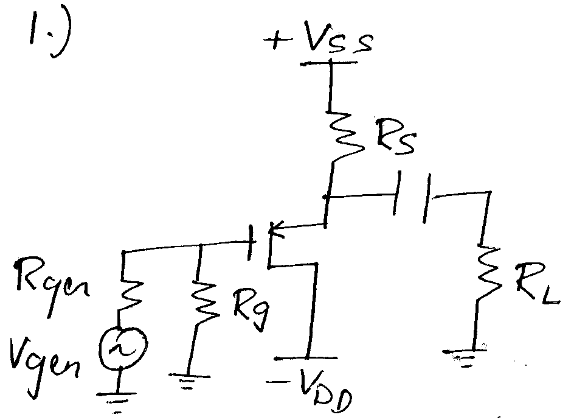
ECE 137A

HW # 4 SOLUTIONS

1.)



$V_t = -0.5\,\text{Volt}$

$v_{sat} C_{ox} W_g = 10\,\text{mA/V}$

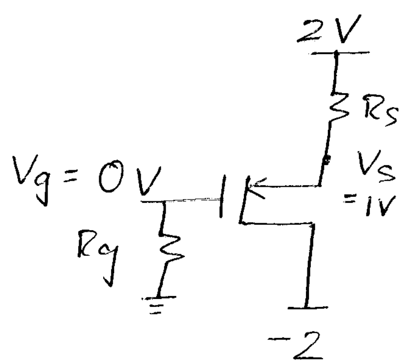
$V_{SS} = -V_{DD} = 2V$

$I_S = 5\,\text{mA}$

$R_g = 10\,\text{M}\Omega$ $\quad R_{gen} = 1\,\text{M}\Omega$

$R_L = 100\,\Omega$

DC Analysis:



$I_S = V_{sat} C_{ox} W_g [-(V_{gs} - V_t)]$

$\Rightarrow 5\,\text{mA} = (10\,\text{mA/V})(-V_{gs} + V_t)$

$\Rightarrow V_{gs} = -0.5 + V_t$

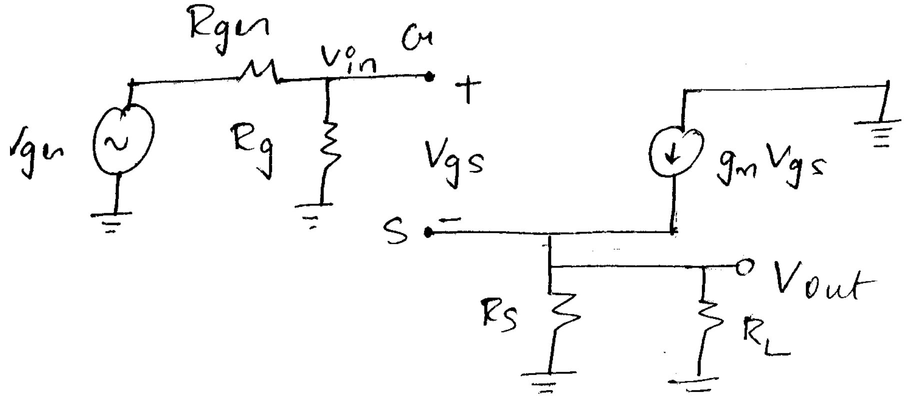
$\Rightarrow V_{gs} = -0.5 - 0.5 = \underline{-1V}$

Now, $V_g = 0V \Rightarrow \underline{\underline{V_s = +1V}}$

$I_S = 5\,\text{mA} \Rightarrow \dfrac{V_{SS} - V_S}{R_S} = 5\,\text{mA} \Rightarrow R_S = \left(\dfrac{V_{SS} - V_S}{5\,\text{mA}}\right)$

$\Rightarrow R_S = \dfrac{2 - (+1)}{5\,\text{mA}} = \dfrac{1V}{5\,\text{mA}} = \boxed{200\,\Omega}$

AC Small Signal Analysis:



