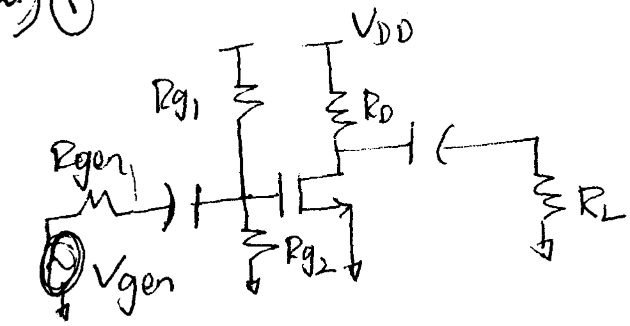
a.) ①




$V_{th} = 0.3$ Volts

$C_{ox} V_{sat} W_g = 5\,mS$

$C_{gd} = C_{gs}/10$

$R_{DS} = \infty$

$f_T = 200$ GHz

$R_{gen} = 200\,k\Omega$

$R_{g1} \| R_{g2} = 1M\Omega$

$I_{DQ1} = 1mA$

$V_{DSQ1} = 1V$

$A_{VMB} = -10$

$R_L = 2R_D$

$$I_{D1} = g_m (V_{gs} - V_{th}) = (5mS)(V_{gs} - 0.3)$$

$$\Rightarrow V_{gs} = \frac{(1mA)}{(5mS)} + 0.3 = \underline{0.5V}$$

Since $V_S = 0$ ; $V_G = 0.5V$

Now, $\frac{V_{DD} \cdot R_{g2}}{R_{g1} + R_{g2}} = 0.5V$ —(1) & $R_{g1} \| R_{g2} = 1M\Omega$ —(2)

solving (1) & (2) simultaneously we get

$R_{g1} = 8 M\Omega$ $R_{g2} = 1.14M\Omega$

where $V_{DD}$ is found as follows:

$$A_{VMB} = -g_m (R_D \| R_L) = -10$$

$$\Rightarrow A_{VMB} = g_m (R_D \| 2R_D) = -10 \} \Rightarrow \frac{2}{3} g_m R_D = 10$$

$$\Rightarrow \underline{R_D = 3000\Omega}$$

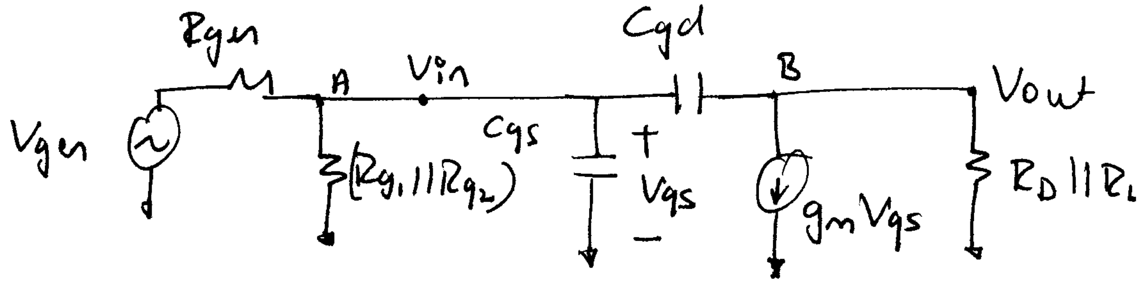
$$V_{DD} = V_{DSQ1} + I_D R_D = 1V + (1mA)(3k\Omega) = \underline{4V}$$

$$f_T = \frac{1}{2\pi}\left(\frac{g_m}{C_{gs} + C_{gd}}\right) \Rightarrow C_{gs} + C_{gd} = \frac{g_m}{2\pi f_T} \quad \text{Also } C_{gd} = \frac{C_{gs}}{10}$$

$$1.1\,C_{gs} = \frac{g_m}{2\pi f_T} \Rightarrow C_{gs} = \frac{g_m}{2\pi \times 1.1 \times f_T} = \underline{3.6\,fF}$$

$$C_{gd} = \underline{0.36\,fF}$$

b).




