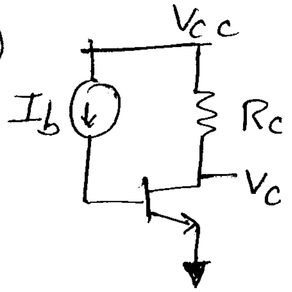
ECE 137A

HW#1 SOLUTIONS

1 a.)



$\beta = 100$

$V_{CC} = 3$ Volts

$I_C = 1mA$

$V_C = 1.5$ Volts

For $I_C = 1mA$

$I_B = \frac{I_C}{\beta} = \frac{1mA}{100}$

$\underline{I_B = 10uA}$ required

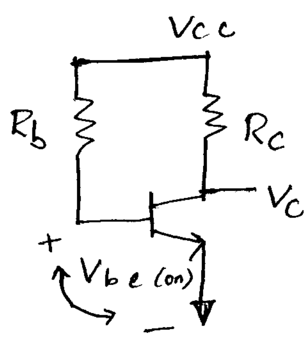
Now, on applying KVL to the output of the circuit we get

$$V_{CC} - I_C R_C - V_C = 0$$

$$\Rightarrow R_C = \frac{V_{CC} - V_C}{I_C} = \frac{(3-1.5)V}{1 mA}$$

$$\Rightarrow \underline{R_C = 1.5K\Omega}$$

b.)



$\beta = 100$

$V_{CC} = 3$ VOLTS

$I_C = 200uA$

$V_{ce} = 1.5$ Volts

$V_{be(on)} = 0.7$ Volts

For $I_C = 200uA$

$I_B = \frac{I_C}{\beta} = \frac{200uA}{100}$

$\underline{I_B = 2uA}$ required.

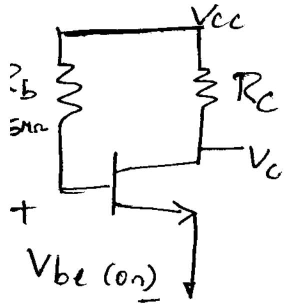
Now, applying KVL to the output of the circuit we get

$V_{CC} - I_C R_C - V_C = 0 \Rightarrow R_C = \frac{V_{CC} - V_C}{I_C} = \frac{3-1.5V}{200uA} = \underline{7.5}$

Applying KVL to the input of the circuit we get

$V_{CC} - I_b R_b - V_{be(on)} = 0 \Rightarrow R_b = \frac{V_{CC} - V_{be(on)}}{I_b} = \frac{3-0.7}{2uA} = \underline{1.15M}$

c) Bias stability of previous circuit:



Here β is increased to 200

Now, applying KVL to input side we get

$$V_{cc} - I_b R_b - V_{be(on)} = 0$$

$$\Rightarrow I_b = \frac{V_{cc} - V_{be(on)}}{R_b} = \frac{(3 - 0.7)V}{1.15\,M\Omega}$$

$$\Rightarrow I_b = 2uA \text{ (same as before)}$$

Now, $I_C = \beta I_B = 200\,(2uA) = 400\,uA$

Using this value of I_c, apply KVL to output loop we get,

$$V_{cc} - I_C R_C = V_C \Rightarrow V_C = 3 - (400\,uA)(7.5\,k\Omega)$$

$$= 3 - 3 = 0 \text{ Volts}$$

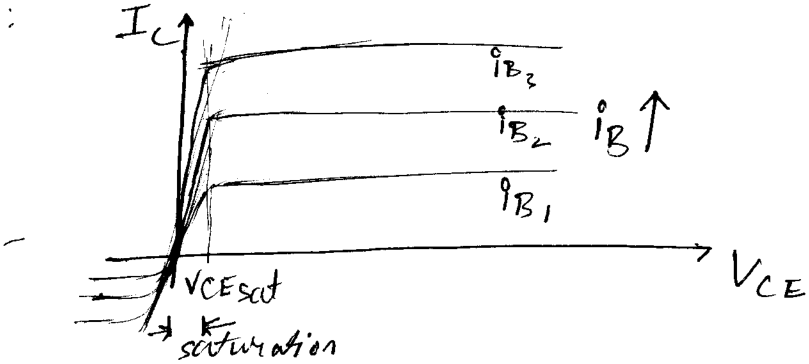
$V_C = 0$ Volts is not practically possible as the minimum V_{CE} is $V_{CE\,sat} = 0.5$ Volts.

In practical situation, as the β increases, the collector current increases (as long as the collector-base junction is reverse biased.)