From the small signal model,

$$-V_{gs} = (V_{out} - V_{in})$$

$$V_{out} = g_m V_{gs} (R_S \| R_L)$$

$$= g_m (V_{in} - V_{out}) (R_S \| R_L)$$

$$V_{out} (1 + g_m (R_S \| R_L)) = g_m V_{in} \cdot (R_S \| R_L)$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{g_m (R_S \| R_L)}{1 + g_m (R_S \| R_L)} = \frac{R_S \| R_L}{\frac{1}{g_m} + R_S \| R_L}$$

Now, $g_m = 10\,mA/V$

$$R_S \| R_L = 200\Omega \| 100\Omega$$

$$= 66.67$$

$$A_V = \frac{66.67}{100 + 66.6} = \underline{0.399}$$

Now, $V_{in} = V_{gen} \cdot \left(\frac{R_g}{R_g + R_{gen}}\right)$

$= V_{gen}\left(\frac{10e6}{10e6 + 1e6}\right)$

$= V_{gen}\,(0.909)$

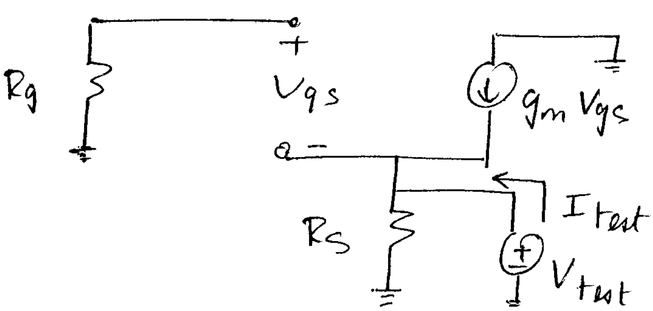
$\Rightarrow \frac{V_{in}}{V_{gen}} = 0.909$

Now, $\frac{V_{out}}{V_{gen}} = \frac{V_{out}}{V_{in}} \cdot \frac{V_{in}}{V_{gen}} = (0.399)(0.909)$

$\therefore \boxed{\frac{V_{out}}{V_{gen}} = 0.362}$

Input impedance $\underline{Z_{in} = R_g = 10 M\Omega}$

<u>Output impedance:</u>



here, $V_{gs} = -V_{test}$

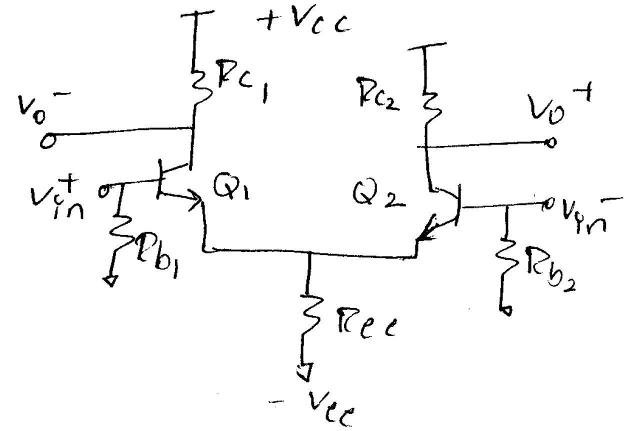
By applying KCL at output node we get

$g_m V_{gs} + I_{test} \mp \frac{V_{test}}{R_s} = 0$

i.e. $\left(-g_m \mp \frac{1}{R_s}\right) V_{test} + I_{test} = 0 \Rightarrow \frac{V_{test}}{I_{test}} = \frac{1}{g_m + \frac{1}{R_s}}$

$Z_{out} = \frac{R_s/g_m}{R_s + \frac{1}{g_m}} = R_s \| \frac{1}{g_m} = 200 \| 100 = \underline{\underline{66.67\Omega}}$

4)



