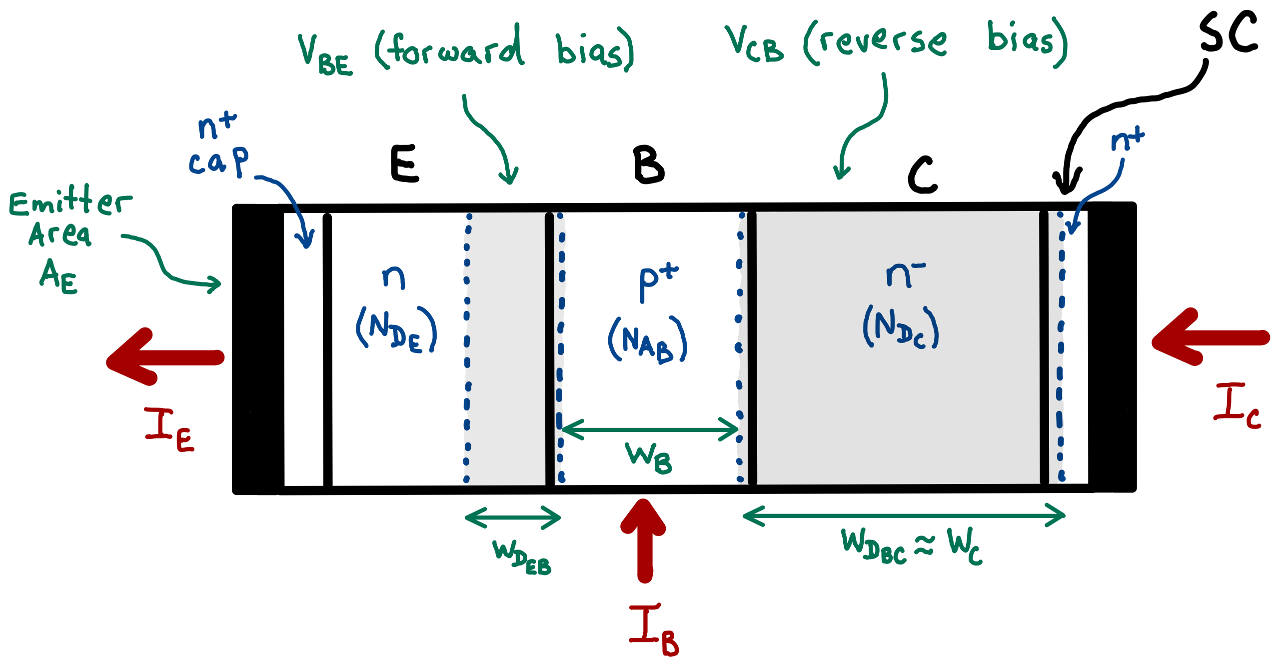
Frequency Response of Bipolar Transistors



- In forward active mode:
  - Currents I_E, I_B, and I_C flow through the emitter, base, and collector, respectively
  - Charge is stored in the emitter base junction, in the base, and in the base-collector junction

- If V_{CE} is constant & V_{BE} is modulated by an amount $\tilde{v}_{BE}$:
  - $\tilde{v}_{CB} = -\tilde{v}_{BE}$
  - Stored charge in each of these regions must change
  - Variation in stored charge is supplied by I_E, I_C, and I_B
  - Finite amount of time required to modulate stored charge → delays

To analyze device frequency response:

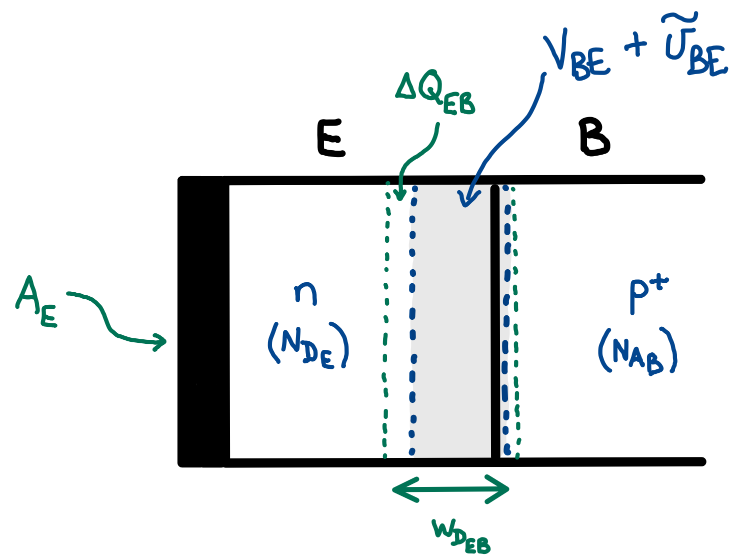
- Calculate all delays in device (charge control model)

