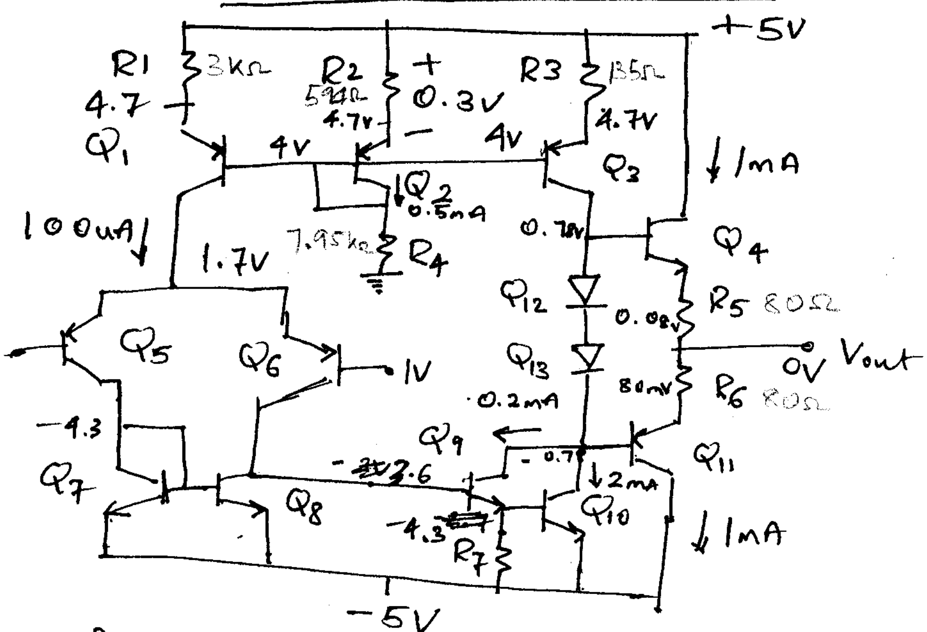
2.) HW #7 SOLUTIONS



DC BIAS DESIGN:

For the O/P stage: $-V_{beQ_{12}} - V_{be_{13}} + V_{be_{11}} + 2(I_{C_4} \cdot R_5) + V_{be_4} = 0$

$$\Rightarrow I_{C_4} \cdot R_5 = V_{be_{12}} - V_{be_4} = V_T \ln\left[\frac{I_{C_{12}}}{I_{S_{12}}}\right] - V_T \ln\left[\frac{I_{C_4}}{I_{S_4}}\right]$$

$$= V_T \ln\left[\frac{I_{C_{12}}}{I_{C_4}} \cdot \frac{I_{S_4}}{I_{S_{12}}}\right]$$

Take $I_{S_{12}} = \frac{1}{10} I_{S_4}$ to get: $I_{C_4} \cdot R_5 = V_T \cdot \ln\left[10 \cdot \frac{I_{C_{12}}}{I_{C_4}}\right]$

(O/P power transistors are typically larger and have a larger I_S)

Now, $I_{C_{12}} = I_{CQ_9} + I_{CQ_{10}} - I_{BQ_{11}}$

$$= 2.2\text{mA} - 5\text{uA} = 2.19\text{mA}$$

$$\Rightarrow I_{C_4} \cdot R_5 = (26\text{mV}) \ln\left[10 \times \frac{2.19\text{mA}}{1\text{mA}}\right] = \underline{80\text{mV}}$$

$$\Rightarrow \boxed{R_5 = 80\Omega}$$

Now, $R_1 = \dfrac{V_{cc} - V_{EQ_1}}{I_{EQ_1}} = \dfrac{(5-4.7)\text{V}}{100\text{uA} \cdot \left(\frac{1}{0.99}\right)} = \boxed{3.03\text{k}\Omega}$

$$R_2 = \frac{0.3\text{V}}{I_{EQ_2}} = \frac{0.3\text{V}}{(0.5\text{mA})\left(\frac{1}{0.99}\right)} = \boxed{594\Omega}$$

