Q1:

sol:

(a)

$$\frac{V_o}{V_{in}}(s) = C \cdot \frac{1 + b_1 s + b_2 s^2 + \cdots}{1 + a_1 s + a_2 s^2 + \cdots}$$

By MOTC, we can redraw the schematic as following

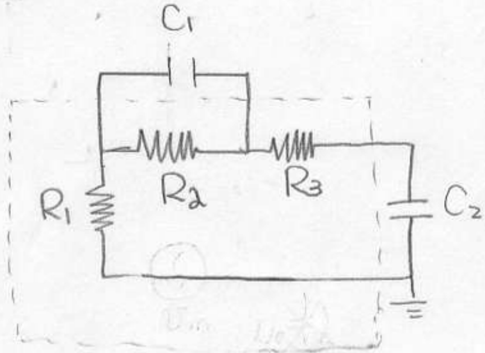
$C_1 = 1\ (fF)$

$C_2 = 2\ (fF)$

$R_1 = 3\ (k\Omega)$

$R_2 = 2\ (k\Omega)$

$R_3 = 1\ (k\Omega)$



$R_{11}^{0} = R_2$

$R_{22}^{0} = R_1 + R_2 + R_3$

$R_{11}^{2} = R_2 // (R_1 + R_3)$

$R_{22}^{1} = (R_1 + R_3)$

$$\therefore a_1 = C_1 R_{11}^{0} + C_2 R_{22}^{0}$$

$$= C_1 R_2 + C_2 (R_1 + R_2 + R_3) = 1(fF) \cdot 2(k\Omega) + 2(fF) \cdot 6(k\Omega)$$

$$= \underline{14\,(ps)}$$

$$a_2 = C_1 C_2 (R_{11}^{0} R_{22}^{1})$$

$$= C_1 C_2 (R_2 \cdot (R_1 + R_3)) = 1(fF) \cdot 2(fF) \left(2(k\Omega) \cdot (3(k\Omega) + 1(k\Omega)) \right)$$

$$= \underline{16\,(ps)^2}$$

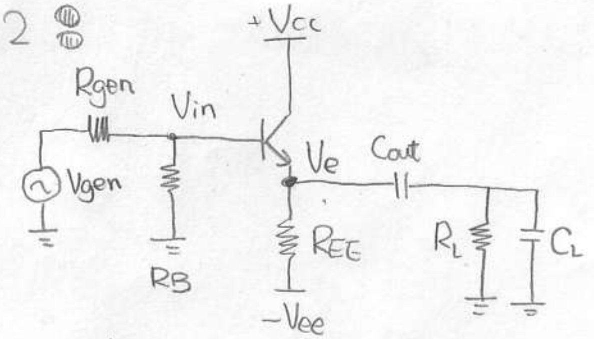
(b) $\frac{a_2}{a_1} = 1.14\ (ps)$ $\therefore a_1 >> \frac{a_2}{a_1}$

$\Rightarrow$ we may use separate pole approximation

$$\therefore f_{p1} = \frac{0.159}{14\,(ps)} = \underline{11.36\,(GHz)}$$

$$f_{p2} = \frac{0.159}{1.07\,(ps)} = \underline{139.5\,(GHz)}$$

Q2



$\tau_f = 1(ps)$, $C_{je} = C_{cb} = 20\ (fF)$

$\beta = 100$, $R_B = 10 \cdot R_{gen} = 10\ (k\Omega)$

$V_{cc} = +3.3(V)$, $-V_{ee} = -3.3(V)$

$R_{gen} = 1(k\Omega)$, $C_L = 50\ (fF)$

$R_L = 100\ (\Omega)$, $I_E \approx I_C = 5(mA)$

- Parameter Calculation

$$g_m \simeq \frac{1}{r_e} = \frac{I_E}{V_T} = \frac{5(mA)}{26(mV)} = \underline{192\ (mS)}$$

$$C_{be} = g_m \tau_f + C_{je} = 192\ (mS) \cdot 1\times10^{-12}\ (sec) + 20\times10^{-15}\ (F) = \underline{212\ (fF)}$$

$$f_T = \frac{g_m}{2\pi\ (C_{cb} + C_{be})} = \frac{192\ (mS) \cdot 0.159}{20\ (fF) + 212 (fF)} = \underline{131.6 (GHz)}$$