$V_{CC} = 10V$

$V_{ee} = -10V$

From datasheet

For 2N3904 BJT,

$\beta = 130$ $\quad R_{CE} = 66.6\,k\Omega$

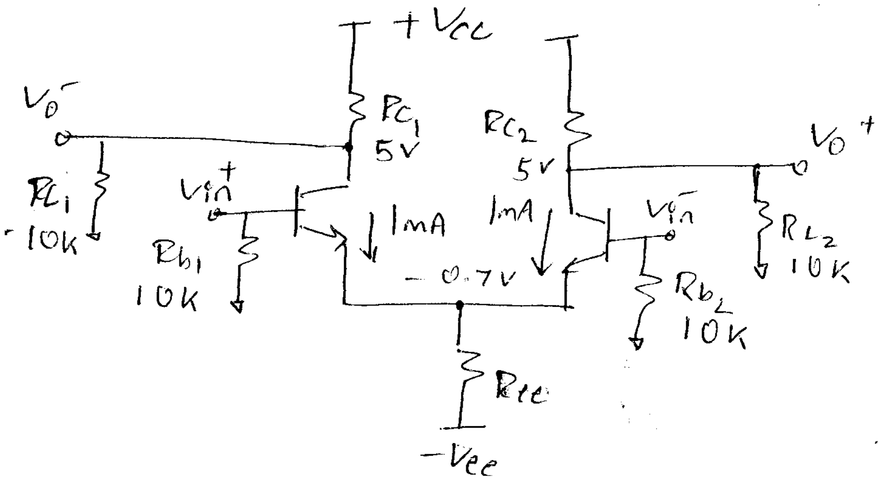
$I_{E_1} = I_{E_2} = 1mA$

$V_{C_1} = V_{C_2} = 5V$

$R_{b_1} = R_{b_2} = 10\,k\Omega$

$R_{L_1} = R_{L_2} = 10\,k\Omega$

DC Analysis:



$I_C = \alpha I_E$ where

$\alpha = \dfrac{\beta}{\beta+1} = 0.99$

$\Rightarrow I_C = 0.992mA$

Now, $I_{RC_1} = I_{C_1} + \dfrac{V_C}{R_L}$

$\therefore I_{RC_1} = 0.992mA + 0.5mA$

$I_{RC_1} = 1.49mA$

$$R_{C_1} = \frac{V_{CC} - V_C}{I_{RC_1}} = \left(\frac{10 - 5V}{1.49mA}\right) = \underline{3.351\,k\Omega}$$

Also,

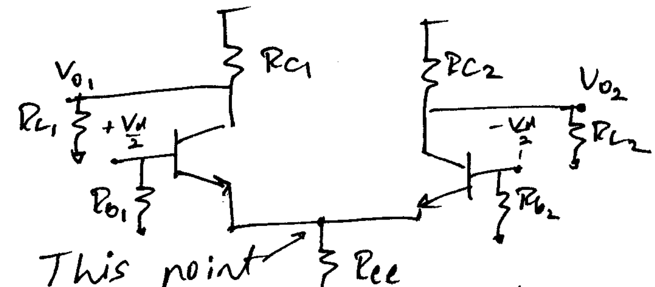
$$V_E - 2 I_E R_{EE} = V_{ee}$$

$$\Rightarrow R_{ee} = \frac{V_E - V_{ee}}{2 I_E} = \frac{-0.7 + 10}{2\,(1mA)} \Rightarrow \boxed{R_{EE} = 4.65\,k\Omega}$$

AC Small Signal Analysis:

Differential input impedance: $\left(\frac{V_d/2}{I_{in}}\right) = (\beta+1)r_e \,||\, R_{b1}$

here $r_e = \frac{1}{g_m} = 26\,\Omega$



$R_{in}(d) = 2\,[131(26) \,||\, 10K]$

$= 5.08\ K\Omega$

$A_D = \frac{V_{out_1}}{V_d} = 2\frac{1}{2}\left(\frac{R_{Leq}}{r_e}\right)$ where $R_{Leq} = R_{C1} \,||\, R_L \,||\, R_{CE} = \cancel{3.19K}$ $\underline{2.4K}$