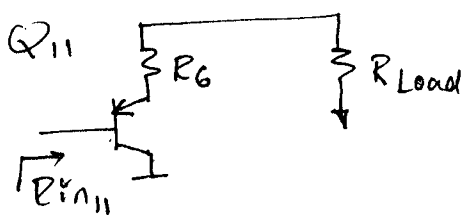
$$R_3 = \frac{V_{CC} - V_{EQ_3}}{I_{EQ_3}} = \frac{(5-4.7)V}{2.2mA \cdot \left(\frac{1}{0.99}\right)} = 135\Omega$$

$$R_4 = \frac{4V}{I_{CQ_2} + I_{BQ_2} + I_{BQ_1}} = \frac{4V}{0.5mA + 2.5uA + 0.5uA} = \boxed{7.952 k\Omega}$$

$$R_5 = \frac{80mV}{1mA} = \boxed{80\Omega = R_6}$$

$$R_7 = \frac{V_{EQ9} - V_{EE}}{I_{E9}} = \frac{-4.3+5}{0.2mA \cdot \left(\frac{1}{0.99}\right)} = \boxed{3.5}$$

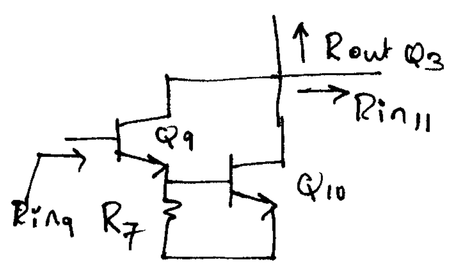
AC Small Signal Analysis:

① 

$$Av_{11} = \frac{R_{Load}}{R_{Load} + R_6 + r_e} = \frac{5k\Omega}{5k\Omega + 80\Omega + 26\Omega}$$

$$\boxed{Av_{11} = 0.979}$$

$$Rin_{11} = (\beta+1)(r_e + R_6 + R_{Load}) = (201)(5106) = \boxed{1.02 M\Omega}$$

② 

$$r_{ce10} = \frac{V_A}{I_{C10}} = \frac{100V}{2mA} = 50k\Omega$$

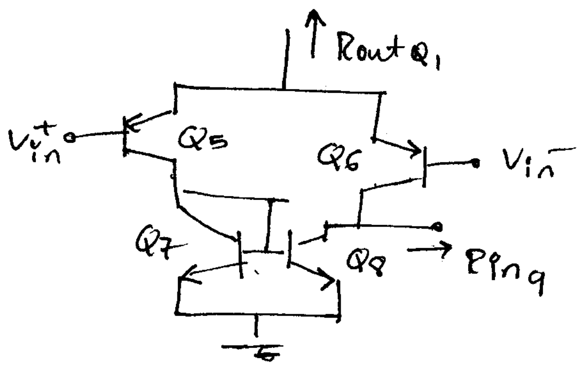
$$Rout_{Q_3} = r_{ce}(1 + g_m \cdot R_{ee})$$
$$= (50k\Omega)\left(1 + \frac{135}{13}\right)$$
$$= 570K$$

$$Av_{9-10} = -\left[Av_9 \cdot \frac{(Rout_3 \| Rin_{11})}{r_{e10}} + \frac{(Rout_3 \| Rin_{11})}{R_7 + r_{e9}}\right]$$
$$= -\left(0.94 \frac{(363k\Omega)}{13} + \frac{363K}{11K+130}\right)$$
$$= -26280$$

$R_{in9} = (\beta+1)(r_e + R_7 \| (\beta+1)(r_{e10}))$

$= (201)(130 + (11.4k) \| (201)(13)) = \boxed{453\,k\Omega}$

3.)