In our case. $V_b = V_{be(on)} = 0.7$ Volts.

Hence, when the collector voltage drops below 0.7 Volts, $V_{cb} = V_C - V_b$ becomes negative and the collector base junction becomes forward biased and the transistor enters saturation wherein both the P-N junctions are now forward biased.

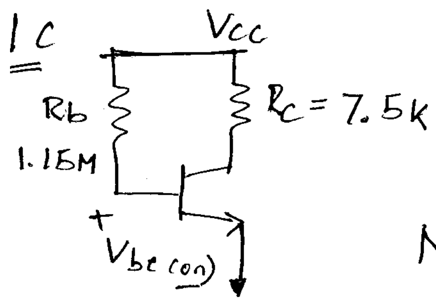
Under saturation, the minimum V_{CE} is $V_{CE(sat)}$ as this graph shows:



Thus, in saturation, $I_C = \dfrac{V_{CC} - V_{CE(sat)}}{R_C}$

$$= \frac{(3 - 0.5)\,V}{7.5k} = 333\ \mu A$$

And $V_C = V_{CE sat} = 0.5$ Volts



$\beta = 100$ and $V_{cc} = 3 + \left(\frac{3 \times 10}{100}\right)$

$V_{cc} = 3.3 \text{ Volts}$

Now, apply KVL to the input loop

$$V_{cc} - I_b R_b - V_{be(on)} = 0$$

$$\Rightarrow I_b = \frac{V_{cc} - V_{be(on)}}{R_b} = \frac{(3.3 - 0.7)\text{ Volts}}{1.15\,M\Omega}$$

$$\Rightarrow I_b = \left(\frac{2.6V}{1.15M\Omega}\right) = 2.26uA$$

Now, $I_c = \beta I_b = 100\,(2.26uA) = 226uA$

Now, apply KVL to output loop :

$$V_{cc} - I_c R_c - V_c = 0 \Rightarrow V_c = V_{cc} - I_c R_c$$

$$= 3.3V - (226uA)(7.5k\Omega)$$

$$= 3.3V - 1.695V$$

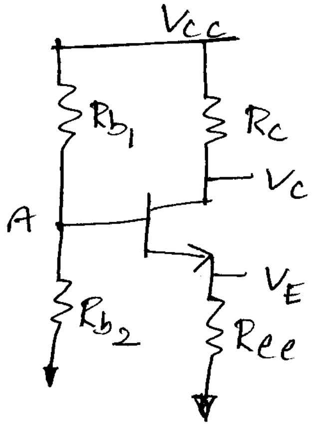
$$= 1.605 \text{ Volts}$$

As a check :

$V_c > V_b$ (= 0.7 Volts) $\Rightarrow$ CB Junction is reverse biased

$\Rightarrow$ Transistor is still in active region

1d.



$\beta = 100$

$V_{CC} = 3.3$ Volts

$V_C = 2$ Volts

$V_e = 1$ Volts

$I_{R_{b_2}} = 200uA$

$I_C = 2mA$

$\beta = 100$

$\Rightarrow \alpha = \frac{\beta}{\beta+1} = \frac{100}{101} = 0.$

Now, $I_E = \frac{I_C}{\alpha} = \frac{2mA}{0.99}$

$\Rightarrow I_E = 2.02mA$

~~$V_E = I_E R_E$~~ $I_B = \frac{I_C}{\beta}$

$\Rightarrow I_B = 0.02mA$

$\underline{I_B = 20uA}$.

Now apply KCL at node A to get

$I_{R_{b_1}} = I_{R_{b_2}} + I_B = 200uA + 20uA = \underline{220uA}.$

~~Now, apply~~ Now, $V_{be(on)} = V_B - V_E = 0.7$ Volts

$\Rightarrow V_B = 0.7 + V_E = 0.7 + 1 = 1.7$ Volts

Apply KVL to the input loop and we get

$V_{CC} - I_{R_{b_1}} R_{b_1} = V_B \Rightarrow R_{b_1} = \frac{V_{CC} - V_B}{I_{R_{b_1}}}$

$\Rightarrow R_{b_1} = \left(\frac{3.3 - 1.7}{220uA}\right) = \underline{7.27k\Omega}.$

Also, $V_B = I_{R_{b_2}} \cdot R_{b_2} \Rightarrow R_{b_2} = \left(\frac{1.7V}{200uA}\right) = \underline{\underline{8.5k\Omega}}$

