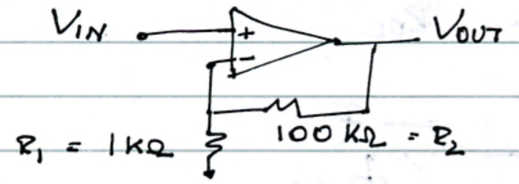
HW #6 SOLUTIONS

1.) 

$A_{diff} = 1{,}00{,}000$

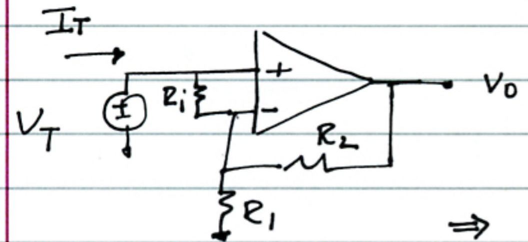
$$V_{OUT} = (V^+ - V^-)\,A_{DIFF} = \left(V_{IN} - \frac{V_{OUT}\cdot R_1}{R_1+R_2}\right)A_{DIFF}$$

$$V_{OUT}\left[1 + \frac{A_{DIFF}\cdot R_1}{R_1+R_2}\right] = V_{IN}\cdot A_{DIFF}$$

a) $$\Rightarrow \frac{V_{OUT}}{V_{IN}} = \frac{A_{DIFF}\,(R_1+R_2)}{R_1+R_2+A_{DIFF}\cdot R_1} = \frac{A_{DIFF}}{1+(A_{DIFF})\left(\frac{R_1}{R_1+R_2}\right)}$$

$$= \frac{10^5}{1+10^5\cdot\left[\frac{1K}{101K}\right]} = \boxed{100.891}$$

b.) $Z_{in\ open\ loop} = \underline{100\ K\Omega}.$



$$I_T = (V^+ - V^-)/R_i$$

$$\Rightarrow I_T = \left[V_T - V_O\left(R_1/(R_1+R_2)\right)\right]/R_i$$

$$\Rightarrow I_T = \left(V_T - \left(\frac{A_D}{1+A_D\beta}\right)V_T\cdot\beta\right)/R_i$$

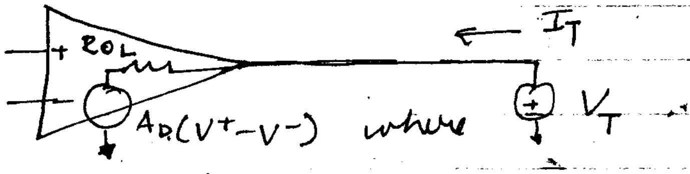
After Substituting $V_O = \left(V_T\cdot\frac{A}{1+A\beta}\right)$ where $\beta = \frac{R_1}{R_1+R_2}$

$$\Rightarrow I_T = \frac{\left(V_T - \left(\frac{T}{1+T}\right)V_T\right)}{R_i} \quad \text{where } T = A_D\beta$$

$$\rightarrow I_T = \frac{V_T}{(1+T)R_i} \Rightarrow \boxed{\frac{V_T}{I_T} = R_i(1+T)}$$

$$\Rightarrow Z_{in(close\ loop)} = (100K\Omega)(1+990.09) = \boxed{99M\Omega}$$

c.) Z_{OUT} open loop $= 10\Omega = R_{OL}$



$V^+ = V_{in} \quad V^- = \beta V_{out}$

$\beta = (R_1)/(R_1 + R_2)$

Here we get:

$$I_T = \frac{V_T - A_D(V^+ - V^-)}{R_{OL}} = \frac{V_T - A_D(0 - \beta V_T)}{R_{OL}}$$

$$\Rightarrow \quad I_T = (1 + A_D\beta)V_T / R_{OL}$$

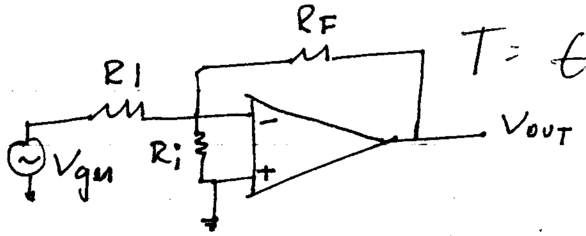
$$\Rightarrow \quad \boxed{\frac{V_T}{I_T} = \left(\frac{R_{OL}}{1 + A_D\beta}\right)} = R_{OUT,\,CL} = \frac{10}{1 + 990.1} = \underline{\underline{0.01\,\Omega}}$$

$\underline{R_{OUT,\,CL} = 0.01\Omega}$

Note: This result can be obtained directly by using the formula $\left(A_\infty \frac{T}{1+T}\right)$ where A_∞ = Loop gain for infinite fwd gain

T = ~~Closed~~ loop gain with effect of loading included

2.)