KCL at node A gives:

$$\frac{(V_{in} - V_{gen})}{R_{gen}} + \frac{V_{in}}{(R_{g1} || R_{g2})} + \frac{(V_{in} - V_{out})}{(1/sC_{gd})} + \frac{V_{in}}{(1/sC_{gs})} = 0$$

$$V_{in}\left[\frac{1}{R_{gen}} + \frac{1}{R_{g1} || R_{g2}} + sC_{gd} + sC_{gs}\right] - V_{out}(sC_{gd}) = (V_{gen}/R_{gen}) \quad -(1)$$

KCL at node B gives:

$$(V_{out} - V_{in})\, sC_{gd} + g_m V_{gs} + \frac{V_{out}}{(R_D || R_L)} = 0$$

Noting that $V_{gs} = V_{in}$ we get

$$V_{in}\left[-sC_{gd} + g_m\right] + V_{out}\left[sC_{gd} + \frac{1}{R_D || R_L}\right] = 0$$

$$\Rightarrow V_{in} = -V_{out}\,\frac{\left[sC_{gd} + \frac{1}{R_D || R_L}\right]}{(g_m - sC_{gd})} \quad -(2)$$

Substitute this into (1).

$$(V_{out})\left\{\left[\frac{sC_{gd} + \frac{1}{R_D || R_L}}{g_m - sC_{gd}}\right]\left[\frac{1}{R_{gen}} + \frac{1}{R_{g1} || R_{g2}} + s(C_{gs} + C_{gd})\right] - sC_{gd}\right\} = \frac{V_{gen}}{R_{gen}}$$

$$\therefore \frac{V_{out}}{V_{gen}} = \left(\frac{1}{R_{gen}}\right) \frac{1}{\left\{\left(\dfrac{sC_{gd} + \dfrac{1}{R_D \| R_L}}{g_m - sC_{gd}}\right)\left[\dfrac{1}{R_{gen}} + \dfrac{1}{R_{g_1} \| R_{g_2}} + s(C_{gs} + C_{gd})\right] - sC_{gd}\right\}}$$

$$\therefore \frac{V_{out}}{V_{gen}} = \left(\frac{1}{R_{gen}}\right) \frac{(g_m - s C_{gd})}{\left(sC_{gd} + \dfrac{1}{R_D \| R_L}\right)\left(\dfrac{1}{R_{gen}} + \dfrac{1}{R_{g_1} \| R_{g_2}} + s(C_{gs} + C_{gd})\right) - sC_{gd}\cdot(g_m - s C_{gd})}$$

$$\therefore \frac{V_{out}}{V_{gen}} = \frac{1}{R_{gen}} \cdot \left[\frac{(g_m - s C_{gd})}{\left(\dfrac{1}{R_D \| R_L}\right)\left(\dfrac{1}{R_{gen}} + \dfrac{1}{R_{g_1} \| R_{g_2}}\right) + s^2\left(C_{gd}(C_{gs} + C_{gd})\right) + s\left[C_{gd}\cdot\left(\dfrac{1}{R_{gen}} + \dfrac{1}{R_{g_1} \| R_{g_2}}\right) + (C_{gs} + C_{gd})\left(\dfrac{1}{R_D \| R_L}\right)\right]}\right]$$

Substituting $R_{g_1} \| R_{g_2} = 1 M\Omega$ $\quad R_D \| R_L = 2000$

$C_{gs} = 3.6\ fF$ $\quad C_{gd} = 0.36\ fF$ we get

$$\frac{V_{out}}{V_{gen}} = \frac{((5e-3) - s(0.36e-15))\ /(200K)}{(3 \times 10^{-9}) + (1.98 \times 10^{-18})\, s + (1.4256 \times 10^{-30})\, s^2}$$

This gives pole frequencies at

$f_{P_1} = 238$ MHz $\qquad f_{P_2} = 219$ GHz

The zero frequency is at: $f_Z = \underline{2.2\ THz}$

→ The zero frequency is high enough to be neglected in analysis.