(a) • Bias Condition   assume $V_{be\,(ON)} = 0.7(V)$

$$I_B = \frac{I_c}{\beta} = 50(\mu A), \quad V_{in\,(DC)} = 0 - I_B \cdot (R_{gen} // R_B) = -46 (mV)$$

$$V_{be} = V_{in} - V_e$$

$$\Rightarrow V_e = V_{in} - V_{be} = -0.746\ (V)$$

$$\therefore R_{EE} = \frac{V_e - V_{ee}}{I_E} = \frac{-0.746(V) - (-3.3(V))}{5\ (mA)} = 511\ (\Omega)$$

- Calculate $\boxed{\text{Mid-band}}$ $\frac{V_{out}}{V_{in}}$, $\frac{V_{in}}{V_{gen}}$, Rout_amp, Rin_amp

$$r_e = \frac{V_T}{I_E} = \frac{26(mV)}{5\ (mA)} = 5.2\ (\Omega)$$

$$R_{L,eq} = R_{EE} // R_L = 511\ (\Omega) // 100\ (\Omega) = 83.6 (\Omega) \quad (\text{assume } R_{ce} = \infty(\Omega))$$

$$\therefore \left.\frac{V_{out}}{V_{in}}\right|_{MB} = \frac{R_{L,eq}}{r_e + R_{L,eq}} = \underline{0.942}$$

$$R_{in,T} = (1+\beta)(R_{L,eq}+r_e) = (101)\cdot(83.6(\Omega)+5.2(\Omega)) \simeq 8.97\ (k\Omega)$$

$$\therefore\ R_{in\_amp} = R_B \,//\, R_{in,T} = \frac{10(k\Omega)\cdot 8.97(k\Omega)}{10(k\Omega)+8.97(k\Omega)} \simeq \underline{4.73\ (k\Omega)}$$

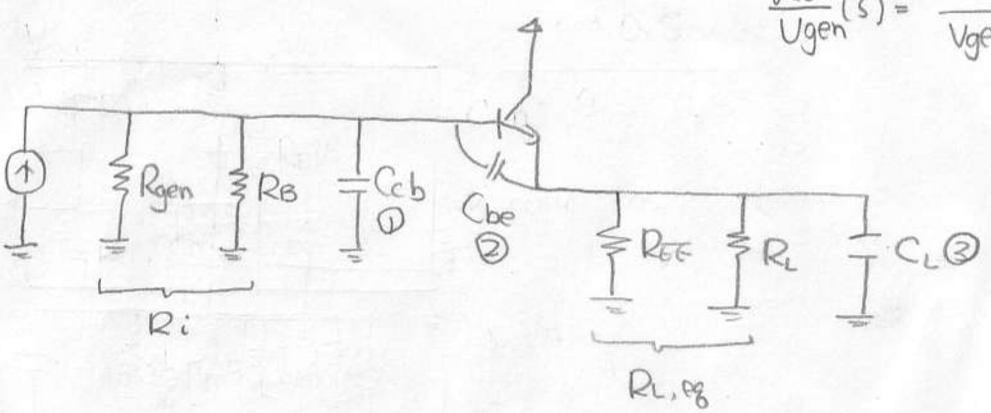
$$\therefore\ \left.\frac{V_{in}}{V_{gen}}\right|_{MB} = \frac{4.73(k\Omega)}{1(k\Omega)+4.73(k\Omega)} = \underline{0.825}$$

$$R_{out,T} = \frac{(R_{gen}//R_B)}{1+\beta} + r_e = \frac{1(k\Omega)//10(k\Omega)}{101} + 5.2(\Omega) = 14.2(\Omega)$$

$$\therefore\ R_{out\_amp} = R_{out,T} \,//\, R_{EE} = \underline{13.82(\Omega)}$$

(b) MOTC

$$\frac{V_{out}}{V_{gen}}(s) = \left.\frac{V_{out}}{V_{gen}}\right|_{MB} \cdot \frac{1+b_1 s}{1+a_1 s+a_2 s^2}$$



$$R_{11}^{o} = R_i \,//\, R_{in,T} = 1(k\Omega)//10(k\Omega)//8.97(k\Omega) \doteq 825(\Omega)$$

$$R_{33}^{o} = R_{L,eq} \,//\, R_{out,T} = 83.6(\Omega)\,//\,14.2(\Omega) = 12.14(\Omega)$$

$$R_{22}^{o} = \frac{(1+\beta)}{g_m} \,//\, \left[R_i\left(1-\frac{R_{L,eq}}{R_{L,eq}+r_e}\right) + r_e//R_{L,eq}\right] = 5.23\ (\Omega)$$

