{V_A}{i_c} = 200\ k\Omega \quad \boxed{\Rightarrow V_o/V_{in} = 26.80}$$

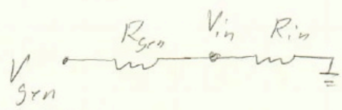
2. b) (cont.) - Using nodal analysis and the S.S. model:

$$i_{in} = -V_{in}/r_\pi + g_m V_{in} + V_{in}/r_o$$ (evaluate for $V_{out} \to \perp$ to find input impedance)

$$\to V_{in}/i_{in} = \left(-\frac{1}{r_\pi} + g_m + \frac{1}{r_o}\right)^{-1}$$

$$\to R_{in} \approx 52.3\ \Omega$$

$\to$



$$\to V_{in} = V_{gen}\left(\frac{R_{in}}{R_{gen} + R_{in}}\right)$$

$$\boxed{\Rightarrow V_{in}/V_{gen} = 0.343} \Rightarrow \boxed{V_{out}/V_{gen} = 9.20}$$

c) From (b), $\boxed{R_{in} = 52.3\ \Omega}$

From the small signal model, it is clear that

$$R_{out} = R_L // R_c // r_o = \left(\frac{1}{R_c} + \frac{1}{r_o}\right)^{-1} = \boxed{4.5\ k\Omega = R_{out}}$$

d) At bias current using the small-signal approximation ($i_{out} \to \infty$),

We have V_{out} (at bias): (V_o-3.3)/R_c+V_o/R_L=0.5mA --> V_o=1.7 V

$V_{out}(max)$ occurs when $i_c = i_E = 0$

$$\Rightarrow V_{out}(max) = V_{cc}\left(\frac{R_L}{R_c + R_L}\right) = 1.0\ V$$

$$\Rightarrow \boxed{V_{out}\uparrow = 1.7\ V - 1.0\ V = 0.7\ V}$$

2. d) (cont.)

$V_{out}(min)$ occurs when the BJT is pushed into saturation. $\Rightarrow V_{CE} = 0.5V$

V_ce=V_o-V_in=0.5V

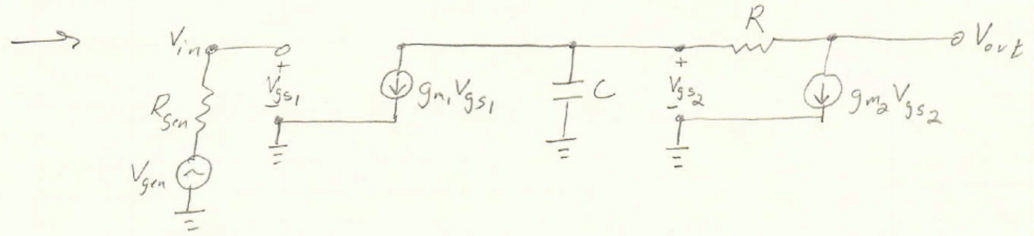
1-V_in/V_o=0.5/V_o

V_o=0.5/(1-1/A_v)=0.52V

V_o_downswing=1.7V-0.52V=1.18V

3. a) Because $C_{gs} = C_{gd} = 0$, and $R_{ds} \to \infty$, we can use the most simple small signal model for the MOSFET's, however C is small and cannot be replaced with a short.

$\rightarrow$



b) Clearly the infinite impedance going into the gate implies $\underline{V_{gen} = V_{in}}$. Obviously $\underline{V_{in} = V_{gs_1}}$.