1.c) Since $f_{P_1} = 238$ MHz is the dominant pole we rewrite the transfer function as:

$$\frac{V_{out}}{V_{gn}} = \frac{-10}{1+\tau_1 s} \quad \text{where} \quad \tau_1 = \frac{1}{\omega_{P_1}} = \underline{0.67\,\text{nsec}}$$

(Neglecting the 2nd pole)

For $v_i(t) = 2mV \cdot u(t) \xrightarrow{\mathcal{L}} v_i(s) = \left(\frac{2mV}{s}\right)$

$$V_{out}(s) = T(s) \cdot V_i(s)$$

$$= \frac{-10}{(1+\tau_1 s)} \cdot \frac{(2mV)}{s} = - \left[\frac{k_1}{1+\tau_1 s} + \frac{k_2}{s}\right]$$

As $s \to -1/\tau_1$ we get $k_1 = +(20mV)\,\tau_1$

As $s \to 0$ we get $k_2 = -(20mV)$

$$\therefore V_{out}(s) = \left[20mV \cdot \left[\frac{\tau_1}{1+\tau_1 s}\right] - \frac{20mV}{s}\right]$$

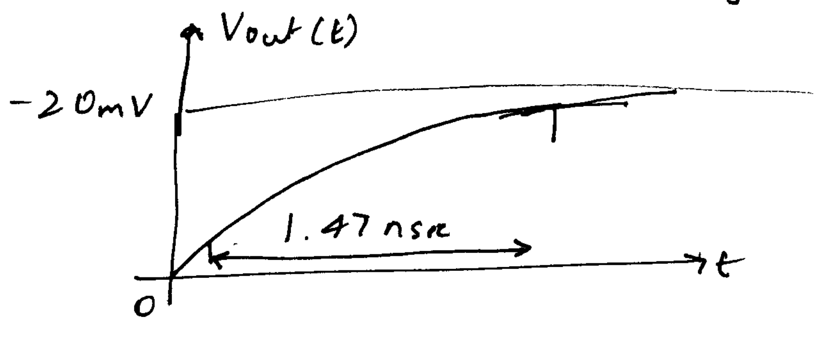
$$= -20mV \cdot \left[\frac{1}{s} - \frac{\tau_1}{1+\tau_1 s}\right]$$

Taking inverse Laplace Transform we get

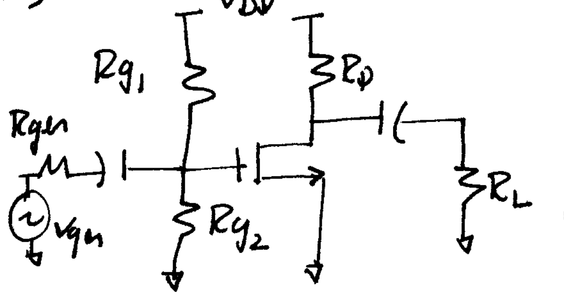
$$V_{out}(t) = -\left[20mV \cdot \left[1 - e^{-t/\tau_1}\right] u(t)\right] \quad \text{where } \tau_1 = 0.67\,\text{nsec}$$

1d)

$$\tau_{10-90\%} = \tau \cdot \ln[0.9 \div 0.1] = \underline{1.47\,\text{nsec}}$$

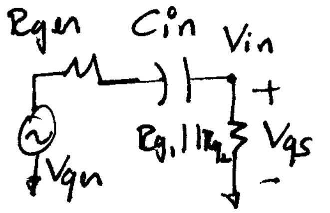


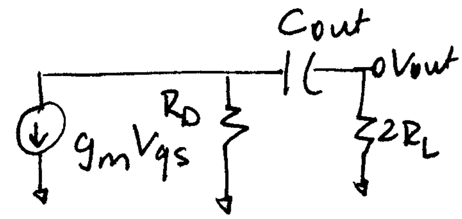
2.) 




=

(Cgs = 0
Cgd = 0)




$$V_{out} = g_m V_{gs} \cdot \left[ R_D \,||\, \left(2R_L + \frac{1}{sC_{out}}\right)\right]$$

$$= g_m V_{gs} \left[ \frac{R_D\left(2R_L + \frac{1}{sC_{out}}\right)}{R_D + 2R_L + \frac{1}{sC_{out}}} \right]$$