- Create small signal model that incorporates all delay effects as well as other extrinsic effects (e.g., contact resistances, etc)

- Nodal analysis

Emitter-base junction:

$\tau_e \equiv$ emitter delay



$$\tau_e = \frac{\Delta Q_{EB}}{\tilde{i}_E}$$

$$\Delta Q_{EB} = C_{jE} \cdot \tilde{v}_{BE}$$

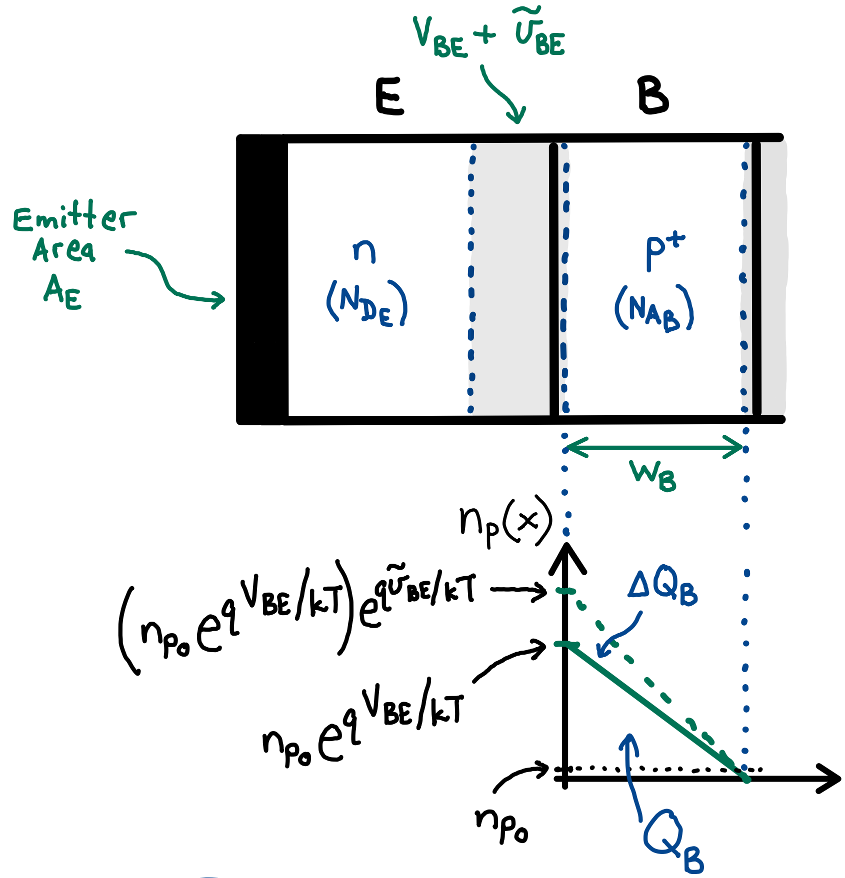
$$\tilde{i}_E = g_m \tilde{v}_{BE}$$

$$g_m = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{V_{CE}=\text{const}} = \frac{qI_E}{kT}$$

$$\boxed{\tau_e = \frac{1}{g_m} C_{jE}} \longrightarrow$$

$$C_{jE} = \frac{\epsilon A_E}{W_{DEB}} = \frac{A_E}{2}\left[\frac{2q\epsilon}{(V_{bi_{BE}} - V_{BE})} \cdot \frac{N_{AB} N_{DE}}{(N_{AB} + N_{DE})}\right]^{1/2}$$

Base:



$\tau_b \equiv$ base delay

$$\tau_b = \frac{\Delta Q_B}{\tilde{i}_c}$$

$$\Delta Q_B = \frac{q}{2} n_{p_0} e^{qV_{BE}/kT}\left(e^{q\tilde{v}_{BE}/kT} - 1\right) \cdot W_B \qquad e^{q\tilde{v}_{BE}/kT} \approx 1 + \frac{q\tilde{v}_{BE}}{kT}$$

$$\approx \frac{q}{2} n_{p_0} W_B e^{qV_{BE}/kT} \cdot \frac{q\tilde{v}_{BE}}{kT}$$

$$\tilde{i}_c = qD_n\left(\frac{n_{p_0} e^{qV_{BE}/kT} \cdot e^{q\tilde{v}_{BE}/kT}}{W_B}\right) - qD_n\left(\frac{n_{p_0} e^{qV_{BE}/kT}}{W_B}\right)$$

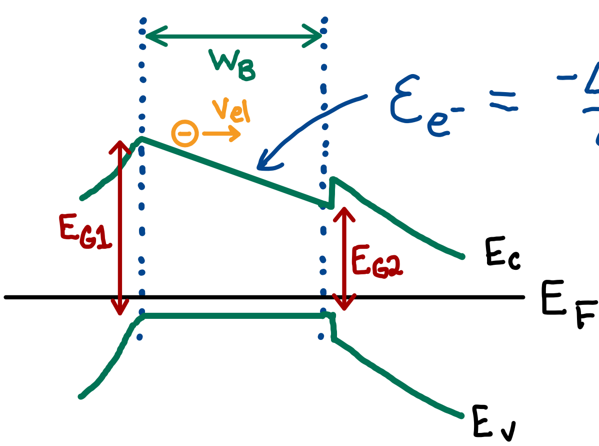
$$= \frac{qD_n n_{p_0}}{W_B} e^{qV_{BE}/kT}\left(e^{q\tilde{v}_{BE}/kT} - 1\right) \approx \frac{qD_n n_{p_0}}{W_B} e^{qV_{BE}/kT} \cdot \frac{q\tilde{v}_{BE}}{kT}$$

$$\boxed{\tau_B = \frac{W_B^2}{2D_n}}$$

- It can be shown that the base delay τ_B is equal to the transit time τ_t that it takes an electron to transit the base width W_B.

- The expression $\tau_t = \frac{W_B^2}{2D_n}$ is valid for electron diffusion across the base.

- τ_t (and therefore τ_B) can be reduced by grading the bandgap in the base, thereby creating a quasi $\mathcal{E}$-field that results in electron drift across the base.



$$\mathcal{E}_{e^-} = \frac{-\Delta E_G}{q W_B}$$

$$\Delta E_G = E_{G1} - E_{G2}$$

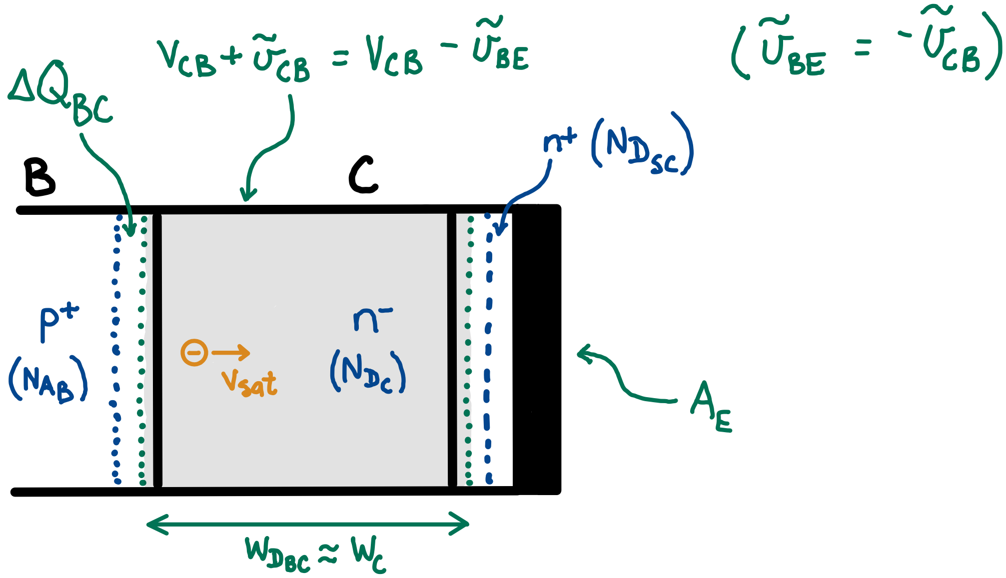
$$v_{el} = -\mu_n \mathcal{E}_{e^-} = \frac{\mu_n \Delta E_G}{q W_B}$$

$$\left(\text{note: } \frac{D_n}{\mu_n} = \frac{kT}{q}\right)$$

In this case: $$\boxed{\tau_t = \frac{W_B}{v_{el}} = \frac{q W_B^2}{\mu_n \cdot \Delta E_G} = \frac{W_B^2}{2D_n} \cdot \frac{2kT}{\Delta E_G}}$$