$A_{V8} = \dfrac{R_{in9} \| r_{ce8} \| r_{out6}}{r_{e6}}$

$= \dfrac{453k \| 2M}{520}$

$= \underline{710.27}$

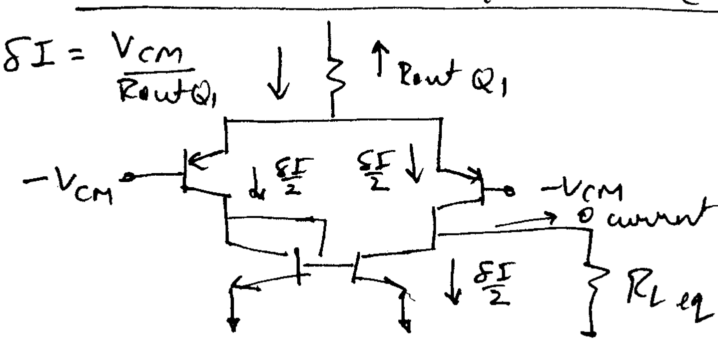
~~r_{ce6}~~ $r_{out6} = r_{ce6}(1 + g_m R_{ee6})$

$= \left(\dfrac{100V}{50uA}\right)\left(1 + \dfrac{\left(\frac{100V}{.1mA}\right)\left(1 + \frac{3000}{260}\right)}{520}\right) = \underline{4.8 \times 10^{10}}$

Total gain $= A_{11} \cdot A_{V9} \cdot A_{V10} \cdot A_{V8} = \boxed{18 \times 10^{6}}$

Common Mode Gain: (Differential Stage):

$\delta I = \dfrac{V_{CM}}{R_{outQ_1}}$



For ideal matched transistors, we see that $A_{CM} = 0 \Rightarrow \underline{CMRR = \infty}$

However in practise there are mismatches in the transistors $Q_5 - Q_6$ & Q_7 & Q_8 giving a finite Common Mode Gain.

Signal swings: (Dominant Factors)

Saturation of Q_{10}: $V_{CEQ} = 4.22V$ $V_{CEsat} = 0.2V$

$\Delta V_{C_{10}} \downarrow = 4.02V$

$\Delta V_{out} \downarrow = (0.979)(4.02V) = 3.93 \downarrow$

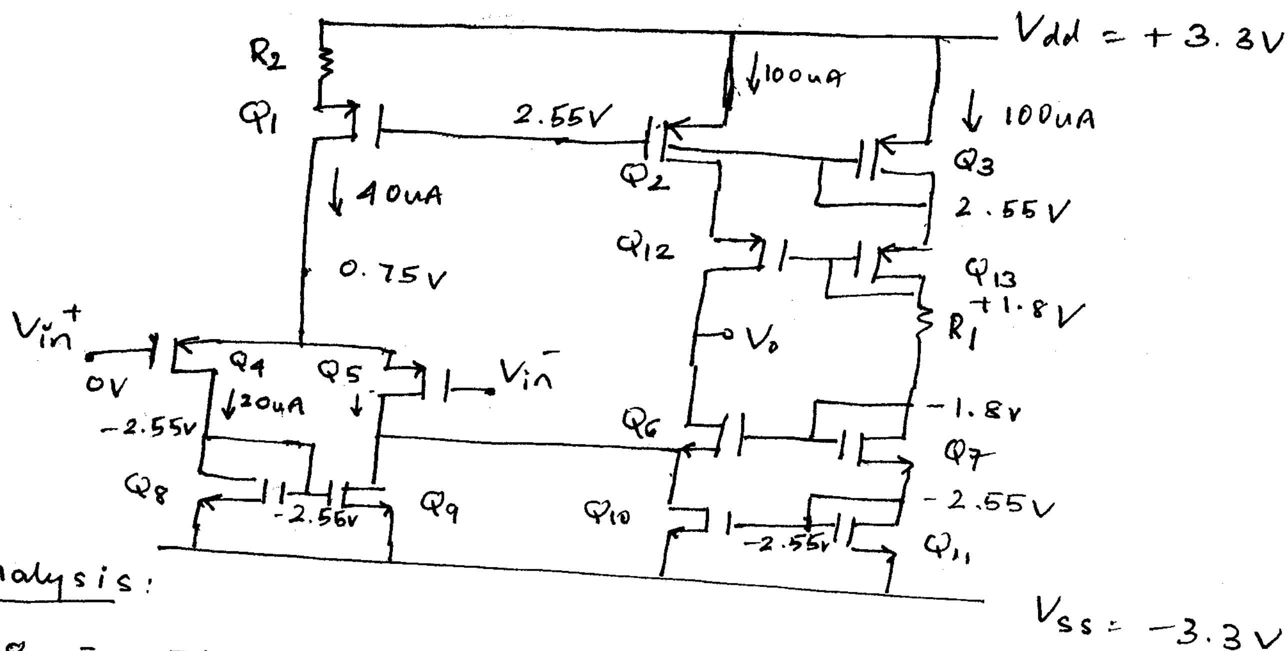
Saturation of Q_3: Q_3 saturation causes it to stop being an ideal current source and it won't be able to give the required drive currents for O/P stage