$= \frac{2.4K}{2 \times 26} = \underline{46.6}$

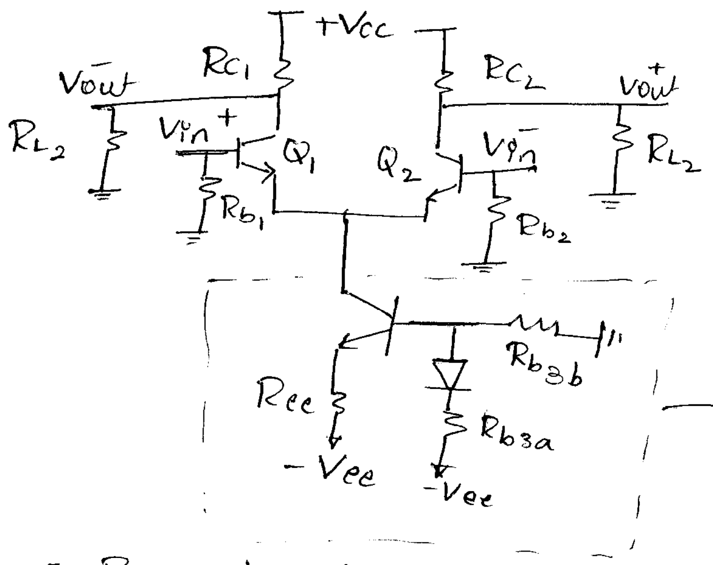
$CMRR = \frac{R_{ee}}{r_e} = \frac{4.65K}{26} = 179.5$

$20 \log (CMRR) = \underline{45.08\,dB}$

$CMRR = \frac{A_d}{A_{CM}}$ where

A_{CM} = Common Mode gain $= \frac{-R_{Leq}}{2R_{EE} + r_e} \approx \frac{-R_{Leq}}{2R_{EE}}$

3.)



The bias currents and bias voltages on the differential pair are the same as before.

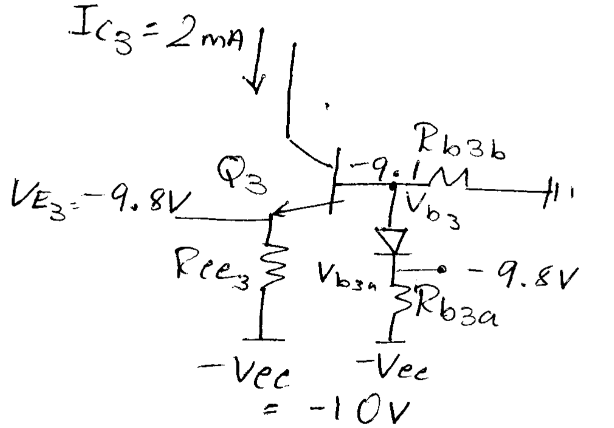
Design of the Biasing circuit is done as follows:

$$R_{C_1} = R_{C_2} = \frac{V_{CC} - V_C}{I_C} = \underline{5.05\,k\Omega}$$

Using the values for the 2n3904 BJT's:

$\beta_3 = 130 \Rightarrow \alpha = 0.99$

$I_E = \frac{I_C}{\alpha} = \underline{\underline{2.02\,mA}}$



$I_{E_3} \cdot Ree_3 = 0.2V$

$\Rightarrow Ree_3 = \frac{0.2V}{2.02mA} = \underline{\underline{99\Omega}}$

Now, assuming drop across the diode to be 0.7 we get

$V_{E_3} = (-10 + 0.2)V = -9.8V$

$V_{b_3} = (-9.8 + 0.7)V = -9.1V$

$V_{b3a} = (-9.1 - 0.7)V = -9.8V$

$\Rightarrow (R_{b3a}) \cdot \left[\frac{1}{10} I_{EQ_3}\right] = (+10 + (\mp 9.8))$

$\Rightarrow R_{b3a} = \frac{0.2 \times 10}{2.02mA} = 990\Omega$

~~Now,~~ ~~Rout collector~~ Q_3 =

$$R_{b3b} = \frac{-V_{b3}}{I_{b3} + I_{Rb3a}} = \frac{9.1}{0.215\,mA} = \underline{\underline{41.8\,k\Omega}}$$

