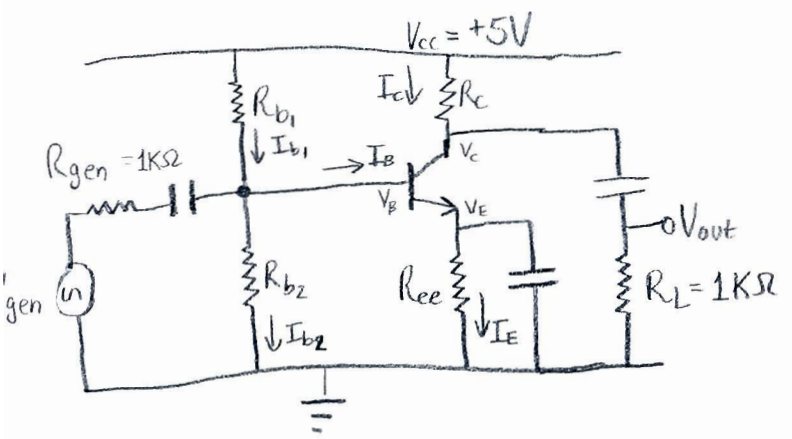
ECE 137A Homework #2
Winter '07

① $\beta = 100$, $V_{cc} = +5V$, $R_L = 1K\Omega$, $R_{gen} = 1K\Omega$

$V_A = 100V$, $V_{ce_{sat}} = 0.5V$, $V_E = 1V$, $V_c = 3V$,

$I_c = 1mA$, $I_{b_2} = 0.1mA$, $V_{BE} = 0.7V$ (on)



a) Find $R_{b_1}, R_{b_2}, R_c, R_{ee}$?

$I_c = \frac{V_{cc} - V_c}{R_c} \Rightarrow \frac{5V - 3V}{R_c} = 1mA \Rightarrow \boxed{R_c = 2K\Omega}$

$I_{b_2} = \frac{V_B}{R_{b_2}}$ ... $V_B - V_E = 0.7V \rightsquigarrow V_B = 1.7V$

$0.1mA = \frac{1.7V}{R_{b_2}} \Rightarrow \boxed{R_{b_2} = 17K\Omega}$

$I_E = \frac{V_E}{R_{ee}}$ w/ $\frac{I_c}{I_E} = \frac{\beta}{\beta+1} \Rightarrow \frac{100}{101} = \frac{1mA}{I_E}$

$1.01mA = \frac{1V}{R_{ee}} \Rightarrow \boxed{R_{ee} = 990.1\Omega}$

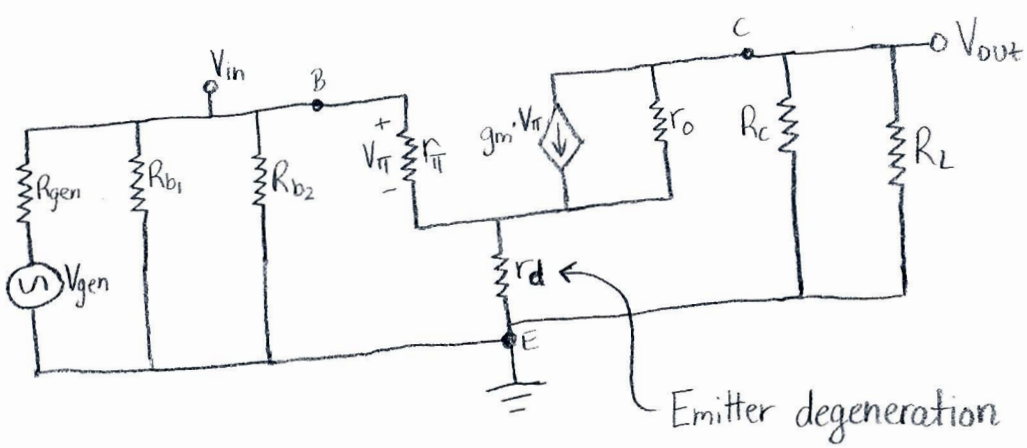
$I_{b_1} = \frac{V_{cc} - V_B}{R_{b_1}}$ & $I_{b_1} = I_{b_2} + I_B \Rightarrow I_{b_1} = 0.1mA + I_B$ w/ $I_B = \frac{I_c}{\beta} = 0.01mA$