$\Delta V_{C_3} \uparrow = (3.3 - 0.78)V = 2.52V$

$\Delta V_{out} \uparrow = 2.46V$

The maximum peak to peak swing which this circuit can deliver will be 4.9V ($\pm$ 2.46V)

3)



<u>DC Analysis:</u>

$I_{DQ_4} = I_{DQ_5} = I_{DQ_8} = I_{DQ_9} = 20uA = g_m \cdot |(V_{gs} - V_t)|$

$\Rightarrow g_m = 8 \times 10^{-5} \qquad = g_m\,(0.25V)$

$= V_{sat} \cdot C_{ox} \cdot W_g = \left(0.5 \frac{mA}{V \cdot micron}\right) W_g$

$\Rightarrow W_g = \left(\frac{8 \times 10^{-5}}{0.5 \times 10^{-3}}\right) um = \boxed{0.16\ um}$

$\therefore \underline{W_{g4} = W_{g5} = W_{g8} = W_{g9} = 0.16um}$

$I_{DQ_2} = I_{DQ_{12}} = I_{DQ_6} = I_{DQ_{10}} = 100uA = g_m\,|V_{gs} - V_t|$

$\Rightarrow g_m = 4 \times 10^{-4} = V_{sat} \cdot C_{ox} \cdot W_g \Rightarrow \boxed{W_g = 0.8um}$

Also, since Q_3, Q_{13}, Q_7 & Q_{11} form a current mirror with Q_2, Q_{12}, Q_6 & Q_{10} respectively their currents are the same and hence the widths too.

$\therefore \boxed{W_{g3,13,7,11} = 0.8um}$

) (contd.)

Now, $(V_{gsQ_1} - V_{th})\, g_{mQ_1} = 40uA$ & $\underline{W_{Q_1} = W_{Q_2} = 0.8um}$

$\Rightarrow V_{gsQ_1} = \frac{40uA}{g_{mQ_1}} + V_{thQ_1} = \frac{40uA}{4\times10^{-4}} + (-0.5) = \underline{-0.}$

Now, $V_{gQ_1} = 2.55V \Rightarrow \underline{V_{sQ_1} = 2.95V}$

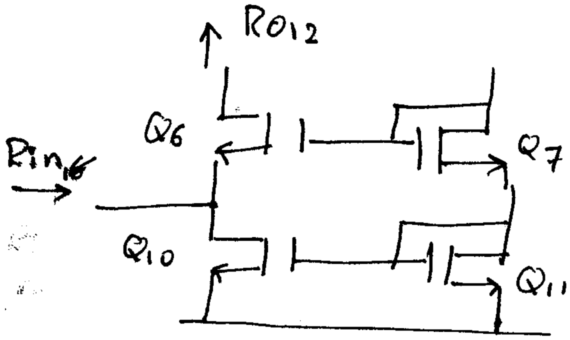
Also, $R_2 = \frac{V_{DD} - V_{SQ_1}}{I_{DQ_1}} = \frac{(3.3-2.95)V}{40uA} = \underline{\underline{8.75k\Omega}}$

Also, $R_1 = \frac{V_{DQ_{13}} - V_{DQ_7}}{I_{D13}} = \frac{3.6V}{100uA} = \underline{\underline{36k\Omega}}$

→ The DC node voltages & currents have been labelled on the schematic. Also note that the DC voltage at the output node V_o is not very well defined and can be assumed to be 0V.