Now, $R_{in\ collector\ Q_3} = r_{ce_3}\left(1 + g_{m_3} R_{EE_3}\left(\frac{r_{be}}{r_{be} + R_B + R_{ee_3}}\right)\right)$

where $r_{ce_3} = 66.6\,k\Omega$ $\quad R_{EE_3} = 99\,\Omega$

$$g_{m_3} = \frac{I_E}{V_T} = \frac{2.02\,mA}{26\,mV} = 0.077\,\mho$$

$$r_{be} = \beta r_e = 1688\,\Omega$$

$$R_B = (R_{diode} + R_{b3a}) \,||\, R_{b3b}$$

$$= \left(\frac{26\,mV}{.202\,mA} + 990\right) || \ 41.8\,k\Omega$$

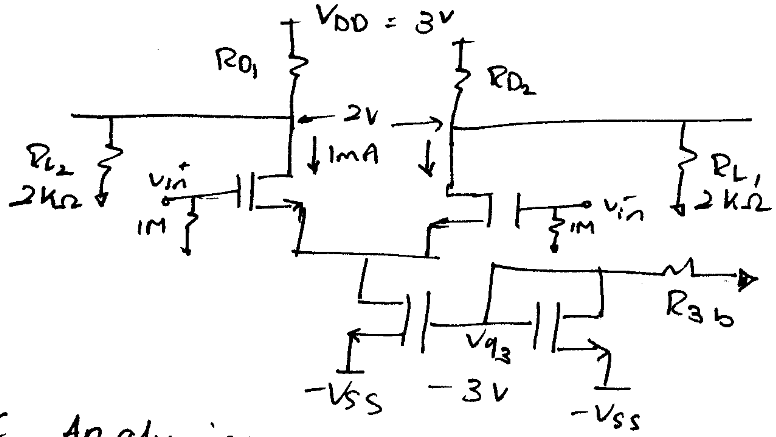
$$= 1089\,\Omega$$

$$R_{in\ collector\ Q_3} = (66.6\,k\Omega)\left(1 + 0.077(99)\left(\frac{1688}{1688 + 1089 + 99}\right)\right)$$

$$= \underline{\underline{364.6\,k\Omega}}$$

Now, $CMRR = \frac{R_{in\ collector\ Q_3}}{r_e} = \frac{364.6\,k\Omega}{26} = \underline{\underline{14 \times 10^3}}$

We thus see that constant current biasing helps to give a higher CMRR on the differential pair.



$V_T = 0.3V$

$V_{sat} C_{ox} W_g = 2mA/V$

$I_{D1} = I_{D2} = 1mA$

$R_{L1} = R_{L2} = 2k\Omega$

$R_{g1} = R_{g2} = 1M\Omega$

DC Analysis:

$$I_{RD1} = I_{D1} + \frac{V_D}{R_{L1}} = 1mA + \frac{2V}{2k\Omega} = 2mA$$

Now, $R_{D1} = \frac{V_{DD} - V_D}{I_{RD1}} = \frac{3-2}{2mA} = \boxed{500\Omega} = R_{D2}$

Now, $I_{DQ3} = V_{sat} C_{ox} W_g (V_{gs3} - V_t) = 2mA$

$\Rightarrow V_{gs3} - V_t = 1V \Rightarrow V_{gs3} = \cancel{0.7}\,1.3V$

Now, $V_{S3} = -3V \Rightarrow V_{g3} = \cancel{0.7}\,1.3V + V_s = -\cancel{2.3}\,1.7V$

Now, $I_{R3b} = 2mA \Rightarrow R_{3b} = \frac{1.7V}{2mA} = \boxed{850\Omega}$

CMRR calculation:

$$V_{S1} = V_{S2} = \frac{-I_{D1}}{V_{sat} C_{ox} W_g} + V_g - V_t = -0.5 + 0 - 0.3 = -0.8V$$

Now, $V_{S1} = V_{D3} = -0.8V \Rightarrow V_{DS3} = +2.2V$

$$R_{DSQ3} = \frac{\frac{1}{\lambda} + V_{DSQ3}}{I_{D3}} (1 + \cancel{g_m R_{ss}}^{\,0}) = \frac{10 + 2.2}{2mA} = 6100\Omega$$

$$CMRR = \frac{R_{ee}}{(1/g_m)} = \frac{6100}{1/(2mA/V)} = 12.2$$

Differential input impedance = $Z_{in(d)} = \frac{V_d}{I_d} = 2R_{g_1} = \underline{\underline{2M\Omega}}$