$A_{DIFF} = 1000$
$R_F = 1M\Omega$
$R_1 = 10k\Omega$
$R_i = 2k\Omega$

$$V_{OUT} = A_{DIFF}\cdot[V^+ - V^-] = -A_{DIFF}\cdot V^-$$
(Since $V^+ = 0$)

Now,

$$\cancel{V^- = V_{gen}\left(\frac{R_i \| R_F}{R_i\| R_F + R_1}\right) + V_{OUT}\cdot\left[\frac{R_1\| R_i}{R_1 \| R_i + R_F}\right]}$$

$$V^- = V_{gen}\cdot\left[\frac{R_i \| R_F}{R_i \| R_F + R_1}\right] + V_{OUT}\left[\frac{R_1 \| R_i}{R_1 \| R_i + R_F}\right]$$

$$\therefore \quad V_{OUT} = -A_{DIFF}\left[V_{gen}\left(\frac{R_i \| R_F}{R_i \| R_F + R_1}\right) + V_{OUT}\left(\frac{R_1 \| R_i}{R_1 \| R_i + R_F}\right)\right]$$

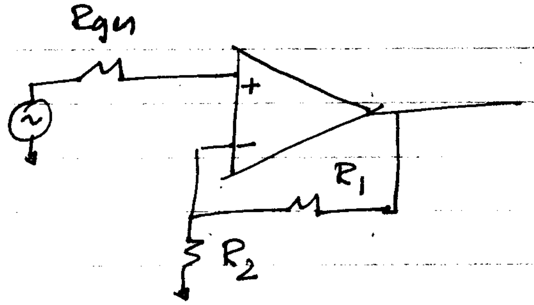
$$\therefore \quad V_{OUT}\left[1 + A_{DIFF}\cdot\left(\frac{R_1 \| R_i}{R_1 \| R_i + R_F}\right)\right] = -A_{DIFF}\cdot V_{gen}\cdot\left(\frac{R_i \| R_F}{R_i \| R_F + R_1}\right)$$

$$\therefore \quad \frac{V_{OUT}}{V_{gen}} = \frac{-A_{DIFF}\cdot\left[\frac{R_i \| R_F}{R_i \| R_F + R_1}\right]}{1 + A_{DIFF}\cdot\left(\frac{R_1 \| R_i}{R_1 \| R_i + R_F}\right)}$$

$$= \frac{-(1000)\left[\frac{1996}{1996 + 10k\Omega}\right]}{1 + 1000\left[\frac{1666}{1666 + 10^6}\right]} = \underline{-62.33}$$

Ans: $\frac{V_{OUT}}{V_{gen}} = -62.33$

3.)



$A_D = 10^6$.

f_{P1} ~~ω_{P1}~~ = 10 Hz

f_{P2} ~~ω_{P2}~~ = 1MHz

f_{P3} ~~ω_{P3}~~ = 10MHz

a) R1 = 0 R2 = 1KΩ → $\beta = 1$

b) R_1 = 9KΩ R2 = 1KΩ → $\beta = 0.9$

a) $\Phi_m = 0°$ GM = 0° Loop B/W = 7.5MHz

b) $\Phi_m = 45°$ GM = < 5dB Loop B/W = 1MHz

If $$T(j\omega) = \frac{1 + (j\omega/\omega_{z_1})}{(1 + j\omega/\omega_{p_1})}$$