AC small signal Analysis :

1)



$$Rin_6 = Rin_6 \,||\, R_{DS10}$$

$$= \left(\frac{1}{g_{m6}}\right) \,||\, \left(\frac{V_{DS10} + 1/\lambda}{I_D}\right)$$

$$= \left(\frac{1}{4\times10^{-4}}\right) \,||\, \left(\frac{2.55 + 20V}{100\,uA}\right)$$

$$= 2500 \,||\, 225.5 \times 10^{3}$$

$$Rin_6 = \boxed{2472\,\Omega}$$

$$Av_6 = \frac{Rout_{12}}{1/g_{m6}}$$

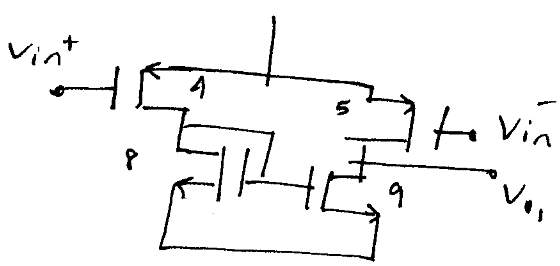
$$= \frac{r_{ds12}(1 + g_m\, r_{in12})}{2.5K}$$

$$= \left(\frac{2.55+20V}{100\,uA}\right)(1 + g_m(r_{ds}))/2.5K$$

$$= \frac{(225K)}{2.5K}(1 + (4\times10^{-4})(225K))$$

$$\boxed{Av_6 = 8190}$$

2)



$$A_D = \frac{R_{eq}}{(1/g_m)} = g_m(Rin_1 \,||\, R_{DS9} \,||\, R_{DS}$$

$$= (8\times10^{-5})(2.4\,k\Omega \,||\, 225\,k\Omega \,||\, 20\,M\Omega)$$

$$= \underline{0.1899}$$

Total Differential gain = $\underline{\underline{1555}}$

As was explained in the previous example, the Common Mode Gain for a Differential Pair with Current Mirror Load is ideally 0 & we need to consider mismatches in the transistors to ~~mea~~ calculate it.

<u>Peak Signal Swings:</u>

1) <u>Knee voltage on Q_6:</u> $V_{dg} > V_{th}$

$\Rightarrow V_d > V_g - V_{th} = -1.8 - 0.5$
$= -2.3V$

$\therefore \boxed{\Delta V_{out} \downarrow = 2.3V}$

2.) <u>Pinch off on Q_5:</u> $\Delta I_0 = 20uA$

$\Delta V = \Delta I_0 \cdot R_{L\,eq} = (20uA)(2.4k\Omega)$
$= 0.048V$

$\Delta V_{out} \uparrow = A_{V_1} \cdot 0.048V = > 6.6V$

$\hookrightarrow$ This doesn't limit the signal swing

3.) <u>Knee voltage on Q_{12}:</u> $V_{Dg_{12}} < V_{th_{12}}$

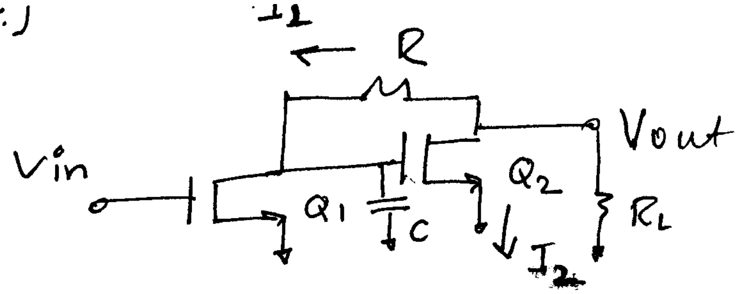
$\Rightarrow V_g \cdot V_{D12} < V_{th_{12}} + V_{g_{12}}$