$\rightsquigarrow I_{b_1} = 0.11mA = \frac{5V - 1.7V}{R_{b_1}} \Rightarrow \boxed{R_{b_1} = 30K\Omega}$

b) $g_m = \frac{I_c}{V_t} = \frac{1mA}{25.8mV} \approx \boxed{0.03876\,S}$ $\quad r_e = \frac{V_t}{I_E} = \frac{25.8mV}{1.01mA} \approx \boxed{25.5\Omega}$

$r_o = \frac{V_{CE} + V_A}{I_C} = \frac{3V - 1V + 100V}{1mA} \approx \boxed{102K\Omega}$ $\quad r_\pi = \frac{\beta}{g_m} \approx \boxed{2.58K\Omega}$

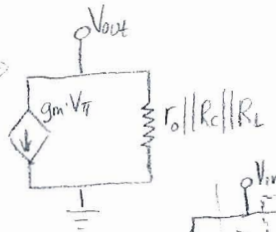
c)



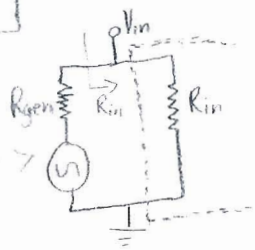
(1) d) $R_{in} = R_{b_1} \| R_{b_2} \| (\beta+1)\cdot r_e$

$\Rightarrow \boxed{R_{in} \approx 2.081\ K\Omega}$

$V_{in} = V_\pi$ & $V_{out} = -g_m \cdot V_\pi \cdot (r_o \| R_c \| R_L)$



$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{-g_m \cdot V_\pi \cdot (r_o \| R_c \| R_L)}{V_\pi} \approx \boxed{-25.7}$



$V_{in} = V_{gen} \cdot \frac{R_{in}}{R_{in}+R_{gen}} \Rightarrow \frac{V_{in}}{V_{gen}} = \frac{R_{in}}{R_{in}+R_{gen}} \approx \boxed{0.675}$

$\frac{V_{out}}{V_{gen}} = \frac{V_{out}}{V_{in}} \cdot \frac{V_{in}}{V_{gen}} = -25.7 \cdot 0.675 \approx \boxed{-17.36}$

e) $I_{C_Q} = 1mA$, $I_{C_{min}} = 0mA$, $\rightarrow \Delta I_{C_{max}} = 1mA$, $\Rightarrow \Delta V_{out_{max}} = \Delta I_{C_{max}} \cdot (r_o \| R_c \| R_L) \approx 0.66\ V$

$V_{CE_Q} = 2V$, $V_{CE_{min}} = V_{CE_{sat}} = 0.5V$, $\Delta V_{CE_{max}} = 1.5V$, $\rightarrow \Delta V_{out_{max}} = 1.5V$

So, for $\Delta V_{out_{max}}$ smaller than cutoff $\Rightarrow V_{p\text{-}p_{max}} = 2 \cdot \Delta V_{out_{max}} = 2 \cdot 0.66V \approx \boxed{1.32\ V_{p\text{-}p}}$

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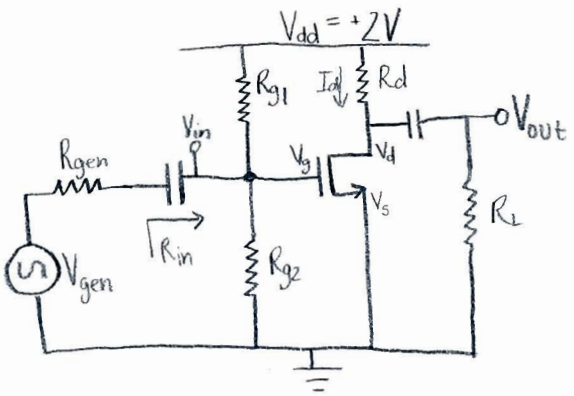
(2) $V_{sat} \cdot C_{ox} \cdot W_g = 10\ mA/V$, $V_{th} = 0.3V$, $\lambda = 0.04\ V$, $V_{dd} = 2V$, $R_{gen} = 50\ K\Omega$, $R_L = 10\ K\Omega$,

$V_D = 1V$, $I_d = 2mA$, $R_{in} = 1\ M\Omega$



a) Find R_{g_1}, R_{g_2}, & R_d?