Our 2 nodal equations are:

$$V_{gs_2}(sC) + g_{m_1}V_{gs_1} + \frac{V_{gs_2} - V_{out}}{R} = 0; \quad \frac{V_{gs_2} - V_{out}}{R} = g_{m_2}V_{gs_2}$$

$$\rightarrow V_{gs_2}\left(sC + \frac{1}{R}\right) + g_{m_1}V_{in} - \frac{V_{out}}{R} = 0 \qquad \rightarrow V_{gs_2} = V_{out}(1 - g_{m_2}R)^{-1}$$

$$V_{out}\left[\frac{\left(sC + \frac{1}{R}\right)}{(1 - g_{m_2}R)} - \frac{1}{R}\right] = -g_{m_1}V_{in}$$

$$V_{out}\left[\frac{RCs + 1 - (1 - g_{m_2}R)}{R(1 - g_{m_2}R)}\right] = -g_{m_1}V_{in}$$

$$V_{out}/V_{in} = -g_{m_1}\left(\frac{1 - g_{m_2}R}{Cs + g_{m_2}}\right) = \frac{g_{m_1}(g_{m_2}R - 1)}{g_{m_2}}\left(\frac{1}{1 + \frac{Cs}{g_{m_2}}}\right)$$

Using the values given, $\boxed{V_{out}/V_{in} = 58\left(\frac{1}{1 + 10^{-10}s}\right)}$

3. (cont.) c) Intuitively some may expect that this circuit would have low-pass behavior. This is reflected in the TF and there is a zero as $s \to \infty$ pole at omega=10^10 $\Rightarrow (f_{c_1} =$ 1.6 GHz $)$

d) ~~See attached~~ plot

e) Lets redo the problem and solve the ODE rather then just using the TF directly.

$$V_{gs_2} = V_{out}(1 - g_{m_2}R)^{-1}$$

and:

$$C\, dV_{gs}/dt + V_{gs_2}/R + g_{m_1}V_{in} - V_{out}/R = 0$$

$$\frac{C}{(1-g_{m_2}R)}\, dV_{out}/dt + \frac{V_{out}}{R(1-g_{m_2}R)} - V_{out}/R = -g_{m_1}V_{in}$$

$$dV_{out}/dt + V_{out}\left(\frac{1}{RC} - \frac{(1-g_{m_2}R)}{RC}\right) = \frac{-g_{m_1}(1-g_{m_2}R)}{C}V_{in}$$

$$dV_{out}/dt + V_{out}\left(g_{m_2}/C\right) = \frac{g_{m_1}(g_{m_2}R-1)}{C}V_{in}$$

define $C/g_{m_2} = \tau$; $\frac{g_{m_1}(g_{m_2}R-1)}{C}V_{in} = K$ for $t \geq 0$

$V_{out} = x$

$\Rightarrow dx/dt + \frac{x}{\tau} = K$; this common differential equation is simple to solve.

3. e) (cont.) $dx/dt + \frac{x}{\tau} = K$

For steady state $dx/dt \to 0$.

define $x_f = K\tau$ (final value of x)

$$\Rightarrow dx/dt = \frac{-(x - K\tau)}{\tau} = \frac{-(x - x_f)}{\tau}$$

$$\frac{dx}{x - x_f} = -\frac{1}{\tau} dt$$

$$\Rightarrow \int_{x(0)}^{x(t)} \frac{dx}{x - x_f} = \int_0^t \frac{-dt}{\tau}$$

$$\ln(x(t) - x_f) - \ln(x(0) - x_f) = -\frac{t}{\tau}$$

$$\Rightarrow \frac{x(t) - x_f}{x(0) - x_f} = e^{-t/\tau}$$

$$\rightarrow x(t) = (x(0) - x_f)e^{-t/\tau} + x_f$$

Because voltage cannot instantaneously change accross a capacitor, $x(0) = 0$

$$\Rightarrow x(t) = x_f(1 - e^{-t/\tau})$$

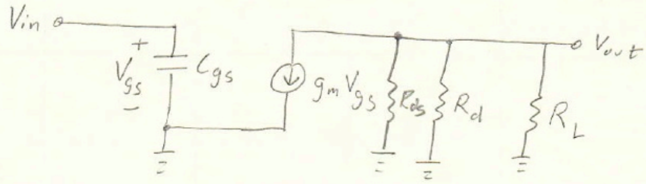
$$\Rightarrow V_{out}(t) = \left(\frac{g_{m_1}(g_{m_2}R - 1)}{C}\right)\left(\frac{C}{g_{m_2}}\right) V_{in}\Big|_{t \geq 0} \left(1 - e^{-g_{m_2}t/C}\right)$$

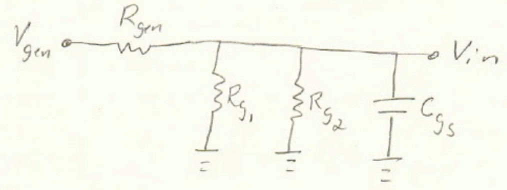
$$\boxed{V_{out}(t) = (5.8\ V)(1 - e^{-t/10\text{^-}10\ \text{sec}})} \quad t \geq 0$$

See attached plot.