$$a_1 = C_{cb}\cdot R_{11}^{o} + C_{be}\cdot R_{22}^{o} + C_L\cdot R_{33}^{o}$$

$$= 20(fF)\cdot 825(\Omega) + 212(fF)\cdot 5.23(\Omega) + 50(fF)\cdot 12.14(\Omega)$$

$$= 18.22(ps)$$

$$a_2 = (R_i \,//\, R_{in,T})\left(1/g_m \,//\, R_{L,eq}\right)\cdot\left(C_{cb}\cdot C_{be} + C_{cb}\cdot C_L + C_{be}\cdot C_L\right)$$

$$= 825(\Omega)\cdot 4.9(\Omega)\cdot\left(20(fF)\cdot 212(fF) + 20(fF)\cdot 50(fF) + 212(fF)\cdot 50(fF)\right)$$

$$= 64\ (ps)^2$$

$$\frac{a_2}{a_1} = 3.153\ (pS)$$

$\because$ a_1 is only 5.78 times bigger than $\frac{a_2}{a_1}$

$\therefore$ Separate pole approximate is invalid here

$\rightarrow$ solve $a_2 \cdot S^2 + a_1 S + 1 = 0$

$$\Rightarrow f_{P1}, f_{P2} = 0.159 \cdot \left[\frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2a_2} \right] = 11.8\ (GHz),\ 33.5\ (GHz)$$

$\therefore$ $f_{P1} = 11.8\ (GHz)$, $f_{P2} = 33.5\ (GHz)$, $f_{3dB} \approx f_{P1} = 11.8\ (GHz)$

two real poles

(c) Bode plot

$$\left.\frac{V_{out}}{V_{gen}}\right|_{MB} = 0.826 \cdot 0.942 = 0.777 = -2.19(dB)$$

the transfer function has a zero at

$$f_z = 0.159 \cdot \frac{g_m}{C_{be}} = 144\ (GHz)$$

$\therefore$ Total Transfer function is

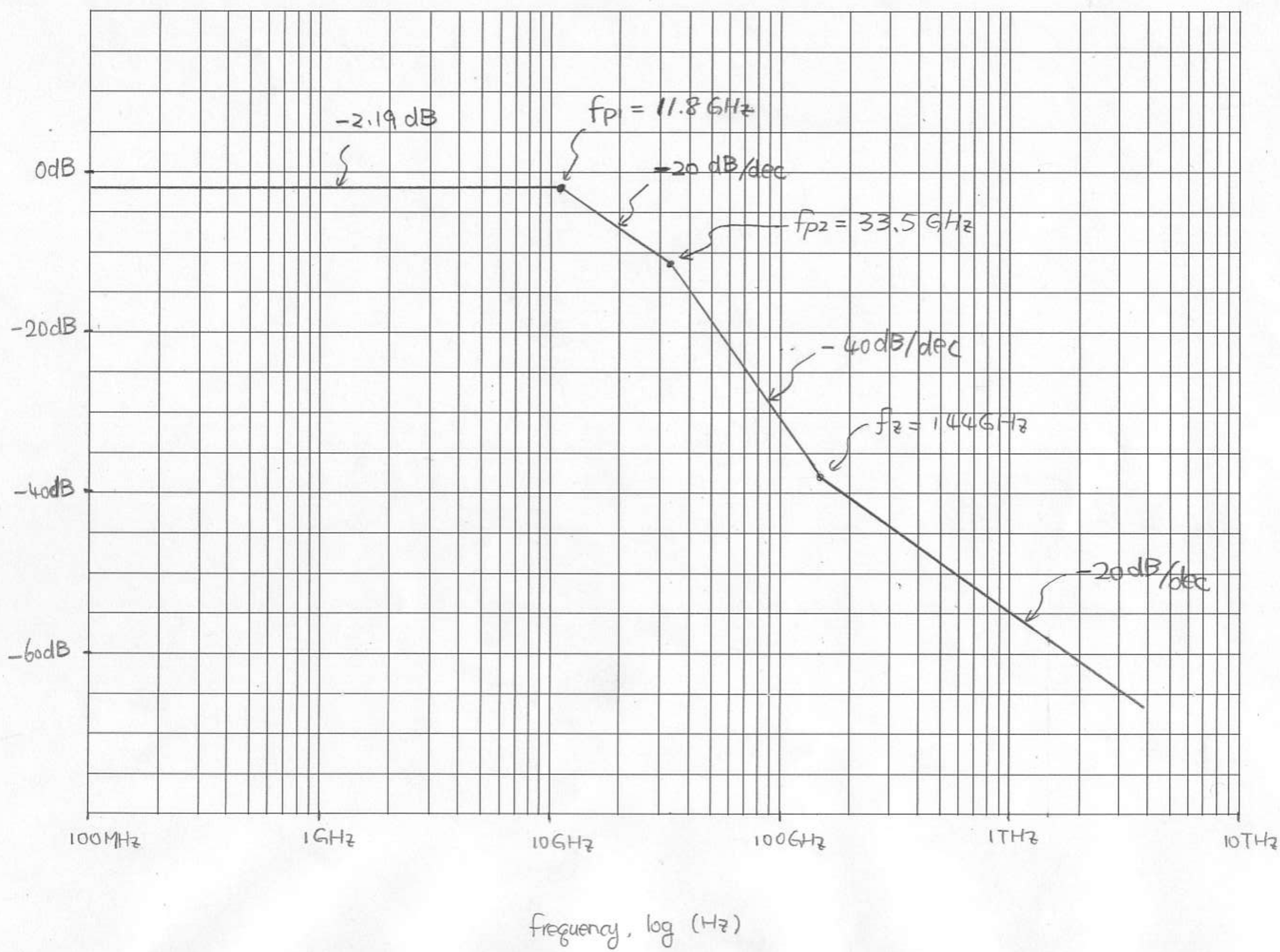
$$\frac{V_{out}}{V_{gen}}(S) = 0.777 \cdot \frac{1 + j\frac{f}{144(GHz)}}{\left(1 + j\frac{f}{11.8(GHz)}\right)\left(1 + j\frac{f}{33.5(GHz)}\right)}$$

