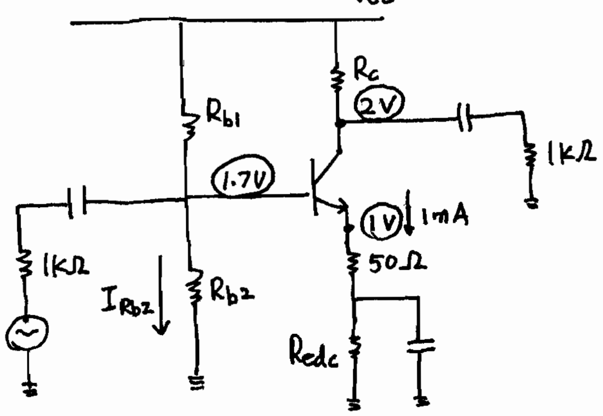
ECE 137A (W07) HW#3

Solutions

P1.

$V_a = 50, \ \beta = 100$



a) Resistor Value.

$$R_c = \frac{V_{cc} - V_c}{I_c} = \frac{5-2}{1\,mA} = \boxed{3\,k\Omega}$$

since $I_c \simeq I_E$

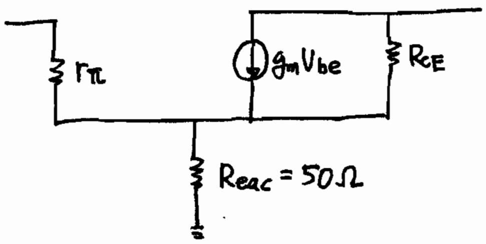
$$V_c = I_c(50 + R_{edc}) \Rightarrow 1V = 1mA\,(50 + R_{edc})$$

$$\Rightarrow \boxed{R_{edc} = 950\,\Omega}$$

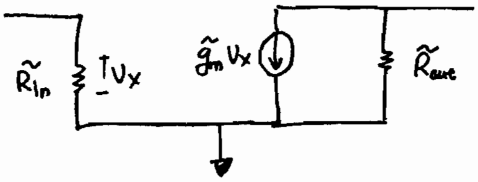
$$I_B = \frac{1}{\beta} I_c = \frac{1\,mA}{100} = 10\,\mu A, \quad I_{Rb2} = 20\,I_B = 200\,\mu A, \quad V_B = 1.7.$$

$$R_{b2} = \frac{1.7}{0.2\,mA} = \boxed{8.5\,k\Omega}, \quad R_{b1} = \frac{V_{cc} - V_B}{I_{Rb2} + I_B} = \frac{5 - 1.7}{21\,I_B} = \boxed{15.7\,k\Omega}$$

b) small signal



$\Rightarrow$ Emitter Degeneration

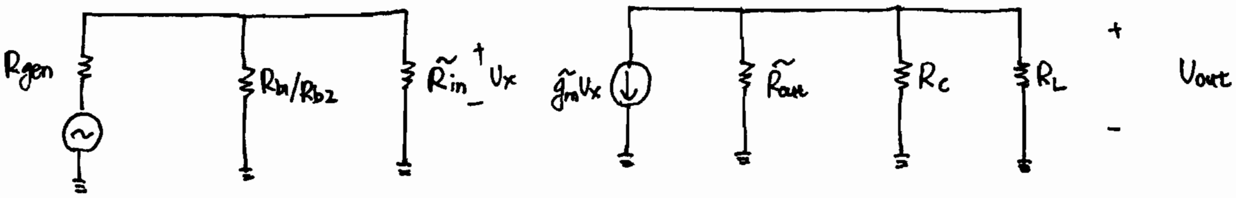


with
$$\tilde{R}_{in} = (1+\beta)(r_e + R_{EAC})$$
$$\tilde{R}_{out} = R_{CE}\left[1 + \frac{R_{EAC}}{r_e}\right]$$
$$\tilde{r}_e = \frac{1}{\tilde{g}_m} = r_e + R_{EAC}$$