Also, $V_E = I_E R_{EE} \Rightarrow R_{EE} = \frac{V_E}{I_E} = \left(\frac{1V}{2.02mA}\right) = \underline{495}$

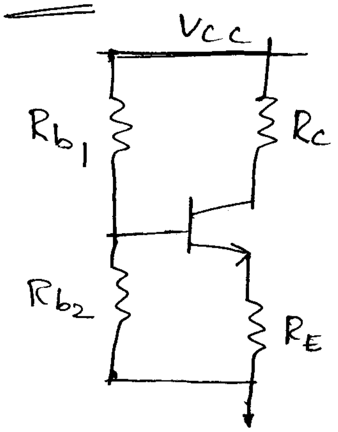
And, $V_C = V_{CC} - I_C R_C = 3.3 - (2mA) R_C = 2$ Volts

$\Rightarrow R_C = \frac{-2 + 3.3}{2mA} = \underline{\underline{650\Omega}}$

Thus, $R_{b_1} = 7.27 k\Omega$ $\quad R_C = 650\Omega$

$R_{b_2} = 8.5 k\Omega$ $\quad R_E = 495\Omega$

1e



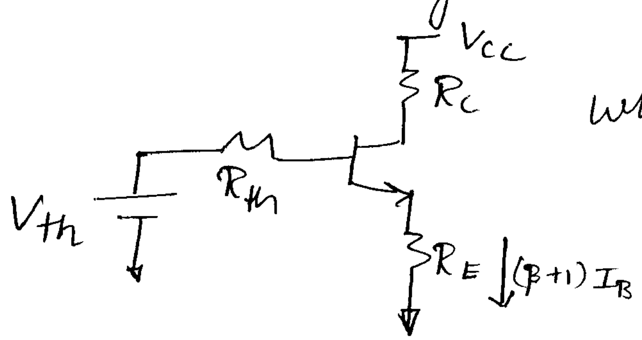
$\beta = 200$

$V_{CC} = 3.3$ Volts

~~To make an approximate analysis, we can neglect I_B to find V_B as~~

$$\cancel{V_B = \frac{V_{CC} \cdot R_{b_2}}{R_{b_2} + R_{b_1}} =}$$

The input to the circuit can be represented using a Thevenin Equivalent as:



where

$$V_{th} = \frac{V_{CC} \cdot R_{b_2}}{R_{b_2} + R_{b_1}} = \underline{1.778 \text{ Volts}}$$

$$R_{th} = R_{b_1} \| R_{b_2} = \underline{3.91 k\Omega}$$

Now, we apply KVL to the input loop to get

$$V_{th} - I_B R_{th} - V_{be(on)} - (\beta+1) I_B \cdot R_E = 0$$

$$\Rightarrow I_B = \frac{V_{th} - V_{be(on)}}{R_{th} + (\beta+1) R_E} = \frac{1.778 - 0.7}{(3.91k) + (201)(495)}$$

$$= \frac{1.078}{\cancel{95.585k}\ 103.405k} = \cancel{11.2uA} \quad \underline{10.4uA}$$

Now, $I_C = \beta I_b = (200)(10.4uA)$

$$= 2.08mA$$

Apply KVL to the output loop:

$$V_C = V_{CC} - I_C R_C = 3.3V - (2.08mA)(650\Omega)$$

$$= 1.948 \text{ Volts}$$

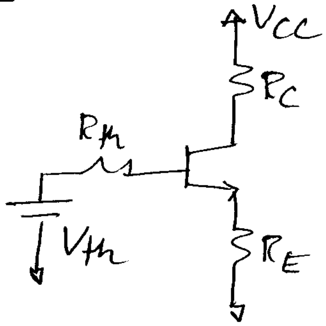
As a check,

$$V_B = V_{th} - I_B R_{th} = 1.77 - (10.4uA)(3.91k\Omega)$$

$$= \underline{1.73 \text{ Volts}}.$$

As $V_C > V_B$, CB junction is reverse biased and the transistor is in the active region.