bode polt is in next page

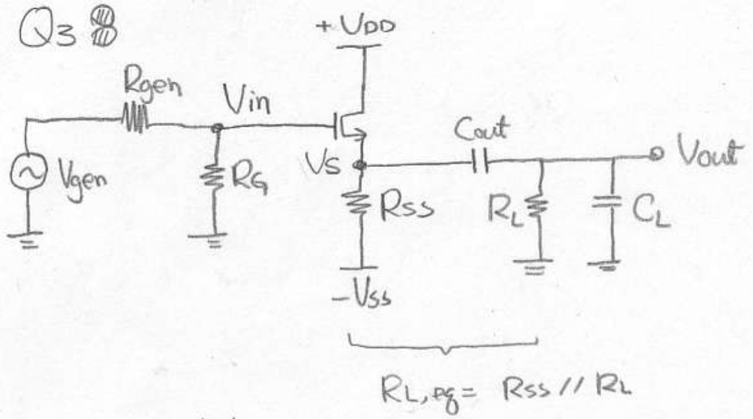
Question 2.

$\left|\frac{V_{out}}{V_{in}}\right|$ dB

3dB frequency $\approx$ 11.8 GHz



Q3 ⑧



$g_m = 10\,(mS)$, $\lambda = 0$, $L_g = 90\,(nm)$

$I_D = I_S = 2mA$, $V_{sat} = 10^5\,(\frac{m}{s})$

$V_{DD} = +1\,(V)$, $-V_{SS} = -1\,(V)$

$V_{th} = 0.3\,(V)$, $\lambda = 0$

$\epsilon_r = 3.8$ $R_L = 2\,(k\Omega)$, $C_L = 200\,(fF)$

$R_G = 20\,(k\Omega)$, $R_{gen} = 2\,(k\Omega)$

$T_{ox} = 1.6\,(nm)$

• Parameter calculation

$$\begin{cases} C_{ox} = \dfrac{\epsilon_r \cdot \epsilon_0}{T_{ox}} = \dfrac{3.8 \times 8.854 \times 10^{-12}\,(\frac{F}{m})}{1.6 \times 10^{-9}\,(m)} = 2.1 \times 10^{-2}\,(\frac{F}{m^2}) \\ W_g = \dfrac{g_m}{V_{sat} \cdot C_{ox}} = \dfrac{10 \times 10^{-3}\,(\frac{A}{V})}{10^5\,(\frac{m}{s}) \cdot 2.1 \times 10^{-2}\,(\frac{F}{m^2})} = 4.76\,um \\ C_{gs} = C_{ox} \cdot L_g \cdot W_g = 9\,(fF) \\ C_{gd} = \dfrac{C_{gs}}{10} = 0.9\,(fF) \end{cases}$$