$$r_e = \frac{V_T}{I_E} = \frac{26\,mV}{1\,mA} = 26\,\Omega$$

$$R_{CE} = \frac{V_A + V_{CE}}{I_c} = \frac{51}{1mA} = 51\,k\Omega$$

$$\therefore \quad \tilde{R}_{in} = 101 \cdot (26+50) = 7.68\,k\Omega$$
$$\tilde{R}_{out} = 51k\left(1 + \frac{50}{26}\right) = 149\,k\Omega$$
$$\tilde{r}_e = 76\,\Omega \,, \quad \tilde{g}_m = 0.013.$$



$$\frac{V_{out}}{V_{in}} = -\tilde{g}_m(\tilde{R}_{out} \,//\, R_c \,//\, R_L) = -0.013 \cdot 746 = \boxed{-9.7}$$

$$\frac{V_{in}}{V_{gen}} = \frac{(R_{b1} // R_{b2} // R_{in})}{R_{gen} + (R_{b1} // R_{b2} // \tilde{R}_{in})} = \frac{3.2\,K}{1\,K + 3.2K} = \boxed{0.76}$$

$$\frac{V_{out}}{V_{gen}} = \frac{V_{in}}{V_{gen}} \cdot \frac{V_{out}}{V_{in}} = \boxed{-7.4}$$

c) $R_{in} = R_{b1} // R_{b2} // \tilde{R}_{in} = \boxed{3.2k\Omega}$

$R_{out} = \tilde{R}_{out} // R_c = \boxed{2.95k\Omega}$

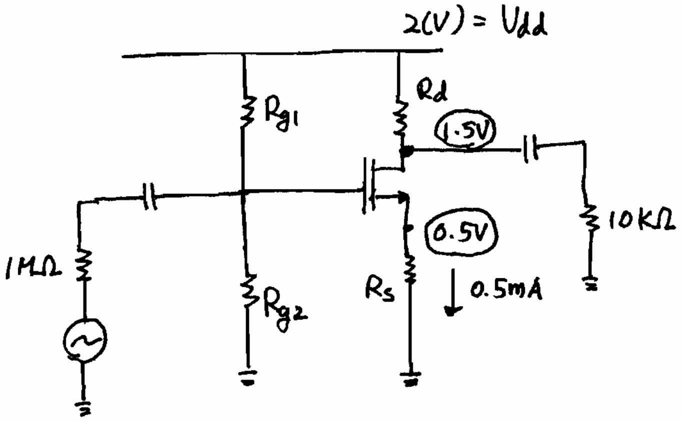
d) saturation.

$V_{CE} = 1\ (V)$ $\therefore \Delta V_{out} \downarrow = 1 - V_{CE(sat)} = 1 - 0.5 = \boxed{0.5\ (V)}$

cutoff

$\Delta V_{out} \uparrow = I_c\ (R_c // R_L) = 1mA\ (0.75k\Omega) = \boxed{0.75\ (V)}$

P2.



$V_{th} = 0.3\,(V)$

$V_{sat} C_{ox} W_g = 1\ mA/V$

$\lambda = \frac{1}{50}$

a) Resistor Value.

$$R_d = \frac{V_{DD} - V_D}{I_D} = \frac{0.5}{0.5mA} = \boxed{1K\Omega}$$

$$R_s = \frac{V_s}{I_D} = \frac{0.5}{0.5mA} = \boxed{1K\Omega}$$

$$I_D = V_{sat} W_g C_{ox} \left(V_g - V_s - V_{th} \right)\left(1 + \lambda V_{DS} \right)$$

$$0.5mA = 1mA/V \left(V_g - 0.5 - 0.3 \right)\left(1 + \frac{1}{50} \right) \Rightarrow V_g = 1.29(V)$$

$$R_{in} = R_{g1} /\!/ R_{g2} = \frac{R_{g1} R_{g2}}{R_{g1} + R_{g2}} = 1M\Omega \quad - ①$$

$$V_g = V_{dd} \frac{R_{g2}}{R_{g1} + R_{g2}} \Rightarrow \frac{R_{g2}}{R_{g1} + R_{g2}} = 0.645 \quad - ②$$

