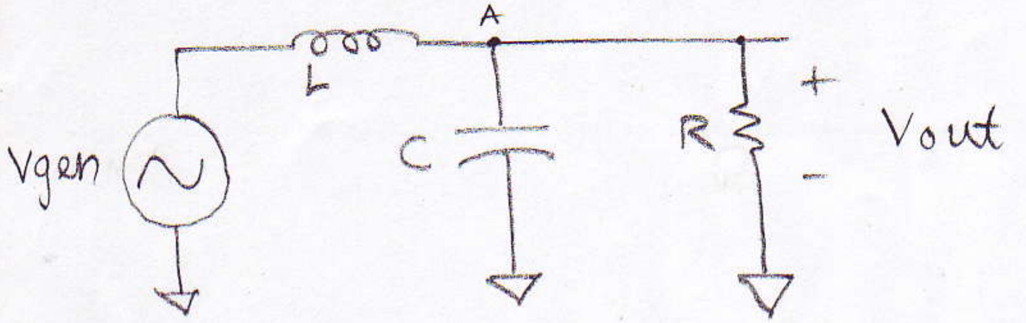
(1)



(a) Considering Node A we have $V_A = V_{out}$

$$\frac{V_{out} - V_{gen}}{Ls} + V_{out} \times sC + \frac{V_{out}}{R} = 0$$

(Equating the sum of the currents at node A to 0)

$$V_{out}\left(\frac{1}{Ls} + sC + \frac{1}{R}\right) = \frac{V_{gen}}{Ls}$$

$$V_{out}\left(\frac{R + s^2LRC + Ls}{RLs}\right) = \frac{V_{in}}{Ls} \quad (\text{since } V_{gen} = V_{in})$$

$$\frac{V_{out}}{V_{in}} = \frac{R}{s^2LRC + sL + R} = \frac{R}{R\left(s^2LC + s\frac{L}{R} + 1\right)}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{s^2LC + s\left(\frac{L}{R}\right) + 1} \rightarrow 1$$

Note that the denominator has no units because LC has units $secs^2$ and $\frac{L}{R}$ has a unit of secs.

(b) $R = 1000\ \Omega$

$f_n = 20\ GHz$.

$\zeta = 0.2$

Comparing equation (1) with that of the standard form of the denominator which is $s^2\left(\frac{1}{\omega n^2}\right) + s\left(\frac{2\zeta}{\omega n}\right) + 1$

we have

$$LC = \left(\frac{1}{\omega n^2}\right)(sec)^2 \rightarrow (2)$$

$$\frac{L}{R} = \left(\frac{2\zeta}{\omega n}\right) sec \rightarrow (3)$$

From (3) we have $L = R\left(\frac{2\zeta}{\omega n}\right)$

$$L = \frac{1000 \times \cancel{2} \times 0.2}{\cancel{2} \times \pi \times 20 \times 10^{9}} \frac{\Omega}{rad/sec}$$

$$= 1000 \times 0.159 \times \frac{2}{10^{11}} \ \Omega\, sec$$

$$L = 3.18 \times 10^{-9} \ H \quad \left(\because \Omega\, sec = H \right.$$

$$V = L \times \frac{di}{dt}$$

units

$$IR = L \times \frac{di}{dt}$$

$$\frac{C}{s}\,\Omega = L \times \frac{C}{s^2} \ ; C - \text{Coulomb}, \ s - \text{second}$$

From 2

$$C = \frac{1}{\omega_n^2 \times L} \left(\frac{sec^2}{H} \right) = \frac{1}{\left(2\pi (20 \times 10^{9}) \right)^2 (3.18 \times 10^{-9})} \ F.$$

$$C = \frac{1}{4\pi^2 \times 400 \times 10^{18} \times \frac{1}{\pi} \times 10^{-8}} \ F$$

$$\left(\because 0.318 = \frac{1}{\pi} \right)$$

$$= \frac{1}{2\pi} \times \frac{1}{2\cancel{\pi}} \times \frac{10^{-10}}{400 \times \frac{1}{\cancel{\pi}}} \ F \quad \left(\frac{sec^2}{H} = \text{Farad} \right.$$

see pg-24)

$$C = \frac{0.159}{800} \times 10^{-10} \ F \quad \left(F \rightarrow \text{Farad} \right)$$

$C = 1.9875 \times 10^{-14}\,F = 19.87 \times 10^{-15}\,F \simeq 20\,fF$

(f - femto)

$L = 3.18 \times 10^{-9}\,H = 3.18\,nH$

(c) The Bode plot and the phase of the frequency response are shown in the next page. The poles are complex and hence there is a single roll off of 40 dB/dec

(d) The root locus is shown next

Note that in the transfer function there is no zero. The poles are given by

$\left(-\omega_n\zeta, \omega_n\sqrt{1-\zeta^2}\right)$ and $\left(-\omega_n\zeta, -\omega_n\sqrt{1-\zeta^2}\right)$

$$\omega_n\zeta = 2\pi \times 20 \times 10^9 \times 0.2 \ \frac{rad}{sec}$$

$$= 8\pi \times 10^9 \ \frac{rad}{sec}$$

$$\omega_n\zeta = 2.51327 \times 10^{10} \ \frac{rad}{sec} \ ; \ f_n\zeta = \underline{4\,GHz}$$

$$\omega_n\sqrt{1-\zeta^2} = 2\pi \times 20 \times 10^9 \times \sqrt{1-0.04} \ \frac{rad}{sec}$$

$$= 1.23125 \times 10^{11} \ rad/sec$$

$$f_n\sqrt{1-\zeta^2} = \underline{19.6\ GHz}$$

The step response is shown on page-9
corresponding to $V_{in}(t) = 2.5\ u(t)$

$$V_{out}(s) = V_{in}(s) \times \frac{1}{s^2\left(\frac{1}{\omega_n^2}\right) + s\left(\frac{2\zeta}{\omega_n}\right) + 1}$$

$$V_{out}(s) = 2.5 \times \frac{1}{s} \times \frac{1}{s^2\left(\frac{1}{\omega_n^2}\right) + s\left(\frac{2\zeta}{\omega_n}\right) + 1}$$

$$V_{out}(t) = 2.5\left[1 - e^{-\sigma t}\left(\cos \omega_d t + \frac{\sigma}{\omega_d}\sin(\omega_d t)\right)\right]$$

where $\sigma = \omega_n \zeta = 2.51327 \times 10^{10}$ rad/sec

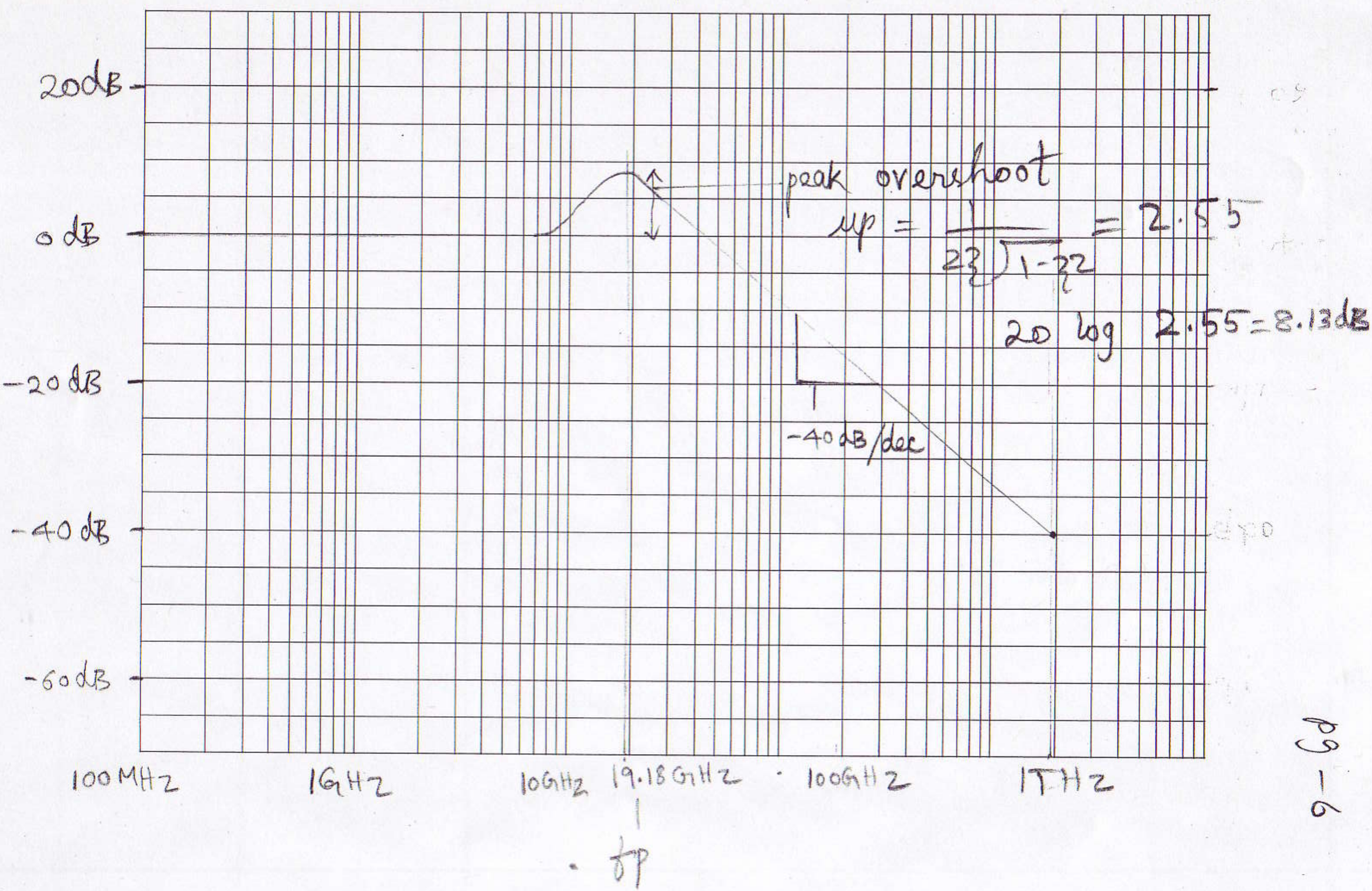
$\omega_d = \omega_n\sqrt{1-\zeta^2} = 1.23125 \times 10^{11}$ rad/sec