Base-collector junction (2 delay terms):

- $\tau_c \equiv$ collector junction charging delay
- $\tau_d \equiv$ collector junction drift delay



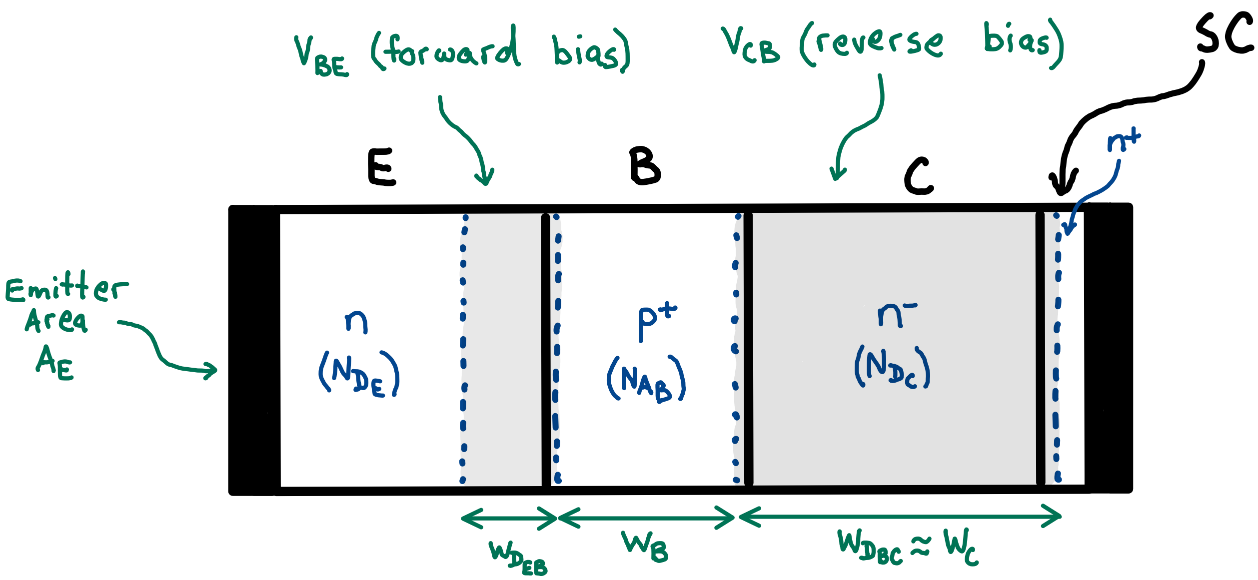
$$\tau_c = \frac{\Delta Q_{BC}}{\tilde{i}_c} \longrightarrow \begin{array}{l} \Delta Q_{BC} = -C_{jc}\cdot\tilde{v}_{BC} = C_{jc}\tilde{v}_{BE} \\ \tilde{i}_c \approx g_m \tilde{v}_{BE} \end{array}$$