1e



$\beta = 100$

$V_{CC} = 3.3 + \frac{3.3 \times 10}{100} = \underline{3.63 \text{ Volts}}$

$V_{th} = \frac{V_{CC} \cdot R_{b2}}{R_{b2} + R_{b1}} = 1.956 \text{ Volts}$

$R_{th} = R_{b1} \| R_{b2} = 3.91 k\Omega$

Apply KVL to the input loop:

$V_{th} - I_B R_{th} - V_{be(ON)} - \overset{\nearrow (\beta+1) I_B}{I_E} R_E = 0$

$\Rightarrow \quad I_B = \frac{V_{th} - V_{be(ON)}}{R_{th} + (\beta+1) R_E} = \frac{1.956 - 0.7}{(3.91k) + (101)(495)}$

$I_B = \frac{1.256}{53905} = \underline{\underline{23.3 \, uA}}$

Now, $I_C = \beta I_b = 100 \cdot (23.3 \, uA)$

$= \underline{2.330 \, mA}$

Apply KVL to the output loop and we get

$V_{CC} - I_C R_C = V_C = 3.63V - (2.33 mA)(650)$

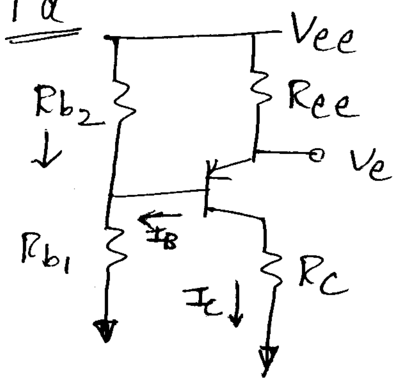
$V_C = \underline{2.12 \text{ Volts}}.$

As a check,

$V_B = V_{th} - I_B R_{th} = 1.956V - (23.3uA)(3.91k) = 1.865 < V_C$

$\Rightarrow$ The transistor is in the active region.

1d



$\beta = 300$

$V_{ee} = 3.3$ Volts

$V_C = 1$ Volts

$V_E = 2$ Volts

$I_C = 2mA$

$I_{Rb_2} = 200uA$

$I_C = 2mA$

$\Rightarrow I_B = \frac{I_C}{\beta} = \frac{2mA}{300} = 6.6$

$I_E = I_B + I_C = 2.006m$

$I_{Rb_2} + I_B = I_{Rb_1}$ (KCL at Base)

$\Rightarrow I_{Rb_1} = \underline{206.6uA}$

Now, $V_C = I_C R_C \Rightarrow \underline{R_C} = \frac{V_C}{I_C} = \frac{1}{2mA} = \underline{500\Omega}$

$V_E = V_{ee} - I_e R_{ee} = 3.3 - (2.006m)(R_{EE}) = 2$

$\Rightarrow R_{ee} = \frac{3.3-2}{(2.006m)} = \underline{648\Omega}$

$V_{be(ON)} = V_b - V_e = -0.7$ ($\because$ PNP transistor)

$\Rightarrow V_b = V_e - 0.7 = 2 - 0.7 = 1.3$ Volts

Now, apply KVL on input loop to get

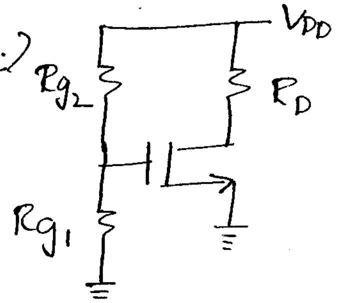
$V_{ee} - I_{Rb_2} \cdot R_{b_2} = V_B \Rightarrow R_{b_2} = \frac{V_{ee} - V_B}{I_{Rb_2}}$

$\Rightarrow R_{b_2} = \frac{3.3 - 1.3}{200uA} = \underline{10k\Omega}$

Also, $I_{Rb_1} \cdot R_{b_1} = V_B \Rightarrow R_{b_1} = \frac{V_B}{I_{Rb_1}} = \frac{1.3V}{206.6uA}$

$\Rightarrow R_{b_1} = \underline{6.29k\Omega}$

2 a.)



$V_{th} = 0.3$ Volt

$v_{sat} C_{ox} W_g = 1 mA/V$

$R_{g1} = 200 k\Omega$

$V_{DD} = 2.0$ Volts

$I_D = 1 mA$

$V_{DS} = 1.5$ Volts

Apply KVL at the output loop: $V_{DD} - I_D R_D = V_D$

Now, $V_{DS} = V_D - V_S = 1.5$ Volts $\Rightarrow$ $V_D = 1.5$ Volts ($\because V_S = 0$)

$$\Rightarrow R_D = \frac{V_{DD} - V_D}{I_D} = \frac{2 - 1.5}{1 mA} = \underline{\underline{500 \Omega}}$$

Also,

$$v_{sat} \cdot C_{ox} \cdot W_g \cdot (V_{gs} - V_{th}) = I_D$$

$$\Rightarrow V_g = \frac{I_D}{v_{sat} C_{ox} \cdot W_g} + V_{th} \quad (\because V_S = 0)$$