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\omega_n^2}{(s+\sigma)^2 + \omega_d^2}$$

$f_n \zeta = 4$ GHz

$f_n\sqrt{1-\zeta^2} = 19.6$ GHz

Problem 1 - Magnitude response

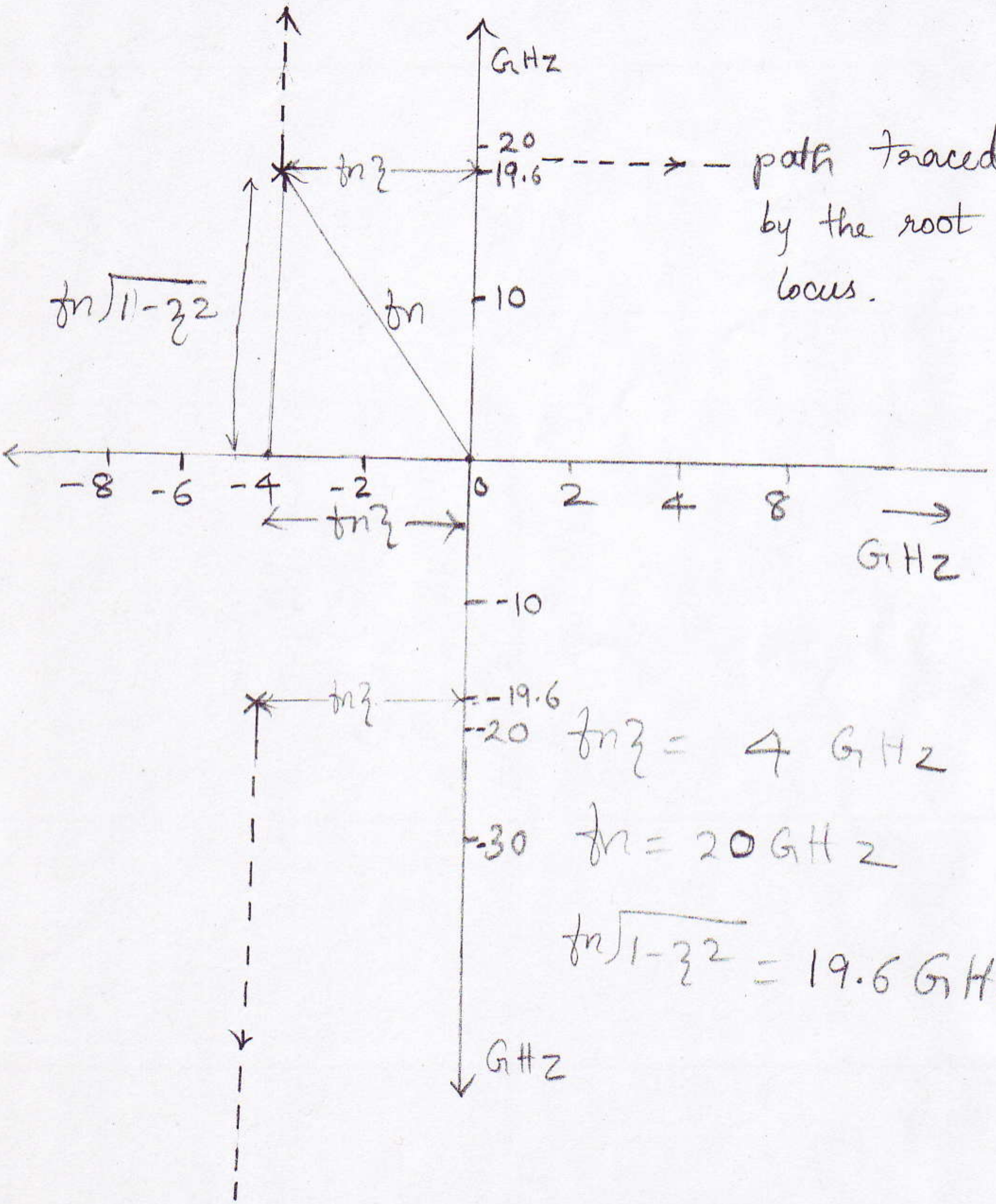


$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 2.55$$

$$20 \log 2.55 = 8.13 dB$$

Problem 1 - Phase Response.