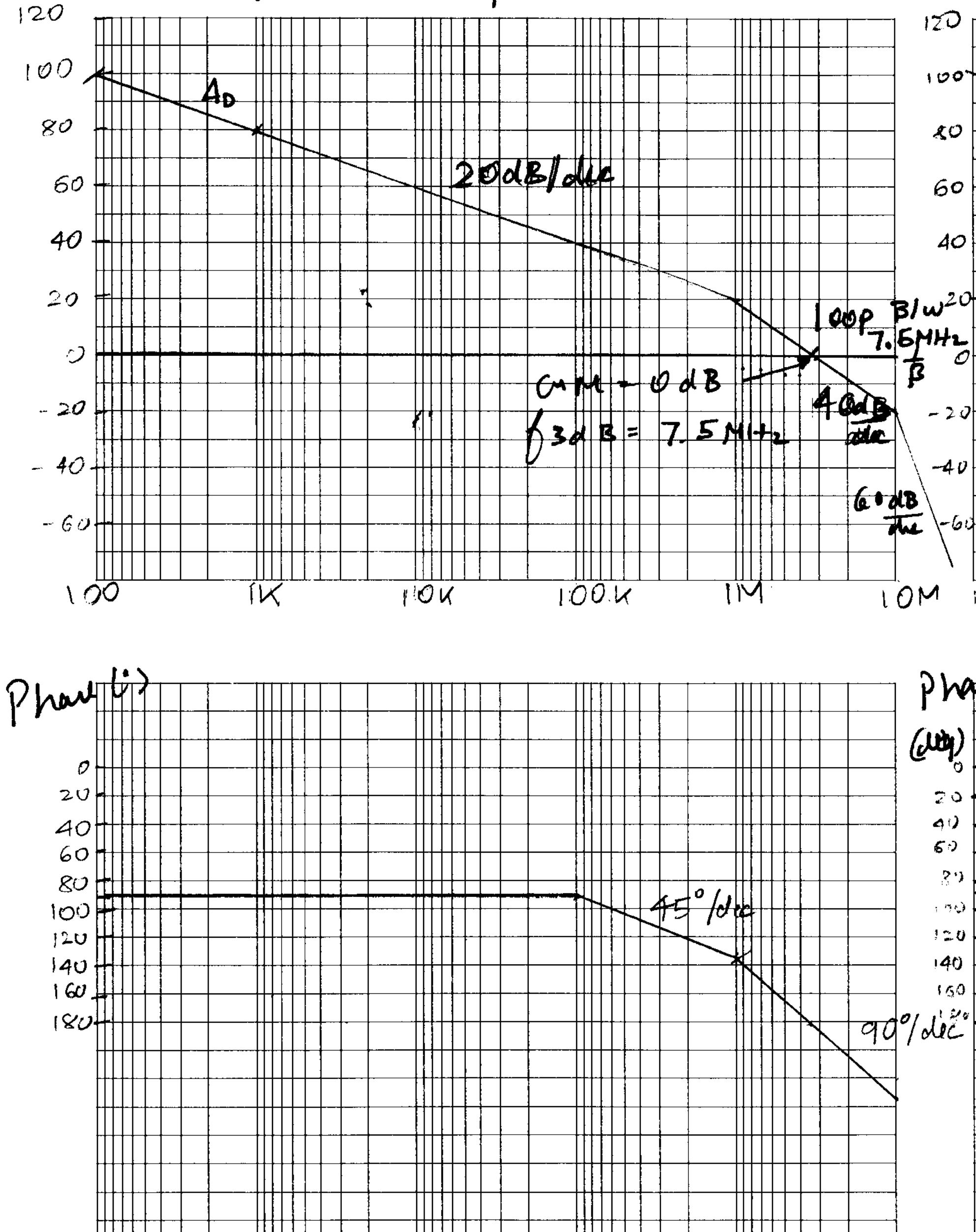
$$\angle T(j\omega) = \tan^{-1}(\omega/\omega_{z_1}) - \tan^{-1}(\omega/\omega_{p_1})$$

This gives −45° phase at pole freq &
+45° phase at zero freq
~~To~~ The tangent is well approximated
by a linear fit from $\frac{\omega_p}{10}$ to $10\omega_p$