(a) • Bias Condition

$I_G = 0$, $V_G = 0$

$\because L_g = 90\,(nm)$ $\therefore$ Velocity model is applied

$\therefore I_D = g_m (V_{GS} - V_{th})(1 + \cancel{\lambda V_{DS}}^{\,0})$ when $V_{gs} \geq V_{th}$

$\Rightarrow V_{GS} = \dfrac{I_D}{g_m} + V_{th} = 0.5\,(V)$

$\therefore V_S = V_G - V_{GS} = -0.5\,(V)$

$\therefore R_{SS} = \dfrac{V_S - V_{SS}}{I_D} = \dfrac{-0.5\,(V) + 1\,(V)}{2\,(mA)} = \underline{250\,(\Omega)}$

$\therefore R_{L,eq} = R_{SS} // R_L = 250\,(\Omega) // 2\,(k\Omega) = 222\,(\Omega)$

$$\therefore \left.\frac{V_{out}}{V_{in}}\right|_{MB} = \frac{R_{L,eq}}{\frac{1}{g_m} + R_{L,eq}} = \underline{0.689}$$

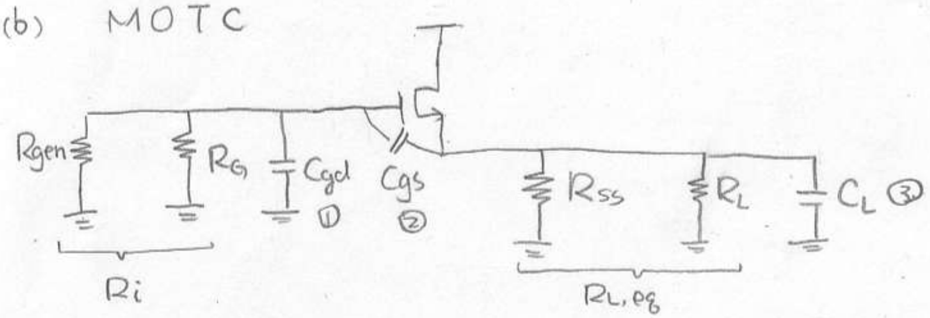
$$R_{in\_amp} = R_G \,//\, R_{in,T} = 20(k\Omega) \,//\, \infty(\Omega) = \underline{20(k\Omega)}$$

$$\therefore \left.\frac{V_{in}}{V_{gen}}\right|_{MB} = \frac{R_{in\_amp}}{R_{gen} + R_{in\_amp}} = \underline{0.909}$$

$$R_{out,T} = \frac{1}{g_m} = 100\Omega$$

$$\therefore R_{out\_amp} = R_{SS} \,//\, R_{out,T} = 71.43(\Omega).$$

(b) MOTC



$$R_{11}^{o} = R_{gen} \,//\, R_G = 1.82(k\Omega)$$

$$R_{22}^{o} = \frac{1}{g_m} \,//\, R_{L,eq} + R_i\,(1 - A_v|_{MB}) = 634.4\ (\Omega)$$

$$R_{33}^{o} = R_{L,eq} \,//\, \frac{1}{g_m} = 68.94(\Omega)$$

$$\therefore a_1 = C_{gd}\cdot R_{11}^{o} + C_{gs}\cdot R_{22}^{o} + C_L\cdot R_{33}^{o}$$

$$= 0.9(fF)\cdot 1.82(k\Omega) + 9(fF)\cdot 634.4(\Omega) + 200(fF)\cdot 68.94(\Omega)$$

$$\simeq 21.03(ps)$$

$$a_2 = R_i\cdot\left(R_{L,eq} \,//\, \frac{1}{g_m}\right)\cdot\left(C_{gd}\cdot C_{gs} + C_{gd}\cdot C_L + C_{gs}\cdot C_L\right)$$

$$= 1.82(k\Omega)\cdot 68.94(\Omega)\cdot\left(0.9(fF)\cdot 9(fF) + 0.9(fF)\cdot 200(fF) + 9(fF)\cdot 200(fF)\right)$$

$$= 249.46\ (ps)^2$$

examine $\frac{a_2}{a_1} = 11.86(ps)$

$\because a_1$ is only 1.77 times bigger than $\frac{a_2}{a_1}$

$\therefore$ Separate pole approximation is invalid here

$\Rightarrow$ solve $a_2 S^2 + a_1 S + 1 = 0$

$$\Rightarrow f_{p1}, f_{p2} = 0.159 \cdot \left[\frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2a_2} \right] = -6.702 \pm j\,7.512 \text{ (GHz)}.$$