$I_d = \frac{V_{dd} - V_D}{R_d} \Rightarrow R_d = \frac{2V-1V}{2mA} \approx \boxed{500\Omega}$

$I_d = V_{sat} \cdot C_{ox} \cdot W_g \cdot (V_g - V_{th})$

$\Rightarrow 2mA = 10mA/V \cdot (V_g - 0.3V) \Rightarrow V_g \approx 0.5V$ & $V_g = V_{dd} \cdot \left(\frac{R_{g2}}{R_{g_1}+R_{g2}}\right)$ & $R_{in} = R_{g1} \| R_{g2}$

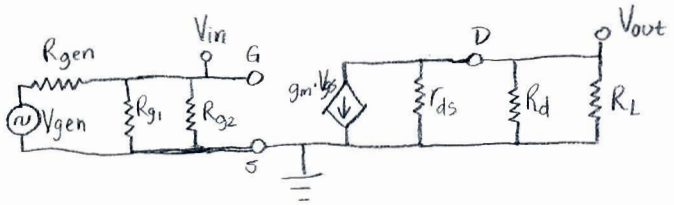
$\Rightarrow \boxed{R_{g1} = 4\ M\Omega}$ & $\boxed{R_{g2} = 1.\overline{3}\ M\Omega}$

b) Find g_m & r_{ds}?

$g_m = V_{sat} \cdot C_{ox} \cdot W_g \Rightarrow \boxed{g_m = 10\ mS}$

$r_{ds} = \frac{1}{\lambda \cdot I_D} \Rightarrow \boxed{r_{ds} = 12.5\ K\Omega}$

(2) c)



d) R_{in} is given as $1M\Omega$

$V_{in} = V_g$ & $V_{out} = -g_m \cdot V_g \cdot (r_{ds} \| R_d \| R_L) \Rightarrow \frac{V_{out}}{V_{in}} = -g_m \cdot (r_{ds} \| R_d \| R_L) \approx \boxed{-4.587}$

$V_{in} = V_{gen} \cdot \left(\frac{R_{in}}{R_{gen}+R_{in}}\right) \Rightarrow \frac{V_{in}}{V_{gen}} = \frac{R_{in}}{R_{gen}+R_{in}} \approx \boxed{0.952}$

$\frac{V_{out}}{V_{gen}} = \frac{V_{out}}{V_{in}} \cdot \frac{V_{in}}{V_{gen}} = -4.587 \cdot 0.952 \approx \boxed{-4.369}$

e) Max -ve swing $\rightarrow V_{ds} = V_g - V_{th} = 0.5V - 0.3V = \underline{0.2V}$ ... $\Delta V_{out_{max}} = V_{ds_Q} - V_{ds} = 1V - 0.2V = \underline{0.8V}$

Max +ve swing $\rightarrow \Delta I_{D_{max}} = 2mA$ & $\Delta V_{out_{max}} = \Delta I_{D_{max}} \cdot (r_{ds} \| R_D \| R_L) \approx \underline{0.917V}$

$\Rightarrow$ Using -ve swing (b/c it's smallest) ... $V_{p\text{-}p_{max}} = 2 \cdot 0.8V = \boxed{1.6V}$

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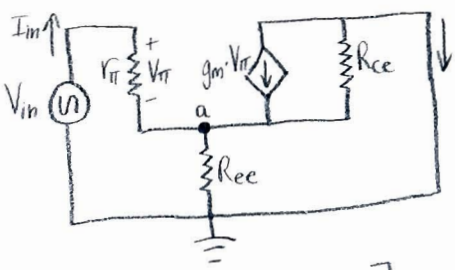
(3)



a) $\underline{V_\pi = I_{in} \cdot r_\pi}$ $\quad V_{in} = V_\pi + V_a \rightarrow V_{in} - V_\pi = V_a$

$\rightarrow \underline{V_a = V_{in} - I_{in} \cdot r_\pi}$

$I_{in} = \frac{V_a}{R_{ee} \| R_{ce}} - g_m V_\pi \rightarrow I_{in} = \frac{V_{in} - I_{in} \cdot r_\pi}{R_{ee} \| R_{ce}} - g_m \cdot I_{in} \cdot r_\pi$

$\Rightarrow I_{in} \cdot \left[\frac{r_\pi}{R_{ee} \| R_{ce}} + g_m \cdot r_\pi + 1\right] = \frac{V_{in}}{R_{ee} \| R_{ce}} \Rightarrow \frac{V_{in}}{I_{in}} = r_\pi + (R_{ee} \| R_{ce}) \cdot (g_m r_\pi + 1) = \boxed{r_\pi + (R_{ee} \| R_{ce})(\beta+1)}$