Problem 1 - Root locus

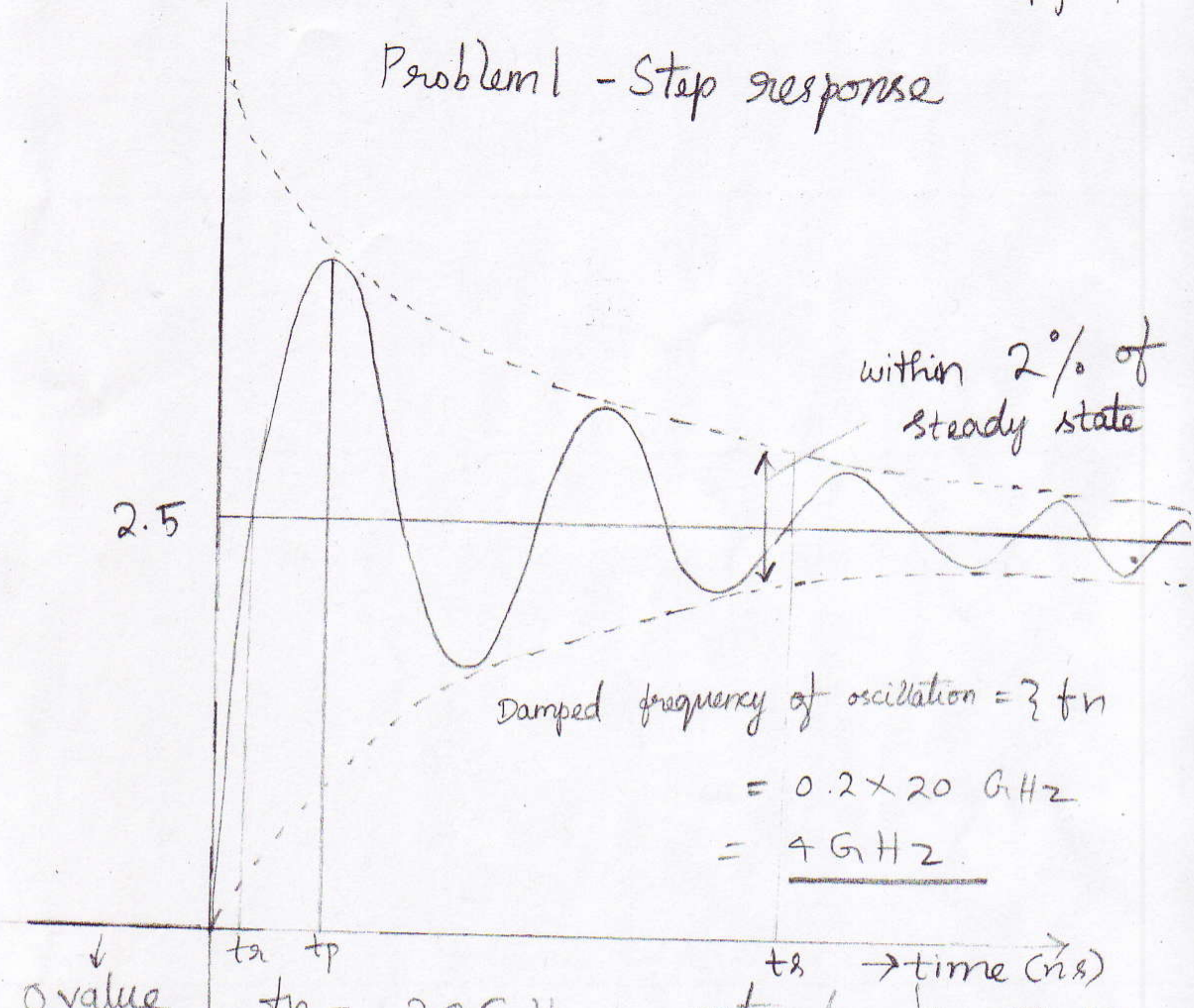


$f_n\zeta = 4$ GHz

$f_n = 20$ GHz

$f_n\sqrt{1-\zeta^2} = 19.6$ GHz

Problem 1 - Step response



Damped frequency of oscillation $= \zeta f_n$

$= 0.2 \times 20$ GHz

$=$ 4 GHz

$f_n = 20$ GHz - natural frequency

Rise time $t_r = \left(\frac{\pi - \theta}{\omega_d}\right)s = \left(\frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)}{\omega_d}\right)s = \left(\frac{3.14 - 1.37}{1.23 \times 10^{11}}\right)s = 0.014$ ns

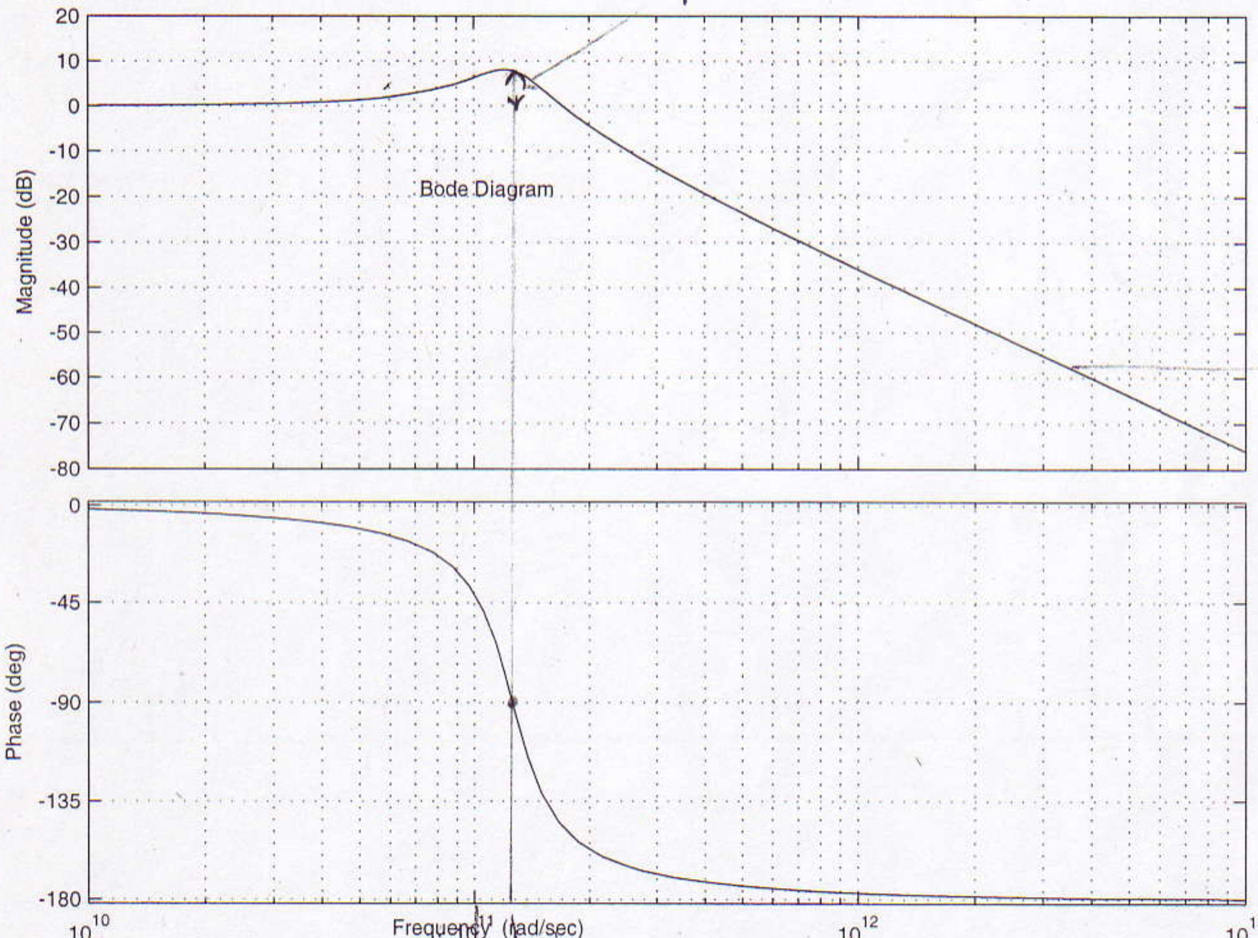
Peak time $t_p = \left(\frac{\pi}{\omega_n \sqrt{1-\zeta^2}}\right)s = \frac{\pi}{\omega_d}s = \frac{3.14}{1.23 \times 10^{11}} = 2.55 \times 10^{-11}s = 0.025$ ns

Settling time $= \left(\frac{4}{\zeta \omega_n}\right)s = \left(\frac{4}{2.5 \times 10^{10}}\right)s = 1.59 \times 10^{-10} s$

$= .159$ ns

Bode plot $\quad M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 2.55$

peak overshoot $\simeq 8.1$ dB



$\rightarrow \frac{40\,\text{dB}}{\text{decade}}$

1) Phase at $\omega_p = -90^\circ$ $\quad 10^{11}$ $\rightarrow 1.20532 \times 10^{11} \frac{\text{rad}}{\text{sec}}$