<u>two complex poles</u>

- $a_2 S^2 + a_1 S + 1 = \left(\frac{1}{\omega_n}\right)^2 \cdot S^2 + \left(\frac{2\xi}{\omega_n}\right) \cdot S + 1$

$\therefore$ $f_n = \frac{0.159}{\sqrt{a_2}} = 10 \text{ (GHz)}$ (natural frequency)

$\xi = \frac{a_1}{2 \cdot \sqrt{a_2}} = 0.667$

$\therefore$ $f_d = f_n \sqrt{1 - \xi^2} = 7.45$ GHz (peak frequency)

and 3 dB BW is roughly at $f_n = 10 \text{(GHz)}$.

(C)

The transfer function has a zero at

$$f_z = 0.159 \cdot \frac{g_m}{C_{gs}} = 176.67 \text{(GHz)}$$

$$\left.\frac{V_{out}}{V_{gen}}\right|_{MB} = 0.909 \times 0.689 = 0.626 = -4.07 \text{ (dB)}$$

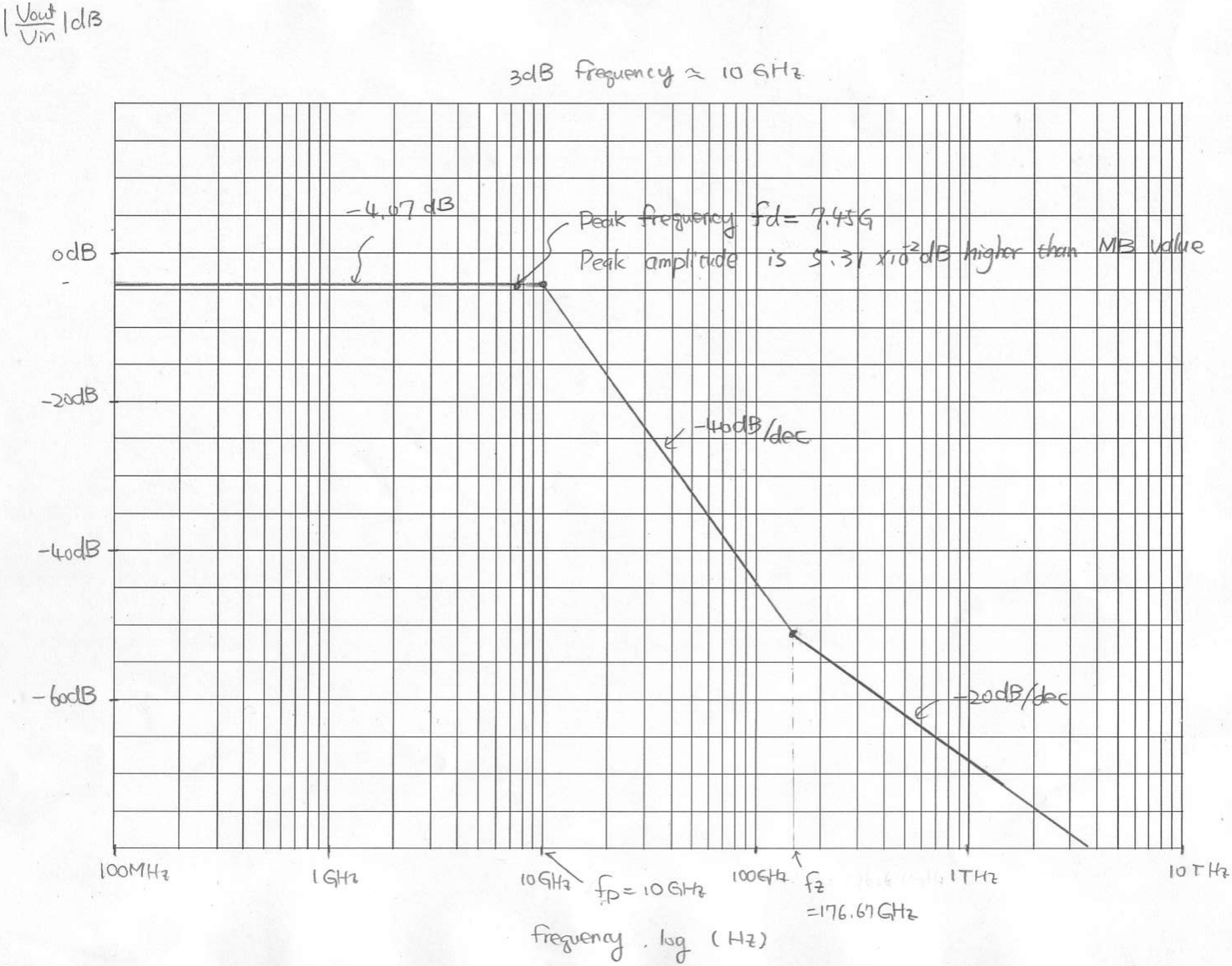
Peak height $= -20 \log_{10} \left(2\xi \sqrt{1 - \xi^2} \right) = 5.31 \times 10^{-2}$ (dB) ... small peak.