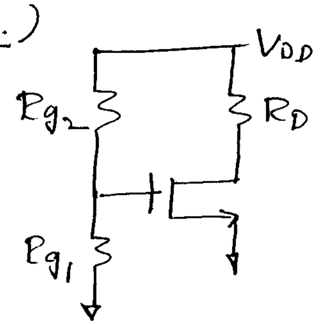
$$= \frac{1 mA}{1 mA/V} + 0.3 V = 1.3 \text{ Volt}$$

Now,

$$V_g = \frac{V_{DD} \cdot R_{g1}}{R_{g1} + R_{g2}} = 1.3 V$$

$$\Rightarrow R_{g2} = \frac{V_{DD} \cdot R_{g1}}{1.3} - R_{g1} = \underline{107.7 k\Omega}$$

2b.)



$V_{DD} = 2$ Volts $\qquad V_{th} = 0.3$ Volts

$R_D = 500\,\Omega$

$R_{g_1} = 200\,k\Omega$

$R_{g_2} = 107.7\,k\Omega$

$\upsilon_{sat} C_{ox} W_g = 1mA/\upsilon + 0.1 mA/V = \underline{1.1 mA/}$

$$V_g = \frac{V_{DD} \cdot R_{g_1}}{R_{g_1} + R_{g_2}} = \frac{2\,(200K)}{200K + 107.7K} = \underline{1.3 \text{ Volts}}$$

$I_D = \upsilon_{sat} C_{ox} W_g \,(V_{gs} - V_{th}) = \left(1.1 \frac{mA}{V}\right)(1.3 - 0.3)$

$I_D = \underline{1.1 mA}$

$V_D = V_{DD} - I_D R_D = 2 - (1.1mA)(500\Omega) = \underline{1.45 V}$

$V_{DS} = 1.45V > V_{gs} - V_{th} = 1.3 - 0.3 = 1$ Volt

$\Rightarrow$ The device is in saturation (Normal Mode of operation for MOSFET.

2b.) $V_{DD} = 2V + 0.2V = 2.2$ Volts $\qquad V_{th} = 0.3V$

$\upsilon_{sat} C_{ox} W_g = 1mA/\upsilon$

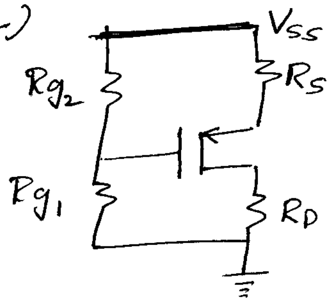
Now, $$V_g = \frac{V_{DD} \cdot R_{g_1}}{R_{g_1} + R_{g_2}} = \frac{(2.2V)(200K)}{200K + 107.7K} = 1.429$$

$I_D = \upsilon_{sat} C_{ox} W_g \cdot (V_{gs} - V_{th}) = (1mA/V)(1.129) = \underline{1.129 mA}$

$V_D = V_{DD} - I_D \cdot R_D = 2V - (1.129mA)(500\Omega) = \underline{1.436 \text{ Volts}}$

$\cancel{V_D} =$ $V_{DS} = 1.436 > (V_{gs} - V_{th}) = (1.3 - 0.3)\,V$

$\Rightarrow$ The device operates in saturation.

c) 

$V_{th} = -0.3$ Volts $\quad V_{SS} = 3.3$ Volts

$v_{sat} C_{ox} W_g = 2mA/V$

$I_{Rg_1} = 1uA \quad V_S = 2.25$ Volts

$I_D = 1mA \quad V_D = 0.5$ Volts

Now,

$V_D = I_D R_D = (1mA)(R_D) \Rightarrow R_D = \frac{V_D}{I_D} = \frac{0.5V}{1mA} = \underline{500\Omega}$

Also,

$V_{SS} - I_D R_S = V_S \quad (\because I_D = I_S)$

$\Rightarrow R_S = \frac{V_{SS} - V_S}{I_D} = \frac{3.3 - 2.25}{1mA} = \underline{1.05 k\Omega}$

Now,

$v_{sat} C_{ox} W_g (-V_{gs} + V_{th}) = I_D$

$\Rightarrow -V_{gs} = \frac{I_D}{v_{sat} C_{ox} W_g} - V_{th} = 0.8 \text{ Volts} = -V_g + V_s$