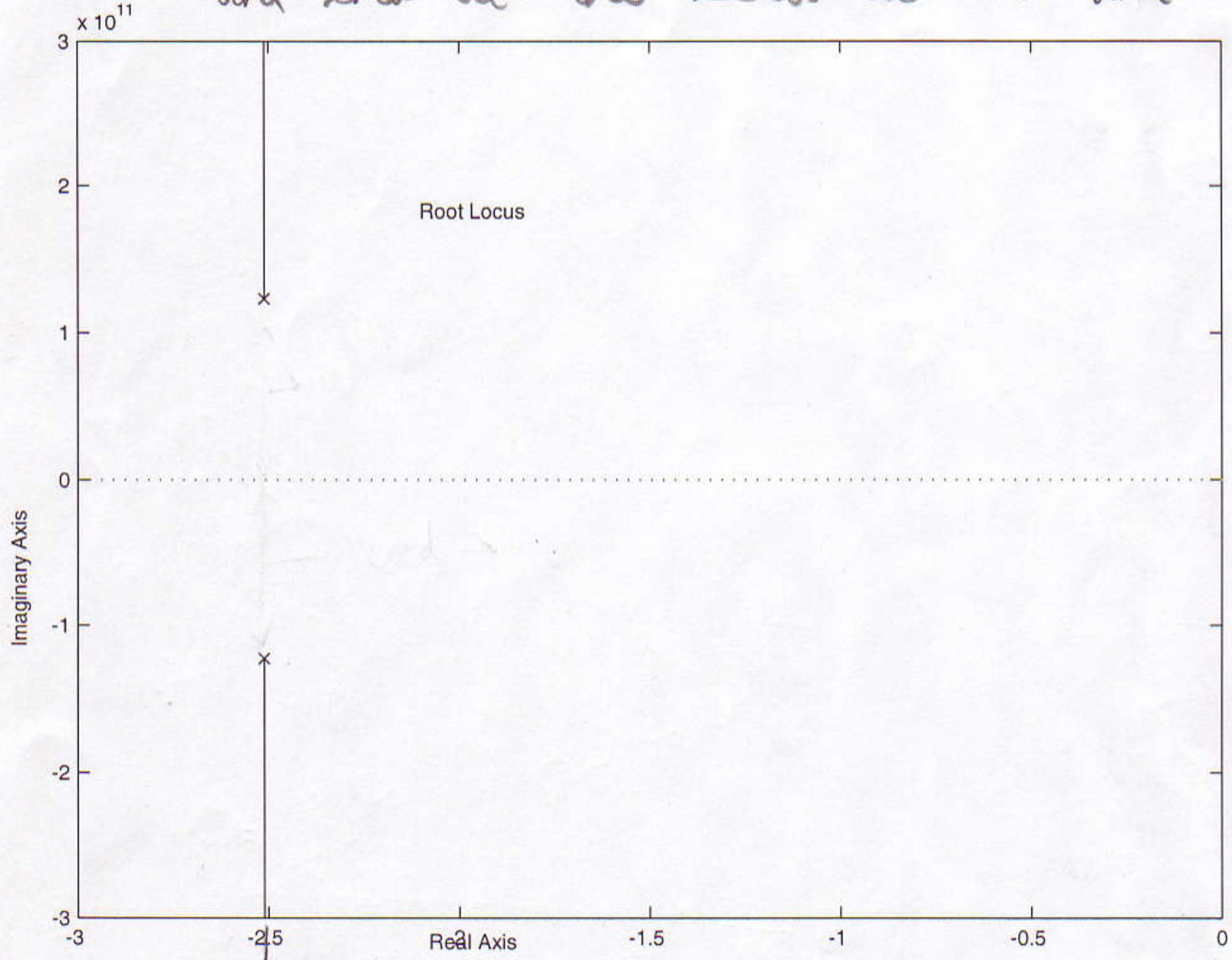
2) There is a single roll off at ω_p

3) $\omega_p = \omega_n\sqrt{1-2\zeta^2} = 2 \times \pi \times 20 \times 10^9 \times \sqrt{1 - 0.08}$

$$= 12.0532 \times 10^{11}$$

$$\omega_p = 1.20532 \times 10^{11} \text{ rad/sec}$$

Root locus - starts at the poles and ends at the zeros at ∞ and $-\infty$.



$-\zeta\omega_n$

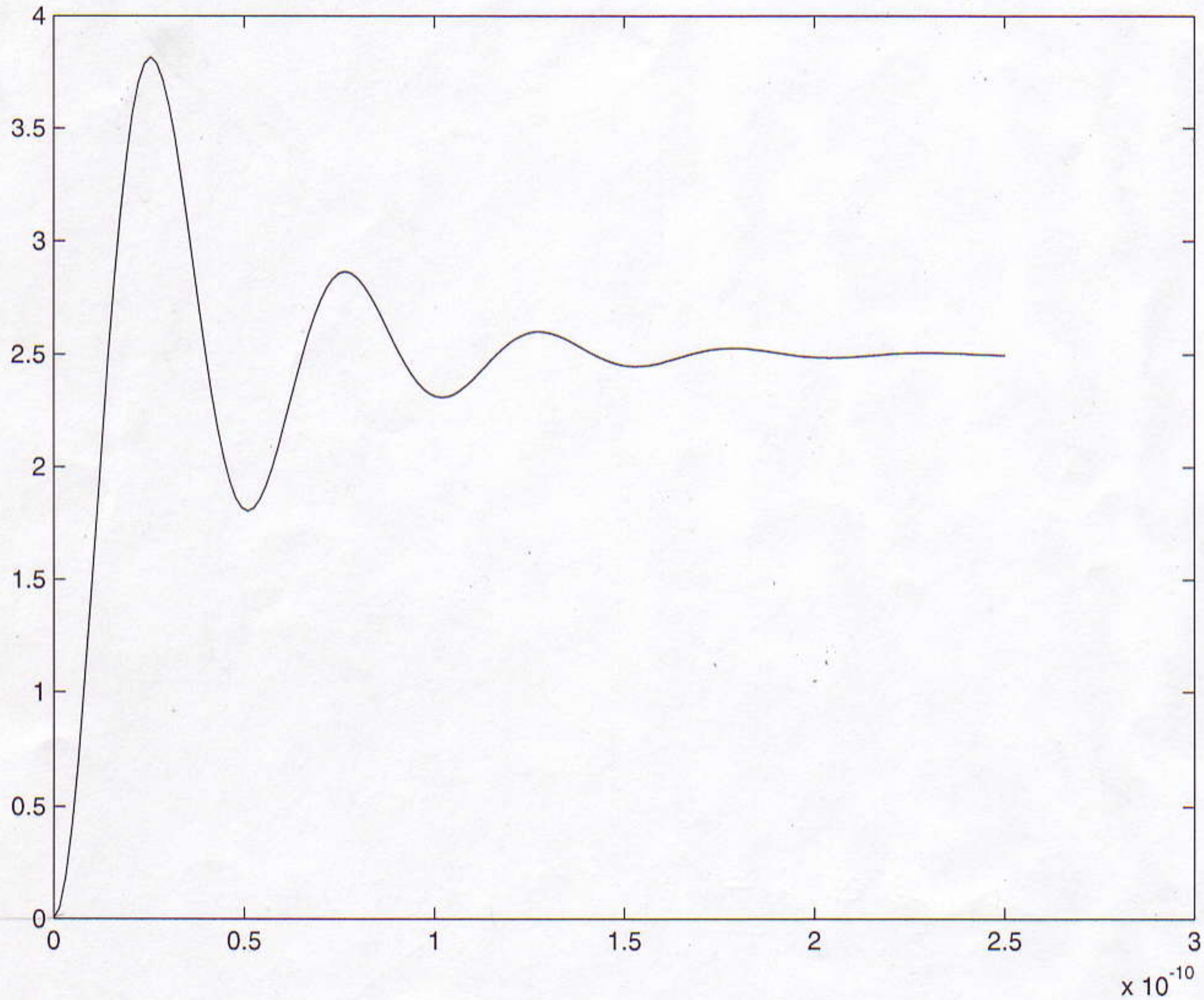
$$-2.51327 \times 10^{10} \frac{rad}{sec} = -\zeta\omega_n$$

X - poles

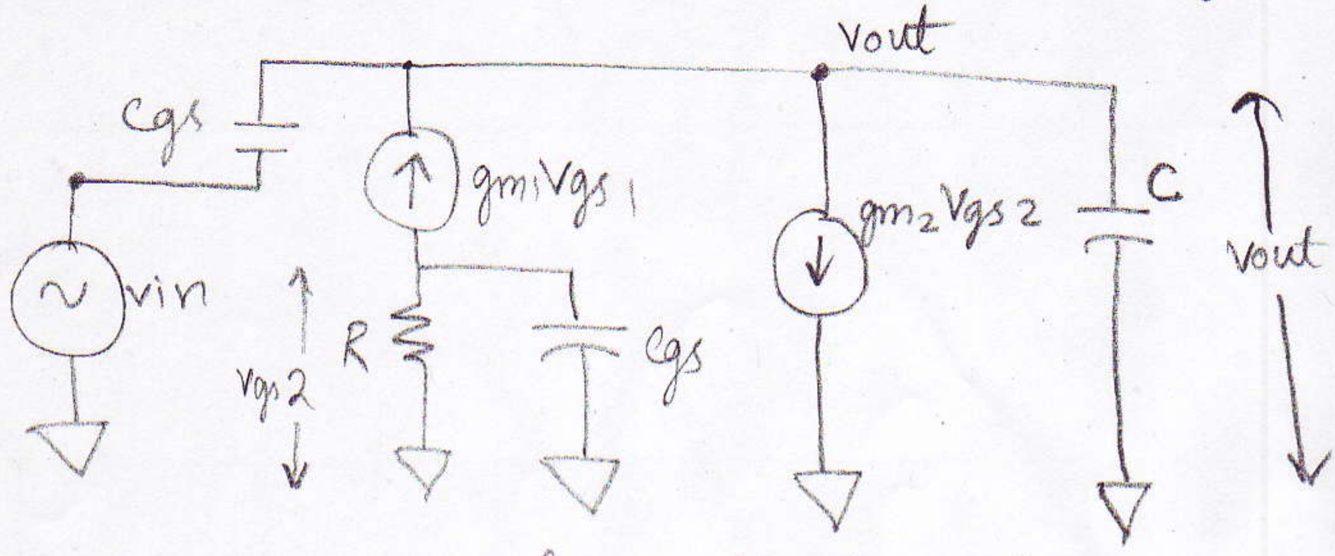
distand of poles from x axis $= \omega_n\sqrt{1-\zeta^2}$