4.

a)





; Note that R_{ds} is given to be 100Ω. This is quite low!

$R_{ds} \approx \frac{1}{\lambda i_D} \Rightarrow$ large i_D or large λ!!

b) Nodal Equations ; $-g_m V_{gs} = V_{out}/R_d + V_{out}/R_L + V_{out}/R_{ds}$

Clearly $V_{in} = V_{gs}$

$$\frac{V_{in}-V_{gen}}{R_{gen}} + \frac{V_{in}}{R_{g_1}} + \frac{V_{in}}{R_{g_2}} + V_{in} s C_{gs} = 0$$

$$\rightarrow V_{in}\left(\frac{1}{R_{gen}} + \frac{1}{R_{g_1}} + \frac{1}{R_{g_2}} + sC_{gs}\right) = \frac{V_{gen}}{R_{gen}}$$

$$\rightarrow V_{in}/V_{gen} = \frac{R_{g_1} R_{g_2}}{R_{g_1}R_{g_2} + R_{gen}R_{g_2} + R_{gen}R_{g_1} + sCR_{gen}R_{g_1}R_{g_2}}$$

Using the values given ; $V_{in}/V_{gen} = \quad = (\; 0.714 \left(\frac{1}{1+ s/7*10^7}\right)$

And $V_{out}\left(\frac{1}{R_d} + \frac{1}{R_L} + \frac{1}{R_{ds}}\right) = -g_m V_{in} \rightarrow V_{out}/V_{in} = \frac{-g_m}{\left(\frac{1}{R_d} + \frac{1}{R_L} + \frac{1}{R_{ds}}\right)} = -1.98$

$$\Rightarrow \boxed{V_{out}/V_{gen} = -1.41/(1+s/7*10^7)}$$

4. (cont.) c) Clearly there is a zero as $s \to \infty$

d) See attached plot   Pole at $s = 7*10^7$

$\Rightarrow (f_{c_1} = 11.1\ \text{MHz})$

e) Resolving, (not required)

$V_{out}/V_{in} = -1.98$

$\left(\frac{1}{R_{gen}} + \frac{1}{R_{S_1}} + \frac{1}{R_{S_2}}\right)$ (nodal analysis)

$$V_{in}\left(7*10^{-6}\ \Omega^{-1}\right) + C_{gs}\, dV_{in}/dt = \frac{V_{gen}}{200\ k\Omega}$$

$$dV_{in}/dt + \left(\frac{V_{in}}{7*10^7\ (sec)}\right) = 5*10^7\, V_{gen}$$

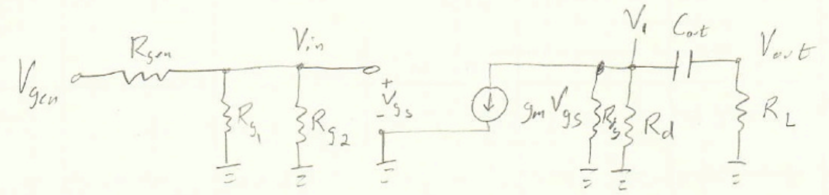
$$\Rightarrow V_{in}(t) = 7.14\ \text{mV}\left(1 - e^{-t/1.42*10^{-8}}\right)$$ (see prob. 3 for justification)

$$\boxed{V_{out} = (-14.1\ \text{mV})\left(1 - e^{-t/1.42*10^{-8}}\right)}$$

for $V_{gen} = 10\ \text{mV}$ step at $t = 0$.

5.

a)



b) Clearly $V_{in} = V_{gs}$; Note: R_{ds} given to be very small!