$\Rightarrow V_g = V_s - 0.8V = 1.45$ Volts

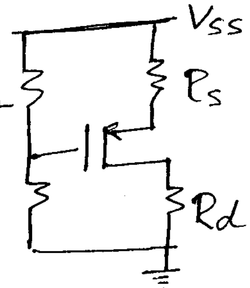
Now, $V_g = I_{Rg_1} \cdot Rg_1 \Rightarrow Rg_1 = \frac{V_g}{I_{Rg_1}} = \frac{1.45}{1uA} = \underline{1.45 M\Omega}$

Also, $\frac{V_{SS}}{Rg_1 + Rg_2} = I_{Rg_1} \Rightarrow Rg_2 = \frac{V_{SS}}{I_{Rg_1}} - Rg_1$

$= \frac{3.3V}{1uA} - 1.45M\Omega$

$Rg_2 = \underline{1.85 M\Omega}$

d.



$R_D = 500\Omega$
$R_S = 1.05k\Omega$
$R_{g_1} = 1.45M\Omega$
$R_{g_2} = 1.85M\Omega$

$V_{SS} = 3.3V$
$V_{th} = -0.3V$

$$\upsilon_{sat} C_{ox} w_g = 2mA/V + 0.2mA/V = 2.2mA/V$$

Now, $$\upsilon_g = \frac{V_{SS} \cdot R_{g_1}}{R_{g_1} + R_{g_2}} = \frac{3.3V(1.45M)}{(3.3M)} = \underline{1.45V}$$

$$I_D = \upsilon_{sat} C_{ox} W_g (-V_{gs} + V_{th}) = \left(2.2\frac{mA}{V}\right)(-V_{gs} - 0.3)$$

$$\therefore I_D = \left(2.2\frac{mA}{V}\right)[-V_g + V_S - 0.3]$$
$$= \left(2.2\frac{mA}{V}\right)[-1.45 + V_{SS} - I_D R_S - 0.3]$$
$$= \left(2.2\frac{mA}{V}\right)[1.55 - I_D R_S]$$

$$\Rightarrow I_D = 3.41mA - 2.31 I_D \Rightarrow I_D = \frac{3.41mA}{3.31} = 1.03mA$$

$$V_D = I_D R_D = (1.03mA)(500) = 0.515 \text{ Volts}$$

$$V_S = V_{SS} - I_D R_S = 3.3 - (1.03mA)(1.05k\Omega)$$
$$V_S = 2.22 \text{ Volts}$$

$$V_{SD} = 1.705 \text{ Volts}$$

$-V_{gs} + V_{th}$

$$V_{dg} = -0.935 < 0.3\ (-V_{th})$$

$\Rightarrow$ The MOSFET is in constant current regime

2d (contd.)

$R_D = 500\Omega$

$R_S = 1.05\,k\Omega$

$R_{g_1} = 1.45\,M\Omega$

$R_{g_2} = 1.85\,M\Omega$

$V_{SS} = 3.3 + 0.33 = 3.63$ Volts

$V_{th} = -0.3$ Volts

$V_{sat}\,C_{ox}\,W_g = 2mA/V$

$$V_g = \frac{V_{SS} \cdot R_{g_1}}{R_{g_1} + R_{g_2}} = \frac{3.63 \times 1.45M}{(1.45M + 1.85M)} = 1.58 \text{ Volts}$$

$$I_D = V_{sat}\,C_{ox}\,W_g\,(-V_{gs} + V_{th}) = \left(\frac{2mA}{V}\right)(-V_{gs} - 0.3)$$

$$I_D = \left(\frac{2mA}{V}\right)(-V_g + V_S - 0.3)$$

$$= \left(\frac{2mA}{V}\right)(-1.58 + V_{SS} - I_D R_S - 0.3)$$

$$= \left(\frac{2mA}{V}\right)(-1.58 + 3.63 - I_D(1.05k) - 0.3)$$

$$= \left(\frac{2mA}{V}\right)(1.75 - I_D(1.05k))$$

$$I_D = 3.5mA - 2.1\,I_D \Rightarrow I_D = \frac{3.5mA}{3.1} = 1.129mA$$

Now, $V_D = I_D R_D = (1.129mA)(500\Omega) = 0.564$ Volts.

$V_S = V_{SS} - I_D R_S = 3.63 - (1.129mA)(1.05k\Omega)$

$= 2.445$ Volts.

Now, $V_{DG} = -1.016 < -V_{th}\ (= 0.3)$

$\Rightarrow$ The device is in constant current regime.