And $$V_{gs} = \frac{V_{gen} \cdot (R_{g1} \,||\, R_{g2})}{(R_{g1} \,||\, R_{g2}) + \frac{1}{sC_{in}} + R_{gen}}$$

$$\therefore \left(\frac{V_{out}}{V_{gen}}\right) = g_m \cdot \left[ \frac{(R_{g1} \,||\, R_{g2})}{(R_{g1} \,||\, R_{g2}) + R_{gen} + \frac{1}{sC_{in}}} \right] \left[ \frac{R_D \cdot \left(2R_L + \frac{1}{sC_{out}}\right)}{R_D + 2R_L + \frac{1}{sC_{out}}} \right]$$

$$\therefore \frac{V_{out}}{V_{gen}} = g_m \cdot \left[ \frac{sC_{in} \cdot (R_{g1} \,||\, R_{g2})}{1 + sC_{in}\,(R_{gen} + (R_{g1} \,||\, R_{g2}))} \right] \left[ \frac{R_D \cdot (1 + s\,2R_L C_{out})}{1 + sC_{out}\,(R_D + 2R_L)} \right]$$

Substitute $R_{g1} \,||\, R_{g2} = 1\,M\Omega$ $\quad C_{in} = C_{out} = 5\,pF$

$R_{gen} = 200\,k\Omega$ $\quad R_L = 6\,k\Omega$ $\quad R_D = 3\,k\Omega$

we get

$$\frac{V_{out}}{V_{gen}} = \frac{(15.)\left[s\,(5\times10^{-6})_{sec}\right]\left[\cancel{(3\times10^{+3})}\,(1 + s\,(6\times10^{-8})_{sec}\right]}{\left[1 + s\,(6\times10^{-6})_{sec}\right]\left[1 + s\,(7.5\times10^{-8})_{sec}\right]}$$

$$= \frac{(15\,s)_{sec}\left[5\times10^{-6} + s\,(3\times10^{-13})_{sec}\right]}{\left[1 + s\,(6\times10^{-6})_{sec}\right]\left[1 + s\,(7.5\times10^{-8})_{sec}\right]}$$

This gives the pole frequencies at

$f_{P_1} = 26\,\text{KHz}$ $\qquad$ $f_{P_2} = 2\,\text{MHz}$

2.) The transfer function can be written as:

$$\frac{V_{out}}{V_{gen}} = \left.\frac{V_o}{V_{gen}}\right|_{MB} \left[\frac{s\tau_{in}}{1+s\tau_{in}}\right]\left[\frac{s\tau_{out}}{1+s\tau_{out}}\right]$$

where

$$\tau_{in} = C_{in}\,(R_{gen} + R_{g_1} \| R_{g_2}) = \underline{6\,\text{usec}}$$

$$\tau_{out} = C_{out}\,(R_D + 2R_D) = \underline{45\,\text{nsec}}$$

$$\left.\frac{V_o}{V_{gen}}\right|_{MB} = -10$$

Step response of amplifier:

$$V_{gen}(s) = \frac{1\,\text{mV}}{s} \longrightarrow V_{gen}(t) = (1\,\text{mV})\,u(t)$$

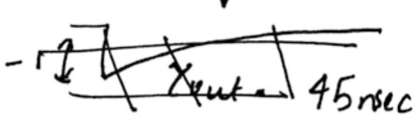
Then

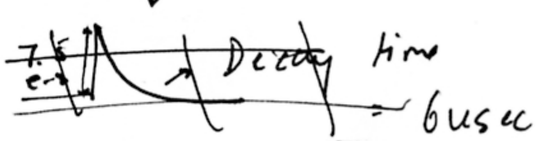
$$V_{out}(s) = \left(\frac{\tau_{in}}{1+s\tau_{in}}\right)\cdot\left(\frac{\tau_{out}}{1+s\tau_{out}}\right)\cdot(s\cdot)\,(A_{VMB})\cdot(1\,\text{mV})$$

$$= (A_{VMB})(1\,\text{mV})\left[\underbrace{\frac{\tau_{in}}{\tau_{in}-\tau_{out}}}_{1.00}\cdot\frac{\tau_{out}}{1+s\tau_{out}} + \underbrace{\frac{\tau_{out}}{\tau_{out}-\tau_{in}}}_{(-7.5\times10^{-3})}\cdot\frac{\tau_{in}}{1+s\tau_{in}}\right]$$

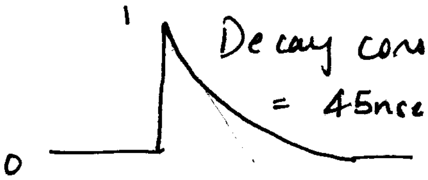
$$\therefore\ V_{out}(t) = -(10\,\text{mV})\left[(1.00)\,e^{-t/\tau_{out}} - (7.5e{-3})\,e^{-t/\tau_{in}}\right]$$