$= 1.23125 \times 10^{11}$ rad/sec

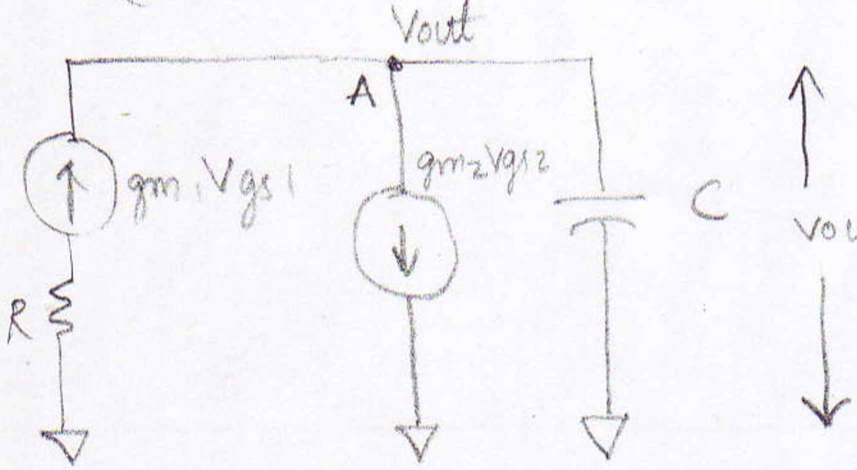
Step response corresponding to $v_{in} = 2.5\,u(t)$



Problem 2



Since $C_{gs} = 0$ we have the circuit reduced to



$V_{gs1} = V_{in} - V_{out}$

$V_{gs2} = -g_{m1}V_{gs1}R$

Applying nodal analysis at node A and equating the currents to 0 we have

$$V_{out}(sC) + g_{m2}V_{gs2} - g_{m1}V_{gs1} = 0$$

$$V_{out} = \frac{V_{gs1}g_{m1} - V_{gs2}g_{m2}}{sC} \rightarrow (1)$$

From the circuit it can be seen that

$V_{gs1} = V_{in} - V_{out}$.

and

$V_{gs2} = -g_{m1} V_{gs1} \times R = -g_{m1} R (V_{in} - V_{out})$

From equation (1)

$$V_{out} = \frac{1}{sC}\left((V_{in} - V_{out}) g_{m1} + g_{m1} R (V_{in} - V_{out}) \times g_{m2}\right)$$

$$V_{out}\left(\frac{1}{Cs}(g_{m1} + g_{m1} g_{m2} R) + 1\right) = \frac{1}{Cs}(g_{m1} + g_{m1} g_{m2} R) V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{g_{m1}}{Cs}\left(1 + g_{m2} R\right)}{1 + \frac{g_{m1}}{Cs}\left(1 + g_{m2} R\right)}$$ no units

$g_{m1} = g_{m2} = 100\,mS$.

$R = 10k\Omega$.

$C = 1\,pF = 10^{-12}\,F$

It can be seen that the transfer function is that of a buffer.

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{100\times10^{-3}}{10^{-12}\times s}\left(1+\underbrace{100\times10^{-3}\times10\times10^{3}}_{\text{no units}}\right)}{1+\frac{100\times10^{-3}}{10^{-12}\times s}\left(1+\underbrace{100\times10^{-3}\times10\times10^{3}}_{\text{no units}}\right)}$$

$$= \frac{10^{14}\ (1001)\times10^{-3}}{s\left(1+\frac{10^{14}(1001)\times10^{-3}}{s}\right)} \quad \text{no units}$$

$$= \frac{10^{14}\ (1.001)}{s+10^{14}\ (1.001)} \quad \text{no units}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{10^{14}}{s+10^{14}}$$

Note: $gm\times R$ – No units

$\frac{gm}{Cs} \equiv \frac{1}{RCs}$ – No units

($RC \to$ units – secs

$s \to$ units – $\frac{1}{\text{secs}}$)

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1+\frac{s}{10^{14}}} \rightarrow (2)$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1+j\times\left(\frac{\omega}{\omega_0}\right)} \rightarrow (3)$$

Comparing 2 and 3 we have

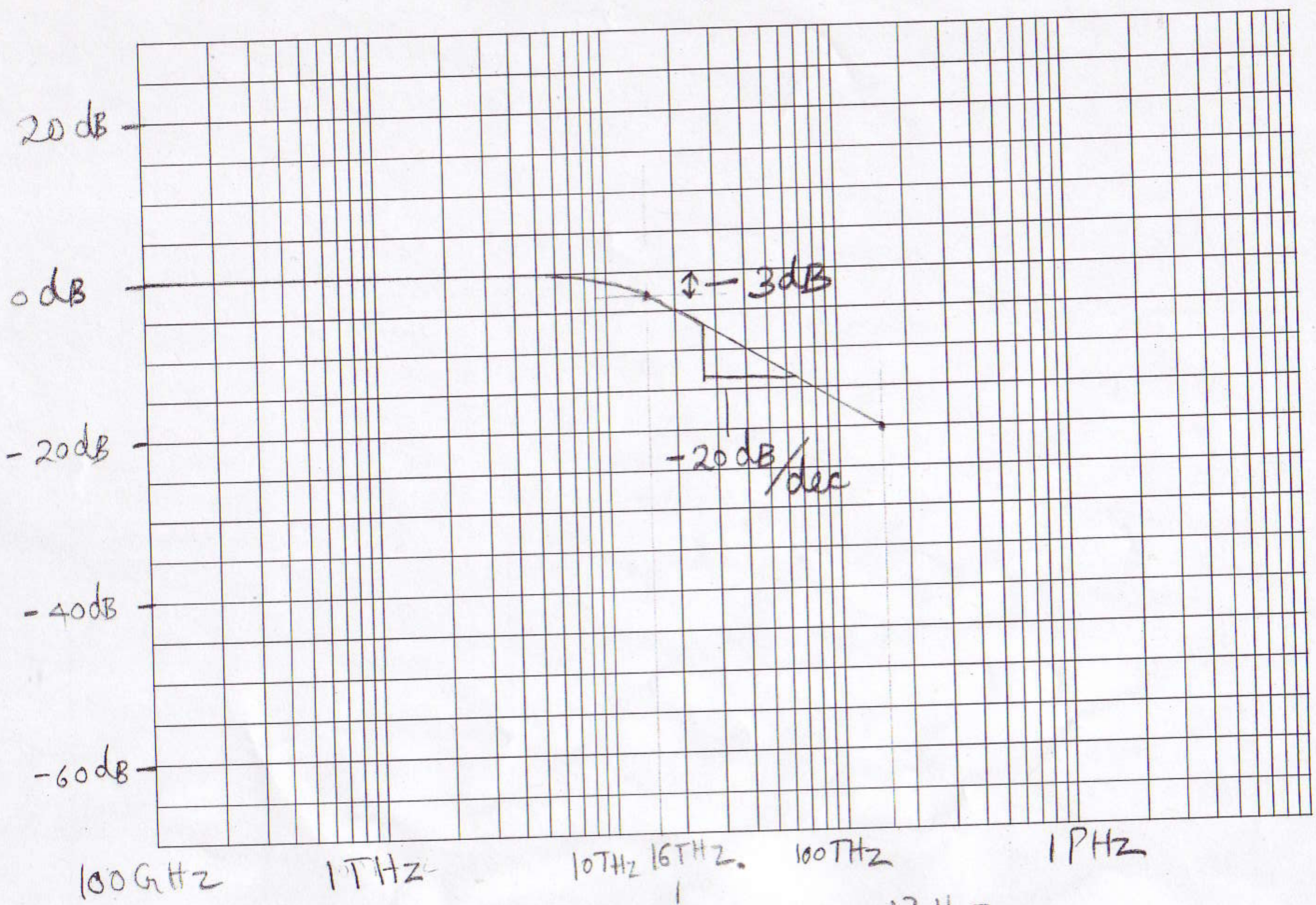
$\omega_0 = 10^{14}$ rad/sec ; $f_0 = 16 \times 10^{12}$ Hz
$f_0 = 16$ THZ

$\omega_p = 10^{14}$ rad/sec. ; $f_p = 16$ THZ

(c) The root locus is shown in the next page followed by (d) the Bode plot.