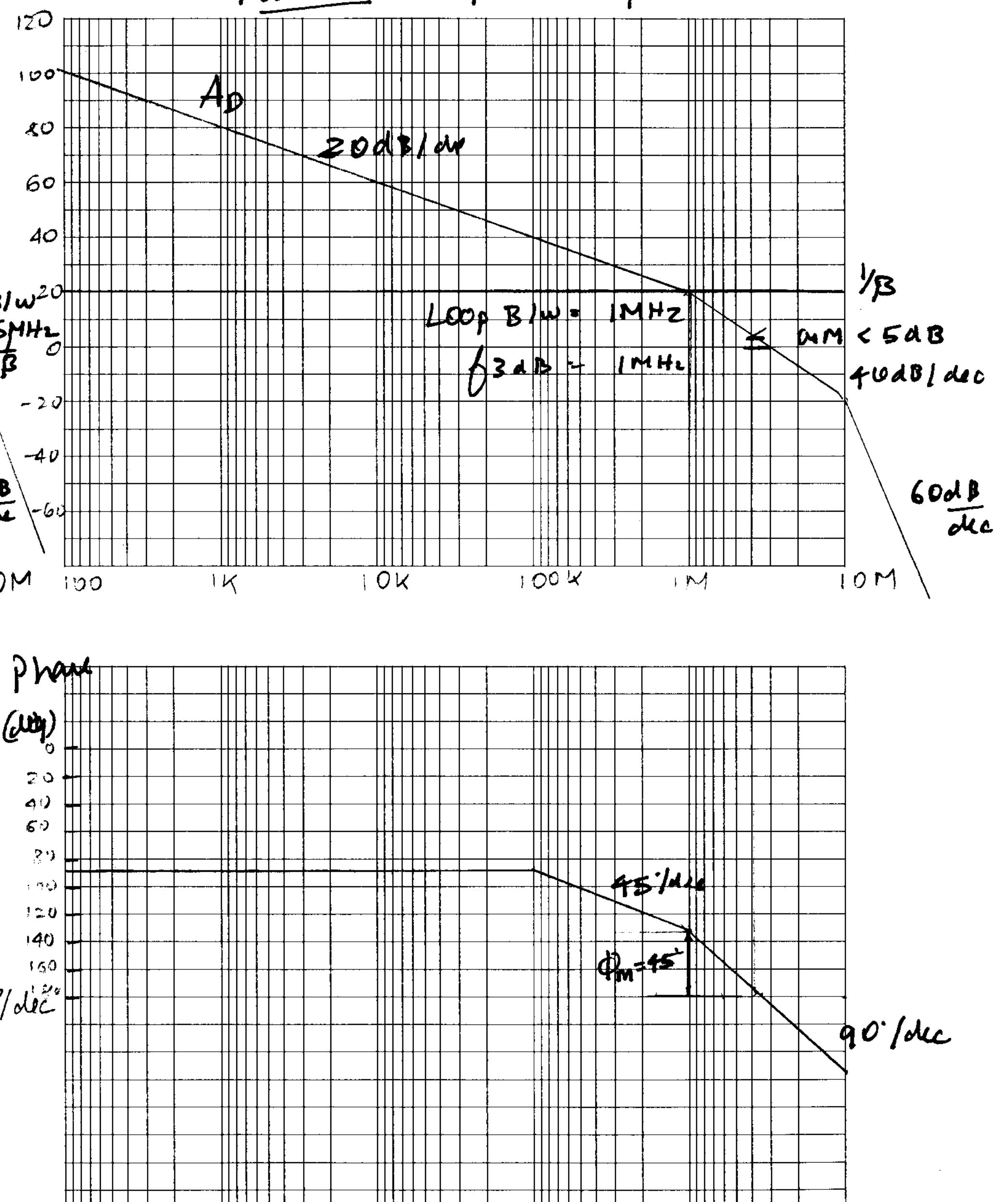
~~us~~ by line whose is −45° / dec
and similarly for a zero by a line
of slope +45° / dec.

The net phase response is obtained by
summing individual responses. (Linear Superposition)

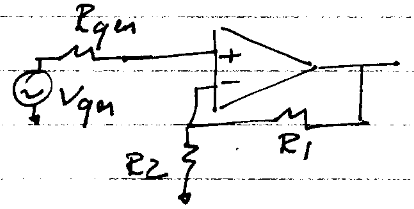
Part a : $\beta = 1$



Part b : $\beta = 0.9$



4)



$A_D = 10^6$

$f_{P_1} = 1Hz \qquad f_{P_2} = 1MHz$

$R_{gen} = 0\Omega$

a) $R_1 = 0 \qquad R_2 = 1K\Omega$

$$\beta = \frac{R_2}{R_2 + R_1} = 1$$

$$A_{OL}(s) = \frac{A_D}{\left(1+\frac{s}{\omega_{P_1}}\right)\left(1+\frac{s}{\omega_{P_2}}\right)} \quad \text{where} \quad \omega_{P_1} = 2\pi f_{P_1}, \quad \omega_{P_2} = 2\pi f_{P_2}$$

$$A_{CL}(s) = \frac{1}{\beta}\cdot\left[\frac{A_{OL}(s)\,\beta}{1+A_{OL}(s)\,\beta}\right]$$