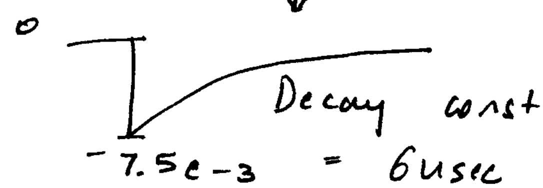
(PTO)



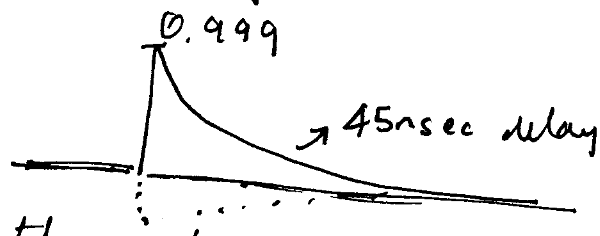
$$V_{OUT}(t) = -10\,mV \cdot \left[ 1.00\, e^{-t/\tau_{out}} - (7.5e-3)\, e^{-t/\tau_{in}} \right]$$



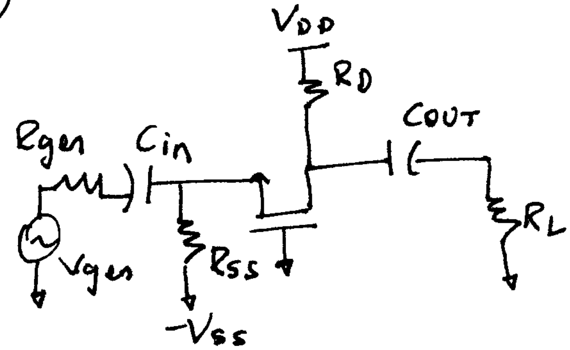




(Curves normalized to gain)

Total step response




The lower of the two frequency poles contributes only a small amount to the step response. → Step response is dominated by the high frequency pole.




$V_{DD} = 1.5V \qquad -V_{SS} = -1.5V$

$V_{sat} C_{ox} W_g = 10mA/V \qquad V_{th} = 0.5V$

$V_D = 0.5V \qquad I_D = 1mA$

$R_L = 3 R_D \qquad R_{gen} = 50\Omega$

$C_{gs} = 10 * C_{gd} \qquad f_T = 150 GHz.$

$G_{ds} = \frac{g_m}{50}$

Design: $V_D = V_{DD} - I_D R_D \Rightarrow R_D = \frac{V_{DD} - V_D}{I_D}$

$\therefore R_D = \left(\frac{1.5V - 0.5V}{1mA}\right) = \boxed{1K\Omega}$

$\Rightarrow \boxed{R_L = 3 R_D = 3 K\Omega}$

Also, $I_D = g_m \cdot [V_{gs} - V_{th}] \Rightarrow V_{gs} = \frac{I_D}{g_m} + V_{th}$

Since $V_g = 0 \Rightarrow V_s = -\left[\frac{I_D}{g_m} + V_{th}\right] = -\left[\frac{1mA}{10mA/V} + 0.5V\right]$

$\Rightarrow V_s = \underline{-0.6 Volts.}$

Now, $V_s = -V_{ss} + I_D R_s \Rightarrow R_s = \frac{V_s + V_{ss}}{I_D}$

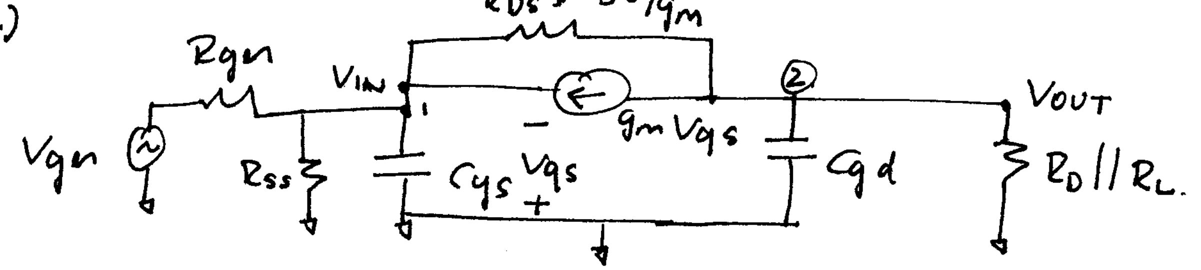
$= \frac{+(-0.6 + 1.5)V}{1mA}$

$\Rightarrow \boxed{R_s = 900\Omega}$

$f_T = \frac{g_m}{2\pi [C_{gs} + C_{gd}]} \Rightarrow 11 C_{gd} = \frac{g_m}{\omega_T}$