Differential gain = $A_D = -\frac{g_m}{2}(R_D \| R_L \| R_{DS})$

Now, $R_{DS} = \left(\frac{\frac{1}{\lambda} + V_{DS}}{I_D}\right)(1 + g_m R_{ss})$

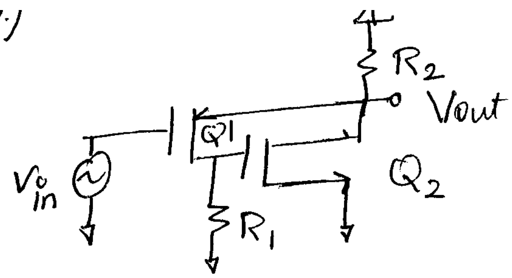
$$= \left(\frac{10V + 2.8V}{1mA}\right)\left(1 + 2\frac{mA}{V} \cdot 6.1k \cdot 2\right)$$

$$= (12.8k)(1 + 24.4) = \underline{325.1k}$$

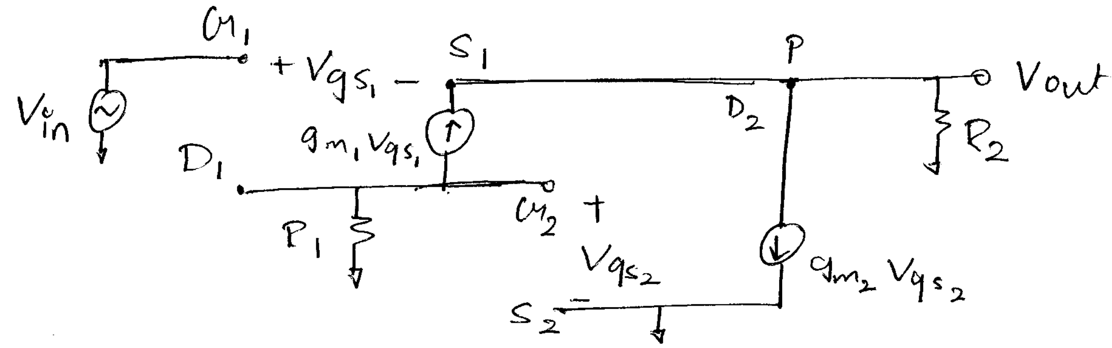
$A_D = -\frac{g_m}{2}(R_D \| R_L \| R_{SS}) = -\frac{2m}{2}(2k \| 500 \| 325.1k)$

$$= \underline{\underline{-0.4}}$$

5.)



Small signal Model:



$$V_{gs_1} = V_{in} - V_{out}$$

$$V_{gs_2} = (g_{m_1} V_{gs_1}) R_1 = g_{m_1} R_1 (V_{in} - V_{out})$$

Apply KCL at node P:

$$\frac{V_{out}}{R_2} + g_{m_2} V_{gs_2} - g_{m_1} V_{gs_1} = 0$$

$$\therefore \frac{V_{out}}{R_2} + g_{m_2} (g_{m_1} R_1)(V_{in} - V_{out}) - g_{m_1} (V_{in} - V_{out}) = 0$$

$$\therefore V_{out}\left(\frac{1}{R_2} - g_{m_1} g_{m_2} R_1 + g_{m_1}\right) + V_{in}(g_{m_1} g_{m_2} R_1 - g_{m_1}) = 0$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{g_{m_1} - g_{m_1} g_{m_2} R_1}{\frac{1}{R_2} + g_{m_1} - g_{m_1} g_{m_2} R_1}$$

Substituting $g_{m_1} = 10\,mS$ $g_{m_2} = 100\,mS$ $R_1 = 1k\Omega$ $R_2 = 500\Omega$

we get $\frac{V_{out}}{V_{in}} = 1.00$