($\xi = 0.7071$ will give "zero peak")

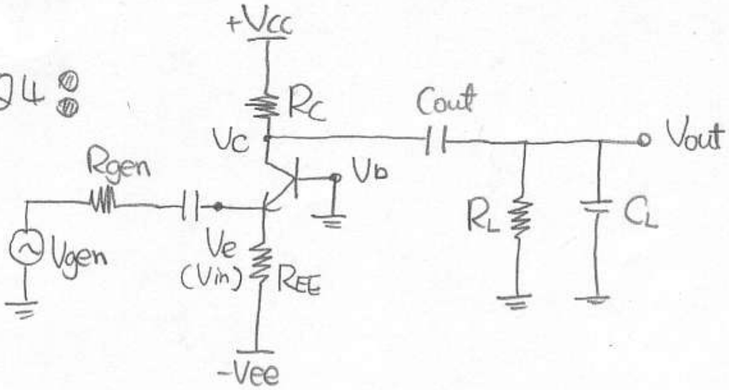
the bode plot is in next page

Question 3

3dB frequency $\simeq$ 10 GHz



Q4 :



$\tau_F = 1\,(ps)$, $C_{je} = 50\,(fF)$

$C_{cb} = 100\,(fF)$, $\beta = 50$

$V_{cc} = 5(V)$, $-V_{ee} = -5\,(V)$

$R_L = 1\,(k\Omega)$, $C_L = 200(fF)$

$R_{gen} = 50(\Omega)$

$I_E \approx I_C = 2mA$, $V_{CE} = 2\,(V)$

· Parameter calculation

$$\begin{cases} g_m \simeq \dfrac{1}{r_e} = \dfrac{I_E}{V_T} = 77\,(mS) \\ C_{be} = g_m \tau_F + C_{je} = 77(mS)\cdot 1(ps) + 50(fF) = 127(fF) \\ f_T = \dfrac{g_m}{2\pi(C_{cb}+C_{be})} = 53.9\,(GHz) \end{cases}$$

(a) · Bias Condition assume $V_{BE(ON)} = 0.7\,(V)$

$$\therefore \begin{cases} V_e = V_b - V_{be(ON)} = -0.7\,(V) \\ V_c = V_e + 2(V) = 1.3(V) \\ R_{EE} = \dfrac{V_e + V_{ee}}{I_E} = \dfrac{-0.7(V) + 5(V)}{2\,(mA)} = 2.15\,(k\Omega) \\ R_C = \dfrac{V_{cc} - V_c}{I_C} = \dfrac{5(V) - 1.3(V)}{2(mA)} = 1.85\,(k\Omega) \end{cases}$$

· Calculate the $\boxed{\text{Mid-band}}$ $\dfrac{V_{out}}{V_{in}}$, $\dfrac{V_{in}}{V_{out}}$, Rout_amp, Rin_amp

assume $R_{ce} = \infty\,(\Omega)$

$$r_e = \frac{V_T}{I_E} = 13\,(\Omega)$$

$\therefore$ $R_{in,T} = r_e$ and $R_{out,T} = R_{ce}(1 + g_m R_{EE}) = \infty\,(\Omega)$

$\therefore$ $R_{in\_amp} = r_e // R_{EE} = 13(\Omega) // 2.15(k\Omega) = \underline{12.92\,(\Omega)}$