The pole is present at -10^{14} rad/sec.
There is no finite zero in the transfer function.

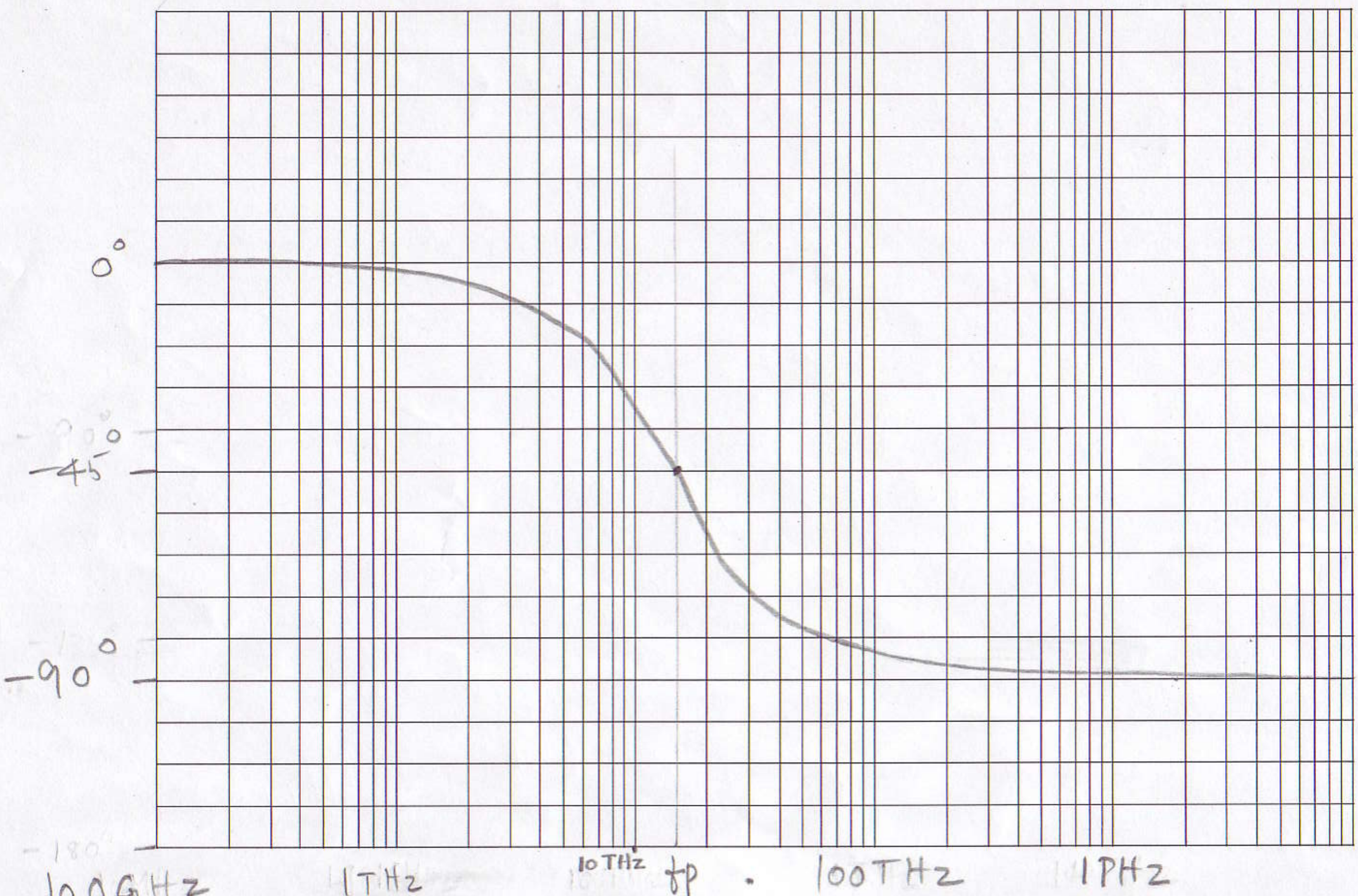
Problem 2 - Magnitude response



$f_P = 16 \times 10^{12}$ Hz

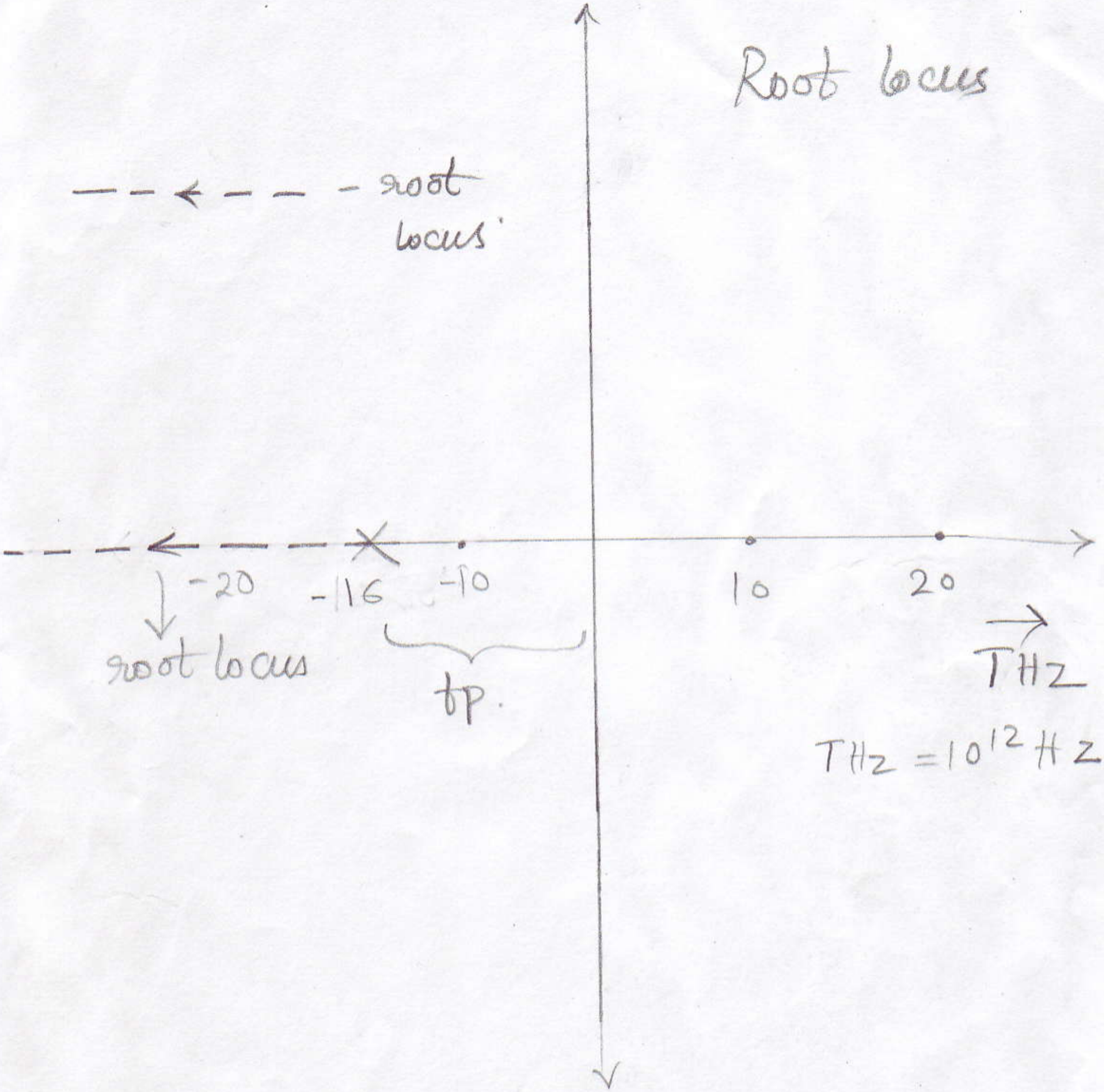
$f_P = 16$ THz

Problem 2 - Phase response



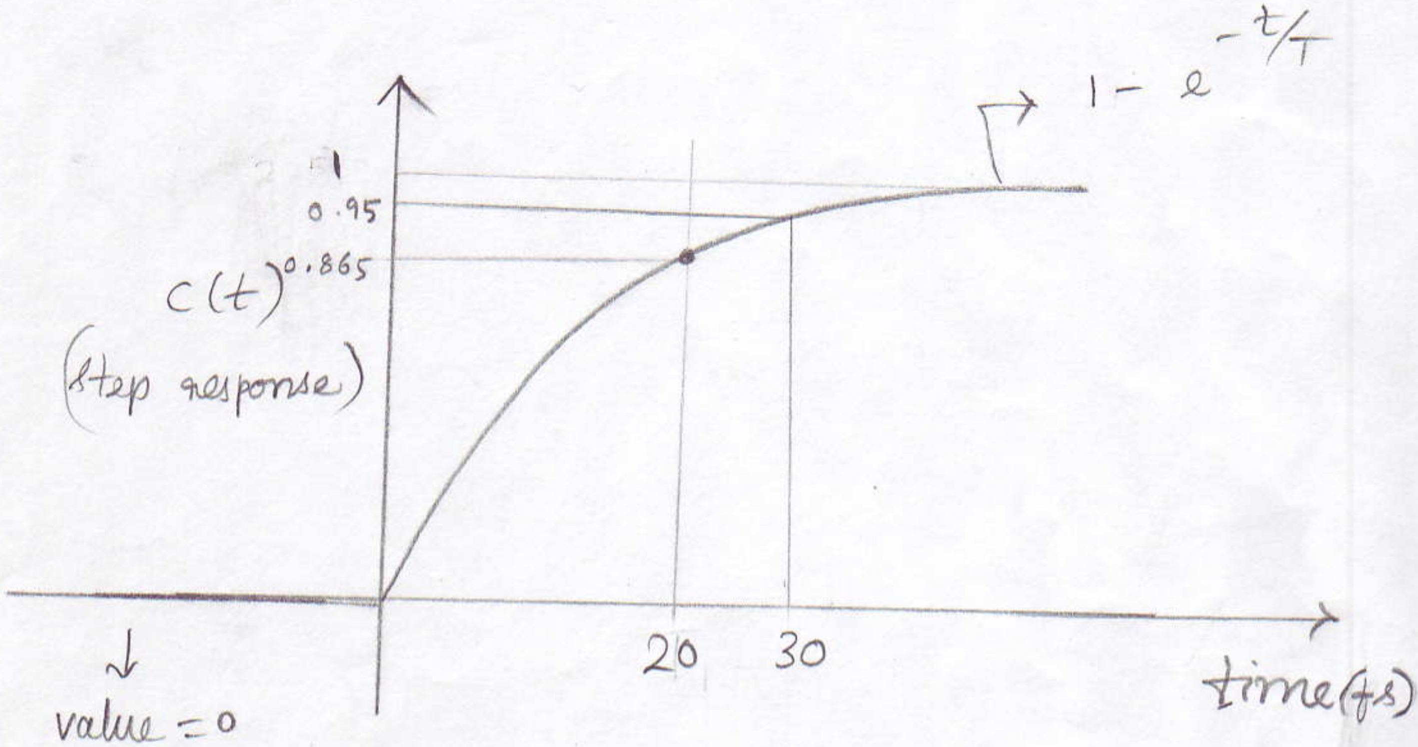
$f_p = 1.6 \times 10^{13}$ Hz $= 16 \times 10^{12}$ Hz $= 16$ THz.