$$R_{g1} = \frac{①}{②} = \boxed{1.55\ M\Omega} \quad - ③$$

plugging ③ into ②, then $\boxed{R_{g2} = 2.81\ M\Omega}$

b. small signal

$$\frac{V_{out}}{V_{in}} = \frac{-R_{Leq}}{R_s + 1/g_m}$$

where $R_{Leq} = R_{in\ drain} // R_d // R_L$

and $R_{in\ drain} = r_{ds}(1 + g_m R_{ss})$

$r_{ds} = (\frac{1}{\lambda} + V_{DS}) / I_D$

$$= \frac{-0.9K}{1K + 1K}$$

$$= \boxed{-0.45}$$

$\Rightarrow$ $r_{ds} = (50 + 1) / 0.5mA = 102K\Omega$

$R_{in,drain} = 102K(1 + (1m)(1K)) = 204K\Omega$

$R_{Leq} = 204K\Omega // 1K\Omega // 10K\Omega = 0.9K\Omega$

$$\frac{V_{in}}{V_{gen}} = \frac{R_{g1} // R_{g2}}{R_{gen} + \underbrace{R_{g1} // R_{g2}}} = \boxed{\frac{1}{2}}$$

$= 1M\Omega$ (from the problem condition)

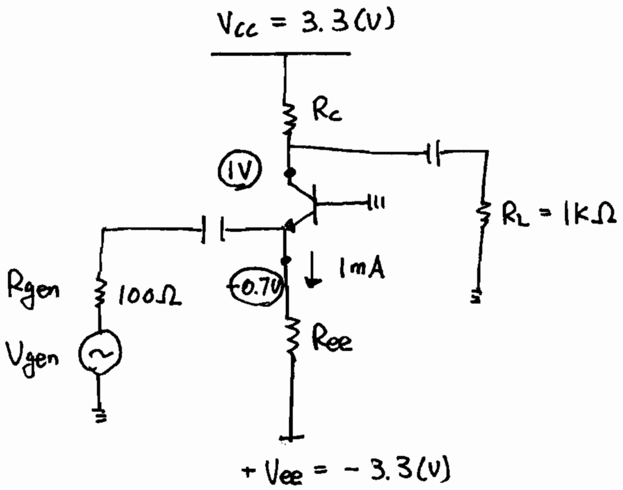
$$\boxed{\frac{V_{out}}{V_{gen}} = -0.23}$$

c. input / output impedance.

$R_{in} = R_{g1} // R_{g2} = \boxed{1 M\Omega}$

$R_{out} = R_d // R_{in,drain} = \boxed{995\Omega}$

3.



$\beta = 50, \quad V_A = 100$

a) Resistor.

$$R_{ee} = \frac{-0.7-(-3.3)}{1\,mA} = \boxed{2.6\,k\Omega}$$

$$I_c = \frac{\beta}{\beta+1} I_E = 0.98\,mA$$

$$R_c = \frac{3.3-1}{0.98\,mA} = \boxed{2.35\,k\Omega}$$

b) small signal

*. Note that Rleq << Rce
so Rin,emitter ~= re + Rb/beta

$$A_v = \frac{V_{out}}{V_{in}} = \frac{R_{Leq}}{R_{in,emitter}} = \frac{R_c /\!/ R_L}{r_e + \cancel{\frac{R_B}{\beta+1}}}$$

where $r_e = \frac{26mV}{I_E} = 26\Omega$

$$= \frac{2.35k /\!/ 1k}{26} = \boxed{27.0}$$

$$\frac{V_{in}}{V_{gen}} = \frac{R_{ee} /\!/ R_{in,emitter}}{R_{gen} + R_{ee} /\!/ R_{in,emitter}} = \frac{25.7}{100+25.7} = \boxed{0.2}$$

$$\frac{V_{out}}{V_{gen}} = \boxed{5.4}$$

c) input, output impedance.