$\Rightarrow V_{D12} < -0.5 + 1.8 = 1.3V$

$\therefore \boxed{\Delta V_{out} \uparrow = 1.3V}$

$\Rightarrow$ Peak swing possible is <u>2.6Vp-p</u>

4.)



$$V_{out} = -(I_1 + I_2) R_L \quad \text{where}$$

$$I_1 = g_{m1} V_{in}$$

$$I_2 = g_{m2}(V_{gs2}) = g_{m2}(V_{out} - I_1 R)$$
$$= g_{m2}(V_{out} - g_{m1} V_{in} R)$$

$$\therefore \quad V_{out} = -\left(g_{m1} V_{in} + g_{m2}(V_{out} - g_{m1} V_{in} R)\right) R_L$$
$$= -\left[\left(g_{m1} - g_{m1} g_{m2} R\right) V_{in} + g_{m2} V_{out}\right] R_L$$

$$\therefore \quad V_{out}(1 + g_{m2} R_L) = -V_{in} \cdot R_L \, (g_{m1} - g_{m1} \cdot g_{m2} \cdot R)$$

$$\therefore \quad \frac{V_{out}}{V_{in}} = -\frac{R_L(g_{m1} - g_{m1} \cdot g_{m2} \cdot R)}{1 + g_{m2} \cdot R_L}$$

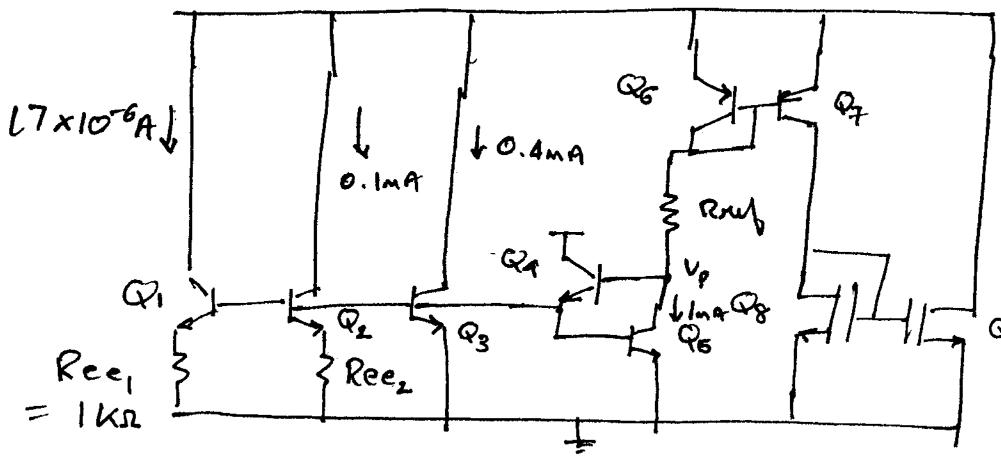
$$= -R_L \cdot g_{m1} \cdot \frac{(1 - g_{m2} \cdot R)}{(1 + g_{m2} R_L)}$$

As R_L becomes very large, we get

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}}{g_{m2}}(1 - g_{m2} \cdot R) = -\frac{20}{10}(1 - (10\,ms)(1k\Omega))$$

$$\underline{A_v = 18 \; V/V}$$

(1)



$V_{cc} = 10V$

$\beta_{NPN} = 200$

$\beta_{PNP} = 10$

$V_T = 0.3V$

$V_{sat} C_{ox} w_g = 2mA/$

$\lambda = 1/100\ V.$

$I_{C5} = 1mA$

$I_{C2} = 0.1mA$

$I_{S6} = 2 I_{S7}$

$I_{S5} = I_{S4}/3 = I_{S3}/4 = I_{S2} = 3 I_{S1}$

Now, $V_{be_3} = V_{be_2} + I_{C_2} R_2$

$\Rightarrow V_T \ln\left[\frac{I_{C_3}}{I_{S_3}}\right] = V_T \ln\left[\frac{I_{C_2}}{I_{S_2}}\right] + I_{C_2} R_2$