Problem 2



Root locus
root locus
-20
-16
-10
10
20
root locus
tp.
THz
THz = 10^{12} Hz

Problem 2 Step response



$$T = \frac{1}{10^{14}} \text{ secs}$$

$$T = 1 \times 10^{-14} \text{ secs.}$$

$$T = 10 \times 10^{-15} \text{ secs} = 10 \text{ f secs}$$

$$f = \text{femto} = 10^{-15}$$

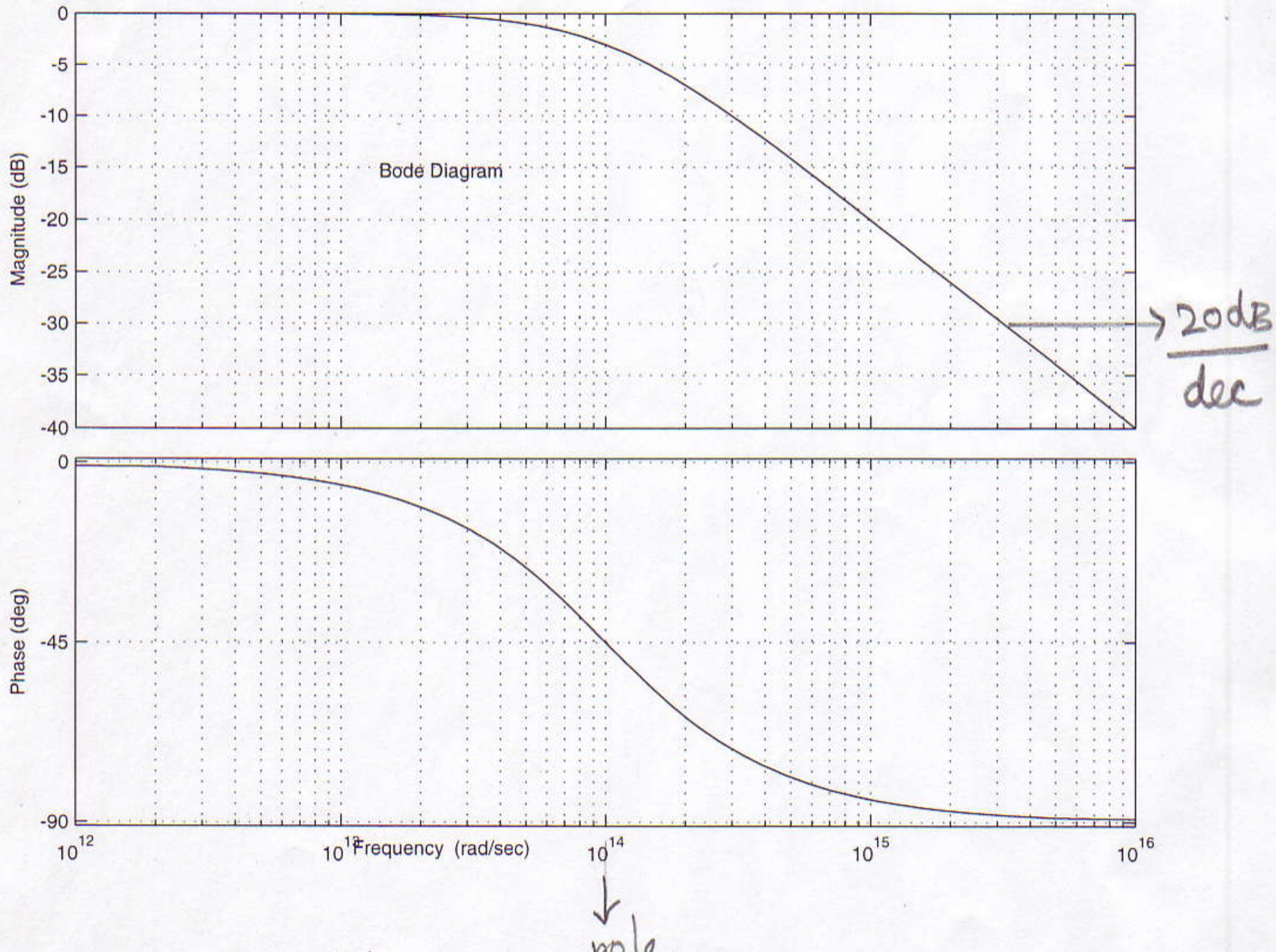
when $t = 1T$, $c(t) = 1 - e^{-1} = 0.632$