$$R_{in} = R_{ee} \,//\, R_{in,\,emitter} = \boxed{25.7\,\Omega}$$

$$R_{out} = R_c \,//\, R_{in,\,collector}$$

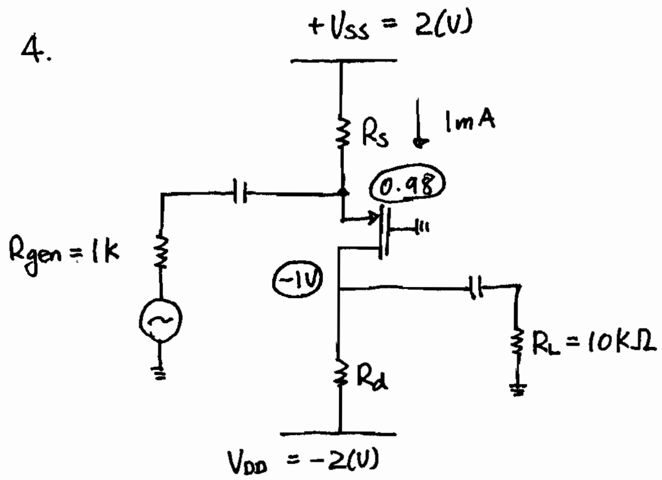
where $R_{in,\,collector} = r_{ce}\left(1 + \frac{R_{gen} // R_{ee}}{r_e}\right)$

$$\left.\begin{aligned} r_{ce} &= \frac{V_a + V_{CE}}{I_c} = 104\,K\Omega \\ r_e &= 26(\Omega) \end{aligned}\right\} \Rightarrow R_{in,\,collector} = 10\,M\Omega$$

$$\therefore \quad R_{out} = 10M \,//\, 2.35K = \boxed{2.35K\Omega}$$

4.

$V_{th} = -0.5(V)$, $\lambda = \frac{1}{50}$, $U_{sat} C_{ox} W_g = 2mA/V$



a) Resistor.

$$R_d = \frac{-1-(-2)}{1mA} = \boxed{1k\Omega}$$

$$I_D = U_{sat} W_g C_{ox} \left(-(V_{gs} - V_{th})\right)\left(1 + \lambda V_{SD}\right)$$

$$\Rightarrow \quad 1mA = 2mA/V \left(-(\underbrace{\cancel{V_g}}_{0} - V_s + 0.5)\right)\left(1 + \frac{1}{50}(V_s - V_D)\right)$$

$$0.5 = (V_s - 0.5)\left(1 + \frac{1}{50}(V_s + 1)\right)$$

solving second order equation : $V_s = \begin{pmatrix} 0.98 \\ \cancel{-51.5} \end{pmatrix}$

$$R_s = \frac{2 - 0.98}{1mA} = \boxed{1.02\,k\Omega} \simeq \boxed{1k\Omega}$$

b) small signal.

$$\frac{V_{out}}{V_{in}} = \frac{R_{Leq}}{R_{in,source}} = \frac{R_D /\!/ R_L}{(1/g_m)} = \boxed{1.82} \qquad R_{in,sourse} = \frac{1}{g_m} = 0.5k\Omega$$

$$\frac{V_{in}}{V_{gen}} = \frac{R_s /\!/ R_{in,source}}{R_s /\!/ R_{in,source} + R_{gen}} = \frac{330}{330 + 1000} = \boxed{0.25}$$

$$\boxed{\frac{V_{out}}{V_{gen}} = 0.46}$$

c) input / output impedance.

$$R_{in} = R_s /\!/ R_{in,source} = \boxed{330\,\Omega}$$

$$R_{out} = R_d /\!/ R_{in,drain}$$

where $R_{in,drain} = r_{ds}\left(1 + \frac{R_s /\!/ R_{gen}}{1/g_m}\right)$

$$r_{ds} = (1/\lambda + V_{DS}) / I_D = (50 + 1.98) / 1mA$$

$$= 52k\Omega$$

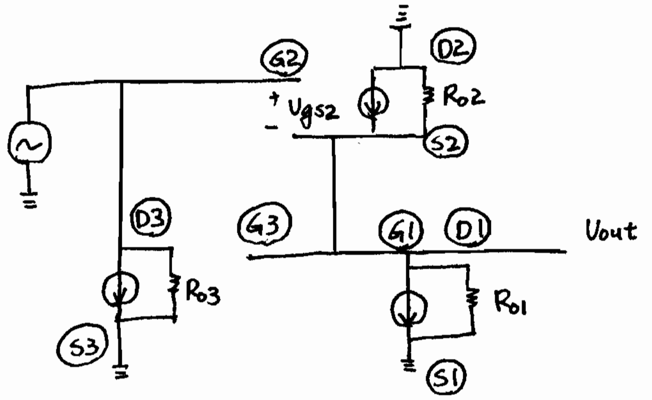
$$\therefore R_{in,drain} = 52K\left(1 + 2mA\,(1k\Omega /\!/ 1k\Omega)\right)$$