$R_{L,eq} = R_C // R_L = 649\,(\Omega)$

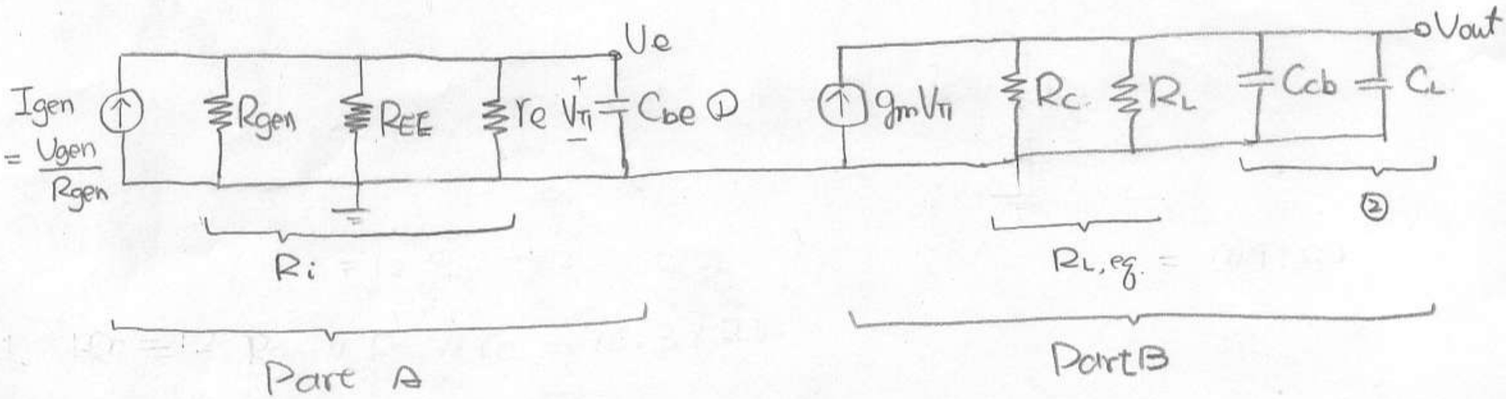
$\therefore$ $R_{out\_amp} = R_{L,eq} // R_{out,T} = \underline{649\,(\Omega)}$

$$\left.\frac{V_{out}}{V_{in}}\right|_{MB} = \frac{R_{y,eq}}{R_{in\_amp}} = \frac{649(\Omega)}{12.92(\Omega)} = \underline{50.23}$$

$$\left.\frac{V_{in}}{V_{gen}}\right|_{MB} = \frac{R_{in\_amp}}{R_{gen} + R_{in\_amp}} = \frac{12.92(\Omega)}{50(\Omega)+12.92(\Omega)} = \underline{0.205}$$

(b)

- Small-signal circuit diagram is shown below



∵ Part A and B have no direct connections

∴ we may consider the whole transfer function as the product of transfer functions of part A and B. due to the dependent current source.

- Part A

$\tau_A = R_i \cdot C_{be} = (R_{gen} // R_{EE} // r_e) \cdot C_{be} = 10.3(\Omega) \cdot 127(fF)$

$= 1.31\ (ps)$

- Part B.

$\tau_B = R_{L,eq} \cdot (C_L + C_{cb}) = 649(\Omega) \cdot (100(fF) + 200(fF)) = 195\ (ps)$

$\therefore\ a_1 = \tau_A + \tau_B = \underline{196.31\,(ps)}$

$a_2 = \tau_A \cdot \tau_B = \underline{255.45\,(ps)^2}$

and
$$\begin{cases} f_{p,A} = \dfrac{0.159}{\tau_A} = \dfrac{0.159}{1.31\,(ps)} = \underline{121.4\,(GHz)} \\ f_{p,B} = \dfrac{0.159}{\tau_B} = \underline{815.38\,(MHz)} \end{cases}$$

$\therefore$ Dominate pole frequency is $\underline{815.38\,(MHz)} \simeq f_{3dB}$ frequency

(C) Bode Plot

- There is no zero frequency in this system

- $\left.\dfrac{V_{out}}{V_{gen}}\right|_{MB} = 50.23 \times 0.205 \simeq 10.3 \simeq 20.57\,(dB)$

- The total transfer function

$$\frac{V_{out}}{V_{gen}}(s) = 10.3 \cdot \frac{1}{\left(1 + j\dfrac{f}{815.38\,(MHz)}\right)\cdot\left(1 + j\dfrac{f}{121.4\,(GHz)}\right)}$$

The bode plot is in next page.

Question 4

3dB frequency ≈ 815.38 MHz