$t = 2T$, $c(t) = 1 - e^{-2} = 0.865$

$t = 3T$, $c(t) = 1 - e^{-3} = 0.95$

$t = \infty$, $c(t) = 1 - e^{-\infty} = 1$

Bode plot



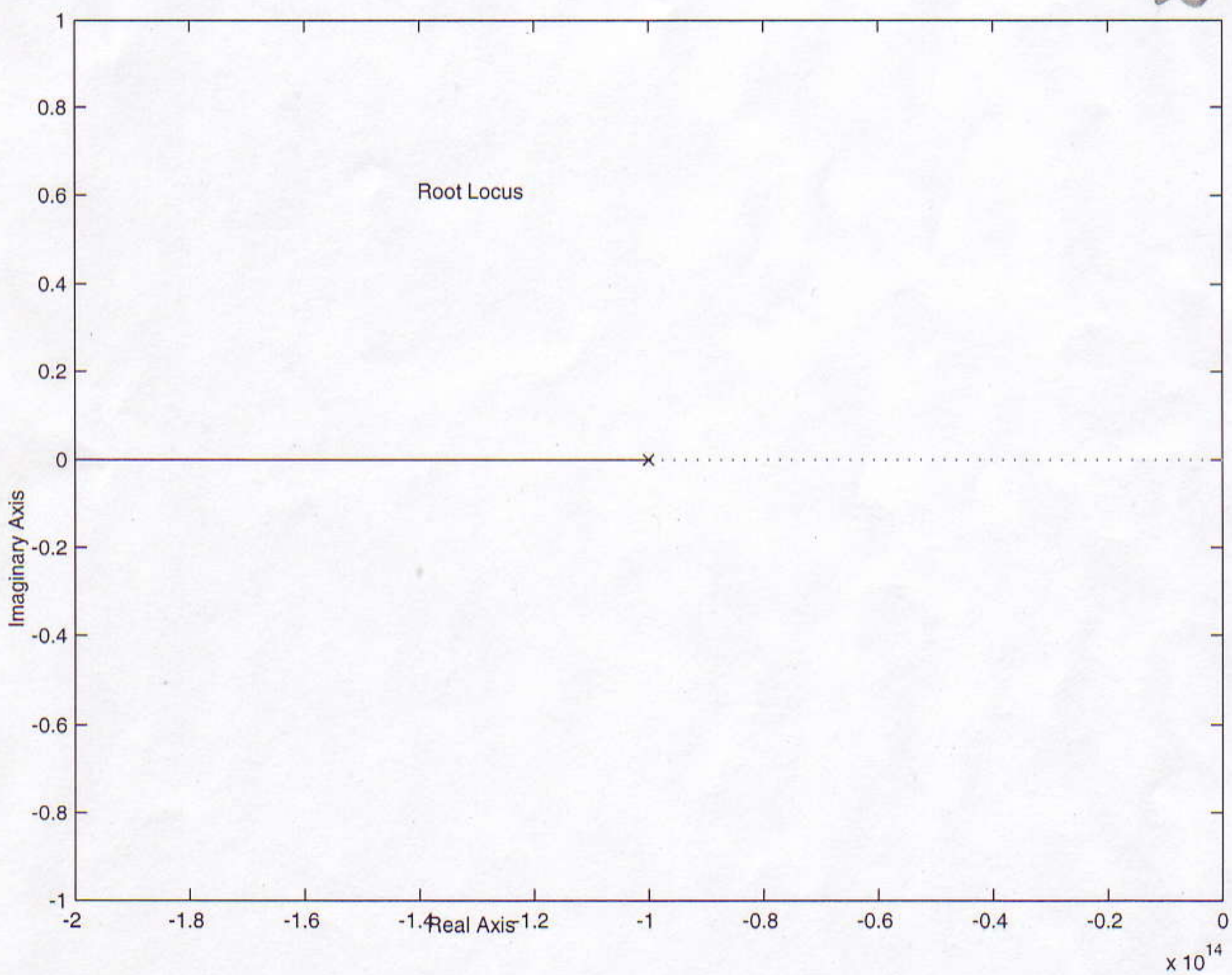
pole $= 10^{14}$ rad/sec $= \omega_p$

Note the phase at $\omega_p = -45^0$

corresponding to $\phi = \tan^{-1}\left(\frac{\omega}{\omega_p}\right)$

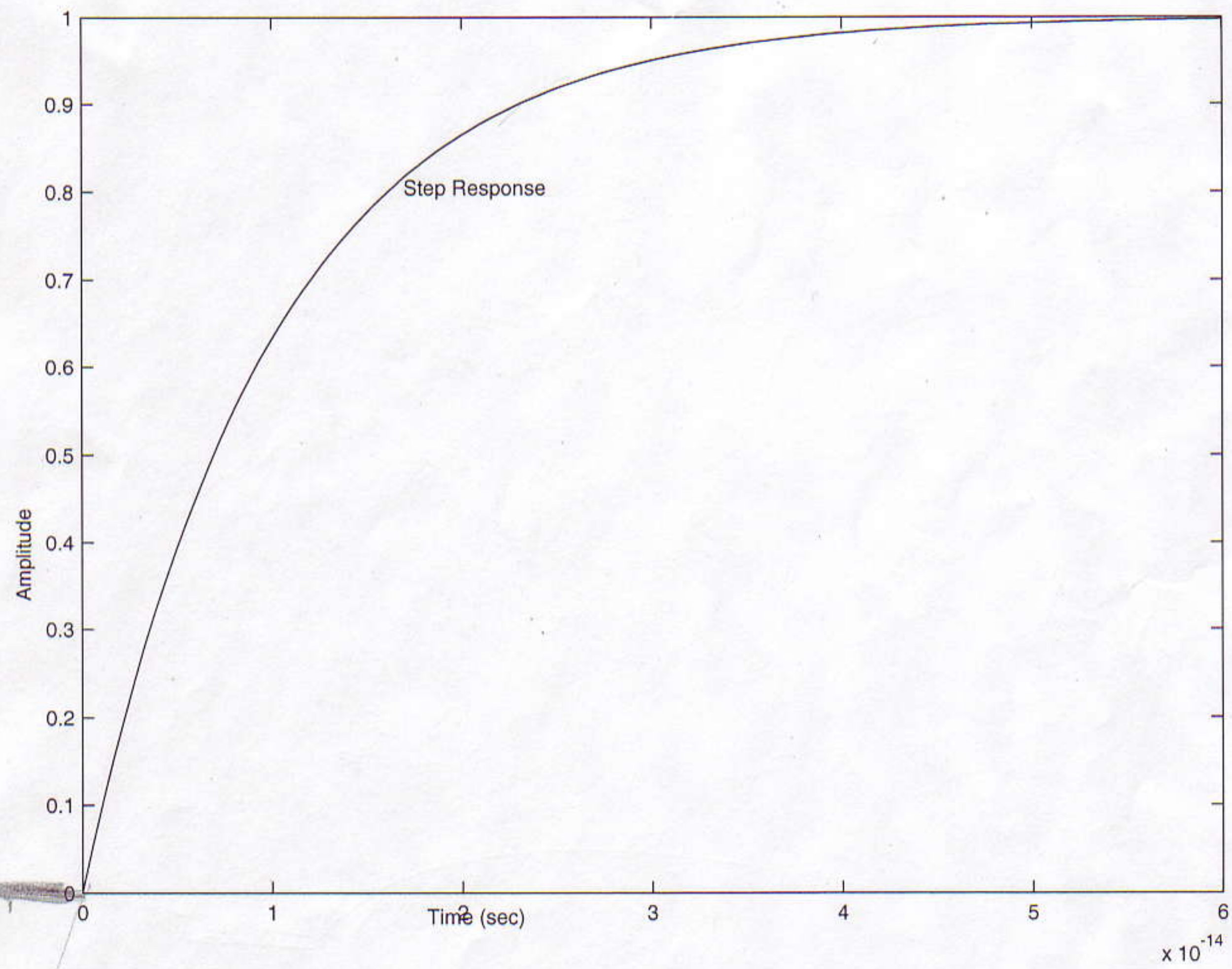
Root locus

Starts at the pole and ends at the zero at $-\infty$.



pole is at -10^{14} rad/sec.

Step response corresponding to Vin = u(t)



$$C = \frac{1}{\omega n^2 \times L} \quad \frac{sec^2}{H} \quad (H - \text{Henry})$$

$$\frac{sec^2}{H} = \frac{sec^2}{\Omega \, sec}$$

$$\frac{sec^2}{H} = \frac{sec}{\Omega}$$

$$i = \overbrace{C}^{\text{capacitance}} \times \frac{dv}{dt}$$

$$\underbrace{C}_{\text{capacitance}} = \frac{i}{\frac{dv}{dt}} = \frac{i}{R \times \frac{di}{dt}} \xrightarrow{\text{units}} \frac{\cancel{C}/s}{\Omega \times \frac{\cancel{C}}{s^2}}$$

(here C - coulomb)

$$F = \frac{sec}{\Omega}$$

$$\text{Hence } \frac{sec^2}{H} = F.$$