$\Rightarrow C_{gd} = 0.964 fF \qquad C_{gs} = 9.64 fF$

b.)




<u>KCL at node 1:</u>

$$\left(\frac{V_{in} - V_{gn}}{R_{gn}}\right) + \frac{V_{in}}{R_{SS}} + V_{in}(sC_{gs}) - g_m V_{gs} + (V_{in} - V_{out}) G_{DS} = 0$$

$$\therefore V_{in}\left[\frac{1}{R_{gn}} + \frac{1}{R_{SS}} + sC_{gs} + g_m + G_{DS}\right] + V_{out}\left[-G_{DS}\right] = \frac{V_{gn}}{R_{gn}} \qquad - (1)$$

<u>KCL at node 2:</u>

$$g_m V_{gs} + (V_{OUT} - V_{IN}) G_{DS} + V_{OUT}(sC_{gd}) + \frac{V_{out}}{(R_D \| R_L)} = 0$$

$$\therefore V_{out}\left[G_{DS} + sC_{gd} + G_L\right] + V_{in}\left[-G_{DS} - g_m\right] = 0 \qquad \left(G_L = \frac{1}{R_D \| R_L}\right)$$

$$\therefore V_{in} = V_{OUT}\left[\frac{G_{DS} + G_L + sC_{gd}}{g_m + G_{DS}}\right] \qquad - (2)$$

Put (2) into (1) to get

$$V_{OUT} \cdot \left\{\left[\frac{G_{DS} + G_L + sC_{gd}}{g_m + G_{DS}}\right]\left[\frac{1}{R_{gn}} + \frac{1}{R_{SS}} + sC_{gs} + g_m + G_{DS}\right] - G_{DS}\right\} = \frac{V_{gn}}{R_{gn}}$$

$$\therefore \frac{V_{OUT}}{V_{gn}} = \left(\frac{g_m + G_{DS}}{R_{gn}}\right)\left[\frac{1}{(G_{DS} + G_L + sC_{gd})\left(\frac{1}{R_{gn}} + \frac{1}{R_{SS}} + sC_{gs} + g_m + G_{DS}\right) - (g_m + G_{DS})G_{DS}}\right]$$

$$\therefore \frac{V_{out}}{V_{gn}} = (2.04 \times 10^{-4})\left[\frac{1}{[1.5e{-3} + s(0.964\,f)(\text{sec})][0.0313 + s\,9.64\,f(\text{sec})] - (2.04e{-6})}\right]$$

b.) NOTE: With $R_{DS}$ very high the circuit can be solved as has been shown in the notes

However, with $R_{DS}$ significantly smaller we will have to use nodal analysis to reach the solution

$$\frac{V_{OUT}}{V_{gn}} = \frac{(2.04 \times 10^{-4})}{(4.7\times10^{-5}) + s(0.0446\times10^{-15}) + s^2(9.3\times10^{-30}) - (2.04e-6)}$$

$$= \frac{2.04\times10^{-4}}{(4.49\times10^{-5}) + s(0.044\times10^{-15})_{(sec)} + s^2(9.3\times10^{-30})_{(sec)^2}}$$

$$= \frac{4.54}{1 + (9.8\times10^{-13})s_{(sec)} + (2\times10^{-25})s^2_{(sec)^2}}$$