$$= 104k\Omega$$

$$\therefore R_{out} = 1k\Omega /\!/ 104k\Omega \simeq \boxed{1k\Omega}$$

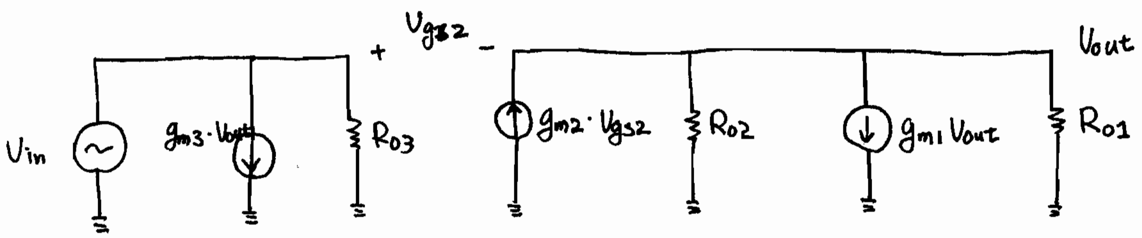
5.

a)



$V_{gs3} = V_{out}$. $V_{gs1} = V_{out}$.

$\Downarrow$



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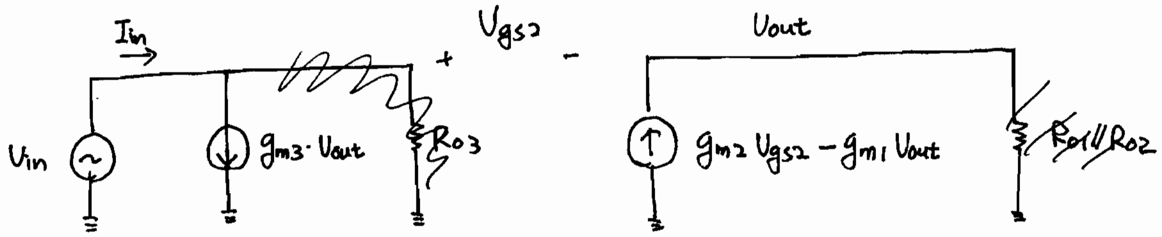

since infinity!

b) ①

$$g_{m2} V_{gs2} - g_{m1} V_{out} = 0 \quad , \quad V_{gs2} = V_{in} - V_{out}$$

$$g_{m2} (V_{in} - V_{out}) - g_{m1} V_{out} = 0$$

$$g_{m2} V_{in} = (g_{m1} + g_{m2}) V_{out} \quad \Longrightarrow \quad \boxed{\frac{V_{out}}{V_{in}} = \frac{g_{m2}}{g_{m1} + g_{m2}}}$$

②

$$I_{in} = g_{m3} \cdot V_{out} \quad , \quad V_{out} = \frac{g_{m2}}{g_{m1} + g_{m2}} \cdot V_{in}$$

$$I_{in} = \frac{g_{m2} g_{m3}}{g_{m1} + g_{m2}} V_{in} \quad \Longrightarrow \quad \boxed{\frac{V_{in}}{I_{in}} = \frac{g_{m1} + g_{m2}}{g_{m2} g_{m3}}}$$

c) numerical

$$\frac{V_{out}}{V_{in}} = \frac{20m}{10m + 20m} = \boxed{\frac{2}{3}}$$

$$\frac{V_{in}}{I_{in}} = R_{in} = \frac{10m + 20m}{(20m)(30m)} = \boxed{50\,\Omega}$$