Nodal Eqn's: $\frac{V_{in}-V_{gen}}{R_{gen}} + \frac{V_{in}}{R_{g_1}} + \frac{V_{in}}{R_{g_2}} = 0 \rightarrow \boxed{V_{in}/V_{gen} = \frac{R_{g_1}R_{g_2}}{R_{g_1}R_{g_2}+R_{gen}(R_{g_1}+R_{g_2})}}$

$V_1/R_{ds} + (V_1 - V_{out})sC_{out} + V_1/R_d + g_m V_{gs} = 0; \; (V_{out}-V_1)sC_{out} + V_{out}/R_L = 0$

$V_1\left(sC_{out}+\frac{1}{R_d}+\frac{1}{R_{ds}}\right) - sC_{out}V_{out} + g_m V_{in} = 0 \qquad \rightarrow \boxed{V_1 = V_{out}\left(\frac{1}{R_L s C_{out}}+1\right)}$

$-g_m V_{in} = V_{out}\left[\left(sC_{out}+\frac{R_{ds}+R_d}{R_dR_{ds}}\right)\left(\frac{1}{R_LsC_{out}}+1\right) - sC_{out}\right] = V_{out}\left(\frac{1}{R_L}+sC_{out}+\frac{R_{ds}+R_d}{R_{ds}R_dR_LsC_{out}}+\frac{R_{ds}+R_d}{R_dR_{ds}} - sC_{out}\right)$

$-g_mV_{in} = V_{out}\left(\frac{R_dR_{ds}sC_{out}+R_{ds}+R_d+(R_{ds}+R_d)R_LsC_{out}}{R_{ds}R_dR_LsC_{out}}\right)$

$\rightarrow \boxed{V_{out}/V_{in} = -400\left(\frac{s}{202s+1005}\right) = -1.98\left(\frac{s}{1+0.2s}\right)}$

Using the values given;

$V_{in}/V_{gen} = 0.714$

$\Rightarrow \boxed{V_{out}/V_{gen} = -1.41*s/(1+0.2*s)}$

5. c) One should predict a zero transfer function at DC and this is indeed present in the TF.
There is no zero this time as $s \to \infty$, reflecting the fact that C_{gd} and C_{gs} cannot always be ignored.
$\Rightarrow$ 1 zero at $s = 0$; pole at $s =$ 1/0.2 $(2\pi f_{c_1} =$ 0.795 Hz$)$

d) See attached plot

e) For fun, lets solve this from the beginning.

at V_o: $\frac{d}{dt}(V_{out} - V_i) C_{out} + V_{out}/R_L = 0 \Rightarrow \frac{d}{dt}(V_i - V_{out}) C_{out} = V_{out}/R_L$

$$dV_i/dt = dV_{out}/dt + V_{out}/R_L C_{out} \Rightarrow V_i = V_{out} + \frac{1}{R_L C_{out}} \int_0^t V_{out}\, dt$$

at V_1:

$$\frac{d}{dt}(V_i - V_{out}) C_{out} + g_m V_{in} + V_1/R_d + V_1/R_{ds} = 0$$

$$\to V_{out}/R_L + g_m V_{in} + \frac{R_d + R_{ds}}{R_d R_{ds}}\left(V_{out} + \frac{1}{R_L C_{out}} \int_0^t V_{out}\, dt\right) = 0$$

This simple integral equation is easy to solve.
Lets use the Laplace transform!

$$\mathcal{L}\{V_{out}\}\left(\frac{1}{R_L} + \frac{R_d + R_{ds}}{R_d R_{ds}}\right) + \frac{R_d + R_{ds}}{R_L R_d C_{out} R_{ds}} \frac{\mathcal{L}\{V_{out}\}}{s} = -g_m \mathcal{L}\{V_{in}\}$$

$\to$

$\longrightarrow$

5.e) (cont.)

$$\rightarrow \boxed{V_{out}(t) = -14.1\text{ mV}\; e^{-0.2*t}} \; ; \; t > 0$$

Email Questions/concerns to:
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