Poles will correspond to s values of

$s_1 = -1.44 \times 10^{12}$ $\quad$ $s_2 = -3.45 \times 10^{12}$

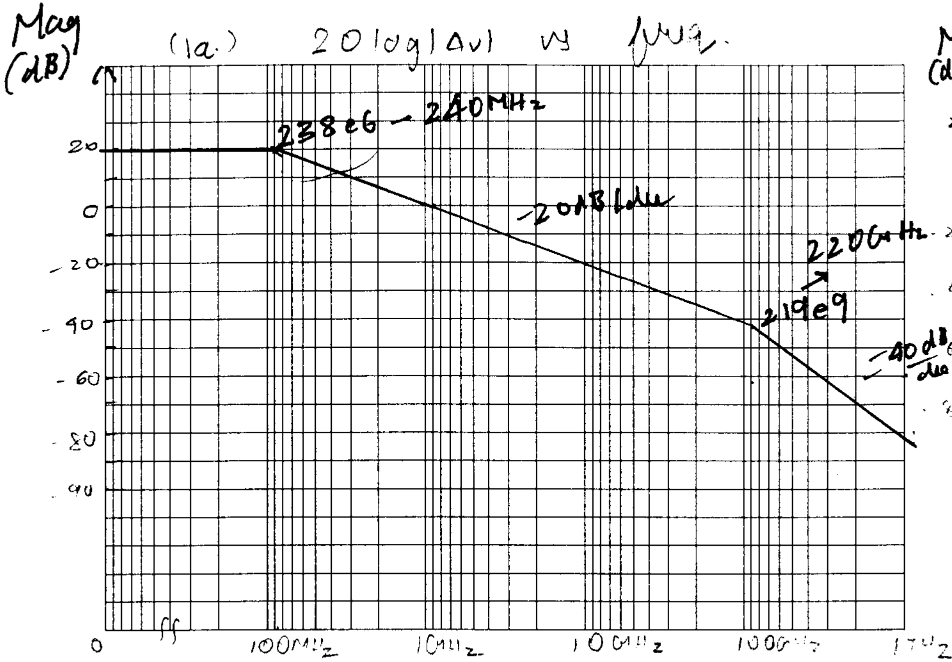
Put $s = j\omega$ to get

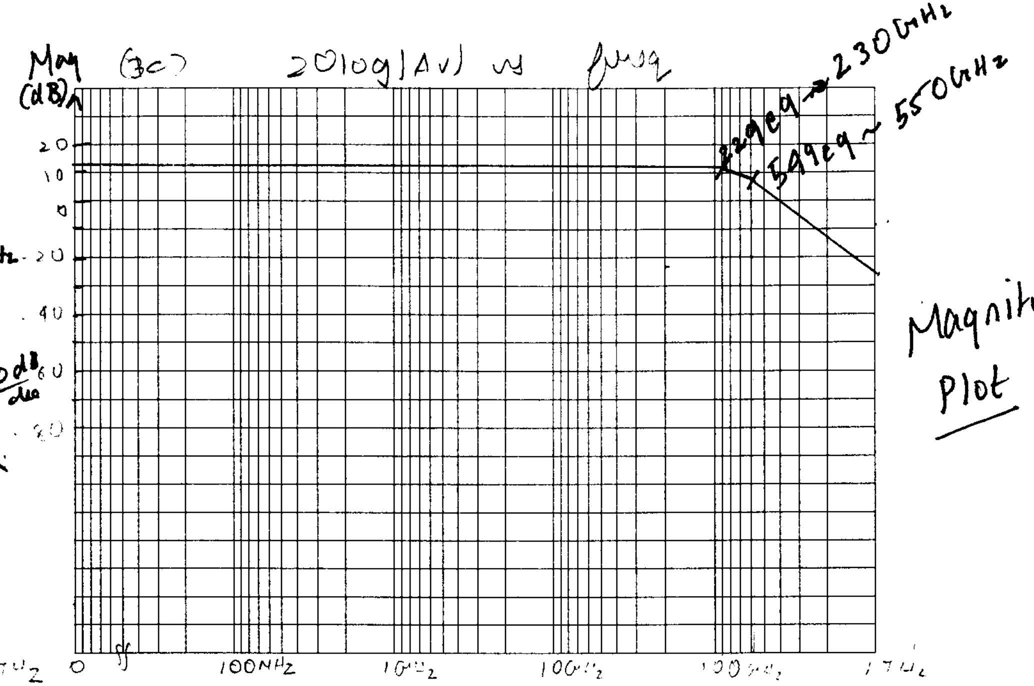
$\omega_{P_1} = 1.44 \times 10^{12}$ R/sec $\quad$ $\omega_{P_2} = 3.45 \times 10^{12}$ R/sec

which corresponds to:

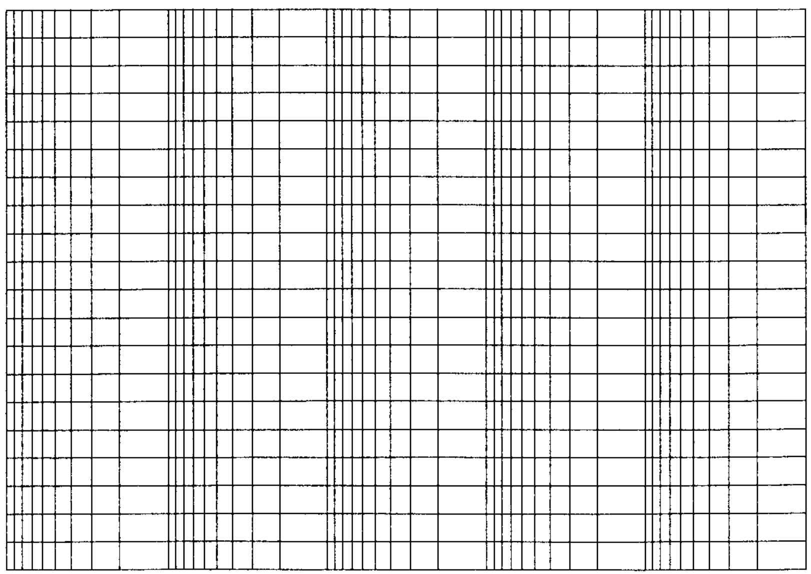
$f_{P_1} = \underline{0.229 \times 10^{12}\ Hz}$ $\quad$ $f_{P_2} = \underline{0.549 \times 10^{12}\ Hz}$

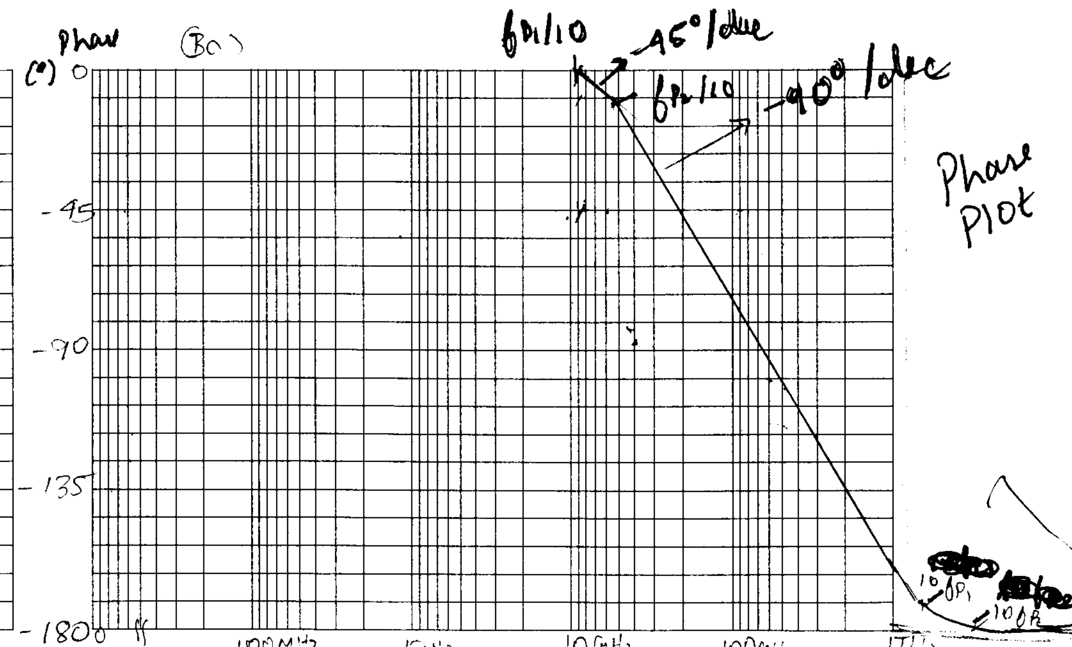
$f_{P_1} = 229$ GHz $\quad$ $f_{P_2} = 549$ GHz




Magnitude Plot






Phase Plot