$\Rightarrow V_T \ln\left[\frac{I_{C_3}}{I_{C_2}} \cdot \frac{I_{S_2}}{I_{S_3}}\right] = I_{C_2} R_2$

Now, $\frac{I_{S_3}}{I_{S_2}} = 4 \Rightarrow V_T \cdot \ln\left[\frac{1}{4} \cdot \frac{I_{C_3}}{0.1mA}\right] = (0.1mA) R_{ee_2}$

If we assume $I_{C_3} = 0.4mA$ $(4 \cdot I_{C_2})$ we get $\underline{R_{ee} \simeq 0}$ — (1)

Also,

$V_{be_3} = V_{be_1} + I_{C_1} R_{ee_1}$

$\Rightarrow V_T \ln\left[\frac{I_{C_3}}{I_{S_3}}\right] = V_T \ln\left[\frac{I_{C_1}}{I_{S_1}}\right] + I_{C_1} \cdot R_{ee_1}$

$\Rightarrow V_T \ln\left[\frac{0.4mA}{I_{C_1}} \cdot \frac{1}{12}\right] = I_{C_1} \cdot R_{ee_1}$

$\Rightarrow$

$26mV \ln\left[\frac{0.033mA}{I_{C_1}}\right] = I_{C_1} \cdot (1k\Omega)$

$\rightarrow$ This is not a closed form solution

$\Rightarrow$ We need to use an ~~int~~ iterative

~~appro~~

approach to find the solution.
Alternatively we can also use the Newton's Method to find roles of non-closed form equations like these.

$I_{S1} \approx 10^{-7}$

Using which we get $\underline{I_{C1} = 17 \times 10^{-6} A}$.

$\Rightarrow V_{B1} = I_{C1} R_{ee1} + V_T \ln\left[\frac{I_{C1}}{I_{S1}}\right] = \underline{0.15 V}$

Now, $I_{C4} = I_{B1} + I_{B2} + I_{B3} + I_{B5} + I_{B4}$

$= \frac{17uA + 0.1mA + 0.4mA + 1mA + I_{C4}}{200}$

$\therefore = \frac{1.4 mA + I_{C4}}{200}$

$\Rightarrow \boxed{I_{C4} = 7uA}$

$I_{C6} = I_{B4} + I_{C5} = \underline{1mA}$

$V_{BE4} = V_T \ln\left[\frac{I_{C4}}{I_{S4}}\right] = 26mV . \ln\left[\frac{7uA}{9 \times 10^{-7}}\right] \approx 53mV$

$\Rightarrow \underline{V_p = 200mV}$

$V_{BE6} = V_T \ln\left[\frac{I_{C6}}{I_{S6}}\right] \approx 0.210V$

(Assuming $I_{S6} \approx 3 \cdot I_{S1} = I_{S5}$)

$\Rightarrow R_{ref} = \frac{9.78 - 0.2}{0.1mA} = \underline{95 k\Omega}$

Now since $V_{be\,Q6} = V_{be\,Q7}$ & $I_{S7} = \frac{I_{S6}}{2}$

$\Rightarrow I_{C7} = \frac{I_{C6}}{2} = 0.05\,mA = I_{D8}$

$I_{D8} = g_m (V_{gs} - V_t)(1 + \lambda V_{gs})$ $\quad (\because V_{gs} = V_{ds})$

$\frac{0.05\,mA}{2\,mA/V} = (V_{gs} - 0.3)(1 + \frac{V_{gs}}{100})$

$0.025 = V_{gs} - 0.003\,V_{gs} + \frac{V_{gs}^2}{100} - 0.3$

$0 = 0.997\,V_{gs} + \frac{V_{gs}^2}{100} - 0.325$

$\Rightarrow \boxed{V_{gs_8} = 0.3249\,V} = V_{gs_9}$

$I_{D9} = g_m (V_{gs} - V_t)(1 + \lambda V_{DS9})$

$= \left(\frac{2\,mA}{V}\right)(0.0249)(1 + 0.1)$

$\boxed{I_{D9} = 0.0547\,mA}$