$I_{out} = -g_m \cdot V_\pi + \frac{V_a}{R_{ce}} = -g_m \cdot I_{in} \cdot r_\pi + \frac{V_{in} - I_{in} \cdot r_\pi}{R_{ce}} \Rightarrow I_{out} \cdot R_{ce} = V_{in} - I_{in} \cdot r_\pi \cdot [g_m R_{ce} + 1]$

$\rightarrow I_{out} \cdot R_{ce} = V_{in} - \frac{V_{in} \cdot r_\pi}{R_{in}} \cdot [g_m \cdot R_{ce} + 1] \Rightarrow I_{out} \cdot R_{ce} = V_{in} - \frac{V_{in} \cdot r_\pi \cdot [g_m \cdot R_{ce} + 1]}{r_\pi + (R_{ee} \| R_{ce})(\beta+1)}$

$\rightarrow I_{out} \cdot R_{ce} \cdot \left[r_\pi + (R_{ee} \| R_{ce})(\beta+1)\right] = V_{in} \cdot \left[r_\pi + (R_{ee} \| R_{ce})(\beta+1) - r_\pi \cdot (g_m \cdot R_{ce} + 1)\right]$

$\rightarrow I_{out} \cdot R_{ce} \cdot \left[r_\pi + (R_{ee} \| R_{ce})(\beta+1)\right] = V_{in} \cdot \left[\cancel{r_\pi} + \frac{R_{ee} \cdot R_{ce} \cdot (\beta+1)}{R_{ee} + R_{ce}} - \beta \cdot R_{ce} - \cancel{r_\pi}\right]$

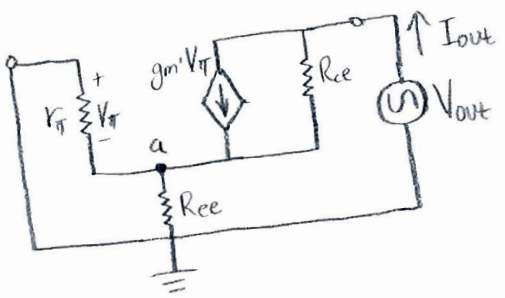
$\rightarrow I_{out} \cdot R_{ce} \cdot \left[r_\pi + (R_{ee} \| R_{ce})(\beta+1)\right] = V_{in} \cdot \left[\frac{R_{ee} \cdot R_{ce} \cdot (\beta+1) - \beta \cdot R_{ce}(R_{ee} + R_{ce})}{R_{ee} + R_{ce}}\right]$

$\rightarrow \frac{I_{out}}{V_{in}} = \left[\frac{R_{ee} \cdot \beta + R_{ee} - \beta \cdot R_{ee} - \beta \cdot R_{ce}}{R_{ee} + R_{ce}}\right] \cdot \left[\frac{r_\pi \cdot (R_{ee} + R_{ce}) + R_{ee} \cdot R_{ce} \cdot (\beta+1)}{R_{ee} + R_{ce}}\right]^{-1}$

3) con'd...

$$\rightarrow \boxed{\frac{I_{out}}{V_{in}} = \frac{R_{ee} - \beta \cdot R_{ce}}{r_\pi \cdot (R_{ee} + R_{ce}) + R_{ee} \cdot R_{ce} \cdot (\beta + 1)}}$$

b)



$V_\pi = -V_a$ $\quad V_a = I_{out} \cdot (R_{ee} \| r_\pi)$

$$I_{out} = \frac{V_{out} - V_a}{R_{ce}} + g_m \cdot V_\pi$$

$$\rightarrow I_{out} = \frac{V_{out}}{R_{ce}} - \frac{I_{out} \cdot (R_{ee} \| r_\pi)}{R_{ce}} - g_m \cdot I_{out} \cdot (R_{ee} \| r_\pi)$$

$$\rightarrow I_{out} \cdot \left[R_{ce} + (R_{ee} \| r_\pi) \cdot (1 + g_m \cdot R_{ce}) \right] = V_{out}$$

$$\Rightarrow \boxed{\frac{V_{out}}{I_{out}} = R_{ce} + (R_{ee} \| r_\pi) \cdot (1 + g_m \cdot R_{ce})}$$