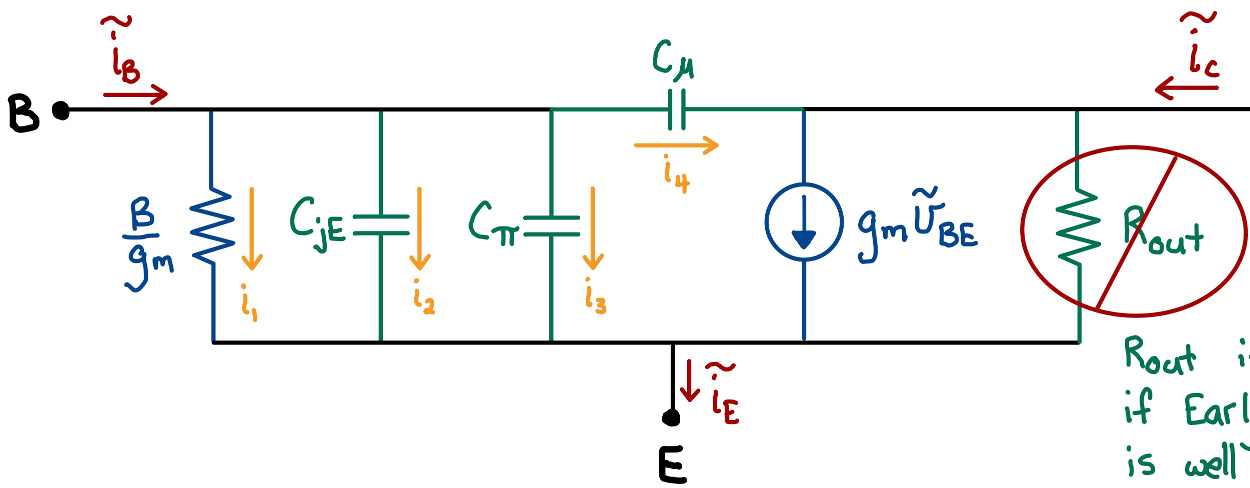
$$\boxed{\tau_c = \frac{1}{g_m} C_{jc}} \longrightarrow \begin{array}{l} g_m = \dfrac{qI_E}{kT} \\ C_{jc} \approx \dfrac{\epsilon A_E}{W_c} \end{array}$$

Note: if we account for contact resistances R_c and R_E, then $\tau_c \approx \left(R_E + R_c + \frac{1}{g_m}\right)C_{jc}$

$$\boxed{\tau_d = \frac{1}{2}\frac{W_{DBC}}{v_{sat}} \approx \frac{1}{2}\frac{W_c}{v_{sat}}}$$

Small-Signal Model





Rout is large if Early effect is well managed

$$C_{jE} = \frac{A_E}{2}\left[\frac{2q\epsilon}{(V_{bi_{BE}} - V_{BE})} \cdot \frac{N_{AB} N_{DE}}{(N_{AB} + N_{DE})}\right]^{1/2} \quad \text{(as before)}$$

$$C_\pi = g_m \tau_t \qquad \left(g_m = \frac{q I_E}{kT}\right)$$

$$C_\mu = C_{jC} + g_m \tau_d$$

as before

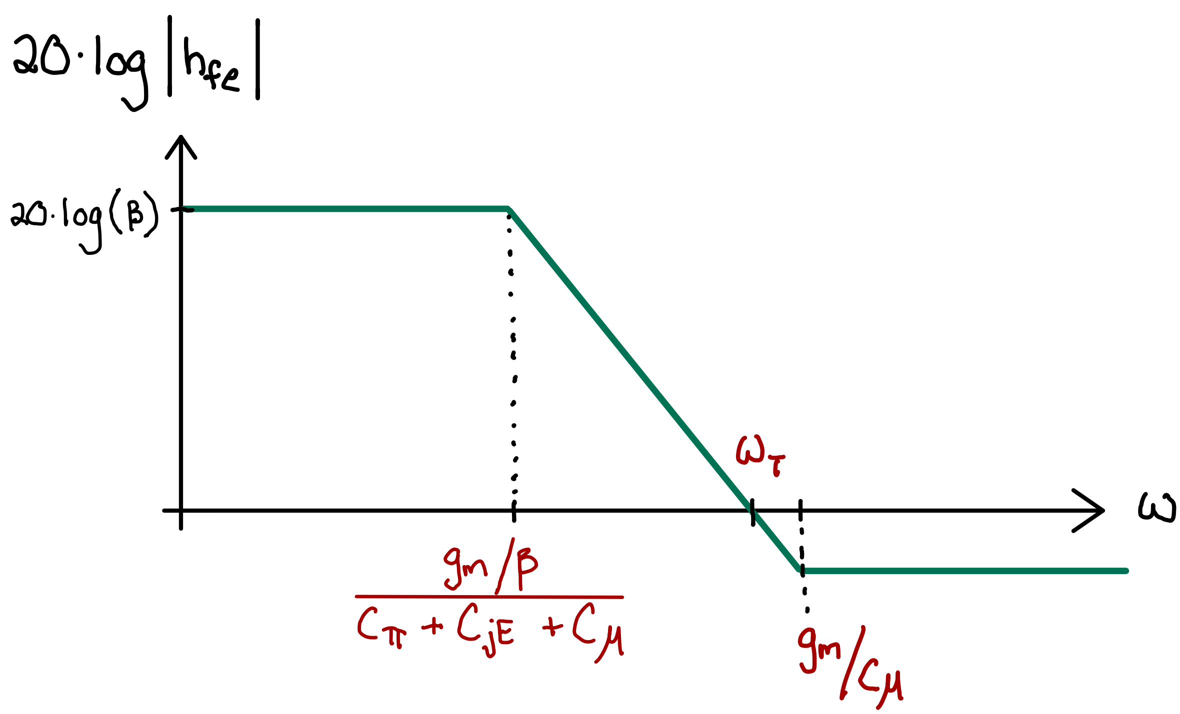
Nodal Analysis:

$$\tilde{i}_B = i_1 + i_2 + i_3 + i_4 = \tilde{v}_{BE}\left[\frac{g_m}{\beta} + j\omega\left(C_\pi + C_{jE} + C_\mu\right)\right]$$

$$\tilde{i}_C = g_m \tilde{v}_{BE} - i_4 = \tilde{v}_{BE}\left[g_m - j\omega C_\mu\right]$$

$h_{fe} \equiv$ short circuit current gain

$$\boxed{h_{fe} = \frac{\tilde{i}_C}{\tilde{i}_B} = \frac{g_m - j\omega C_\mu}{g_m/\beta + j\omega\left(C_\pi + C_{jE} + C_\mu\right)}}$$



ω_T = angular frequency for unity current gain
$(|h_{fe}| = 1)$

$$g_m^2 + \cancel{\omega_T^2 C_\mu^2} = \cancel{\frac{g_m^2}{\beta^2}} + \omega_T^2 (C_\pi + C_{jE} + C_\mu)^2$$

$\ll g_m^2$ $\ll g_m^2$

$$\omega_T = \frac{g_m}{C_\pi + C_{jE} + C_\mu}$$

$f_T \equiv$ current gain cut-off frequency

$$\boxed{f_T = \frac{\omega_T}{2\pi} = \frac{g_m}{2\pi(C_\pi + C_{jE} + C_\mu)} = \frac{1}{2\pi \tau_{tot}}}$$

$$\boxed{\tau_{tot} = \tau_e + \tau_t + \tau_c + \tau_d}$$

$$\tau_e = \frac{1}{g_m} C_{jE}$$

$$\tau_c = \frac{1}{g_m} C_{jC} \qquad \left(C_{jC} = \frac{\epsilon A_E}{W_C} \right)$$

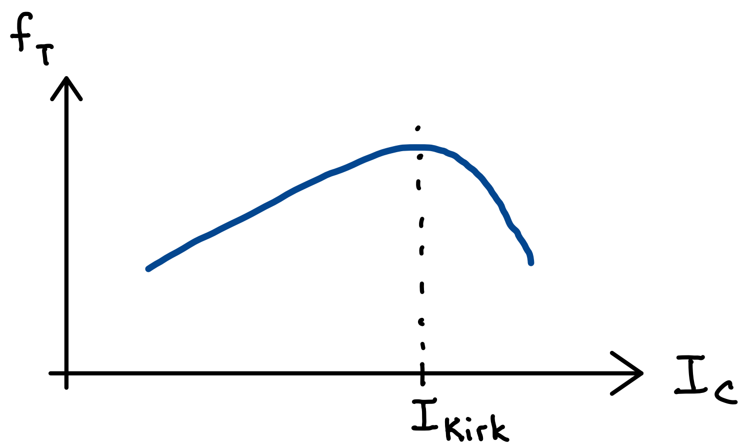
$$\tau_d = \frac{1}{2} \frac{W_C}{v_{sat}}$$

$$\tau_t = \frac{W_B^2}{2D_n} \cdot \frac{2kT}{\Delta E_G}$$

for graded bandgap in base

To maximize f_T:

- Make W_B small and grade bandgap in base (minimizes τ_t)
- Optimize W_C to minimize $\tau_c + \tau_d$
- Operate at high current (maximizes g_m, thereby minimizing τ_e and τ_c)



$$J_{Kirk} = q N_{D_C} v_{sat}$$

$$I_{Kirk} = A_E \cdot J_{Kirk}$$

Kirk Effect:

- High-Level injection of electrons into base-collector depletion region

- Base push-out