Put

$$A_{OL}(s)\cdot\beta = T(s) = A_{OL}(s)\cdot(1)$$

$$T(s) = \frac{N_T}{D_T} = \frac{A_D\cdot(1)}{\left(1+\frac{s}{\omega_{P_1}}\right)\left(1+\frac{s}{\omega_{P_2}}\right)} = \frac{A_D}{\frac{s^2}{\omega_{P_1}\cdot\omega_{P_2}} + s\left(\frac{1}{\omega_{P_1}}+\frac{1}{\omega_{P_2}}\right)+1}$$

$$\Rightarrow A_{CL}(s) = \frac{1}{\beta}\left[\frac{T(s)}{1+T(s)}\right] = \frac{1}{\beta}\cdot\left[\frac{N_T/D_T}{N_T/D_T+1}\right] = \frac{1}{\beta}\left[\frac{N_T}{N_T+D_T}\right]$$

$$= \frac{1}{\beta}\cdot\left[\frac{A_D}{A_D+1+s\left(\frac{1}{\omega_{P_1}}+\frac{1}{\omega_{P_2}}\right)+s^2\left(\frac{1}{\omega_{P_1}\cdot\omega_{P_2}}\right)}\right]$$

$$= \frac{1}{\beta}\cdot\left[\frac{T_0}{1+T_0+s\left(\frac{1}{\omega_{P_1}}+\frac{1}{\omega_{P_2}}\right)+s^2\left(\frac{1}{\omega_{P_1}\cdot\omega_{P_2}}\right)}\right]$$

where $T_0 = A_D\cdot\beta$ here $\underline{\beta = 1}$

$$\therefore A_{CL}(s) = \frac{1}{\beta}\cdot\frac{T_0}{1+T_0}\left[\frac{1}{1+s\left[\frac{1}{(1+T_0)\omega_{P_1}}+\frac{1}{(1+T_0)\omega_{P_2}}\right]+\frac{s^2}{(1+T_0)\omega_{P_1}\cdot\omega_{P_2}}}\right]$$

$$= \frac{1}{\beta}\cdot\frac{T_0}{1+T_0}\left[\frac{1}{1+\frac{a_1}{1+T_0}\cdot s+\frac{a_2}{(1+T_0)}s^2}\right]$$

In general for $\beta \neq 1$

$$A_{CL}(s) = \frac{1}{\beta} \cdot \frac{T_O}{1+T_O} \cdot \frac{1}{1+\left(\frac{a_1}{1+T_O}\right)s + \left(\frac{a_2}{1+T_O}\right)s^2}$$

For part a: $\beta = 1$ $f_{P_1} = 1Hz$, $f_{P_2} = 1MHz$.

$$a_1 = \frac{1}{\omega_{P_1}} + \frac{1}{\omega_{P_2}} = 0.159$$

$$a_2 = \frac{1}{\omega_{P_1} \cdot \omega_{P_2}} = 2.53 \times 10^{-8} \ sec^2$$

$$T_O = A_D \cdot \beta = 10^6$$

$$\therefore A_{CL}(s) = \left(\frac{10^6}{1+10^6}\right) \frac{1}{1 + 1.58 \times 10^{-7} s + (2.53 \times 10^{-14}) s^2}$$

$$\therefore \cancel{AC}\ A_{CL}(s) = 0.9999 \cdot \frac{1}{1 + (1.58 \times 10^{-7} s) + (2.58 \times 10^{-14}) s^2}$$

$$D_A = \cancel{1 + (1.58 \times 10^{-7})} s + \cancel{(2.58 \times 10^{-14})} s^2$$

$$A_{CL}(s) = \frac{3.87 \times 10^{13}}{s^2 + (6.12 \times 10^6) s + 3.87 \times 10^{13}}$$

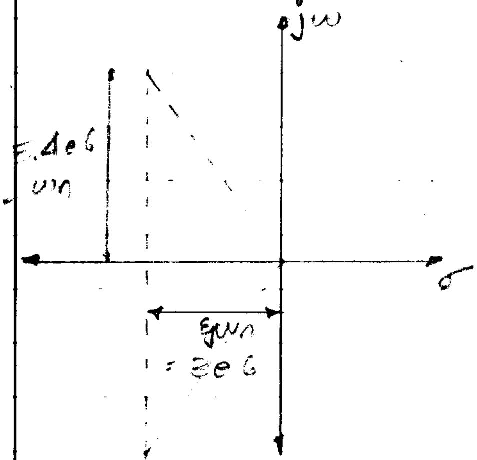
Comparing the denominator with $s^2 + 2\xi\omega_n s + \omega_n^2$ we get

$\omega_n = 6.22 \times 10^6$ Rad/s $\rightarrow$ Natural freq

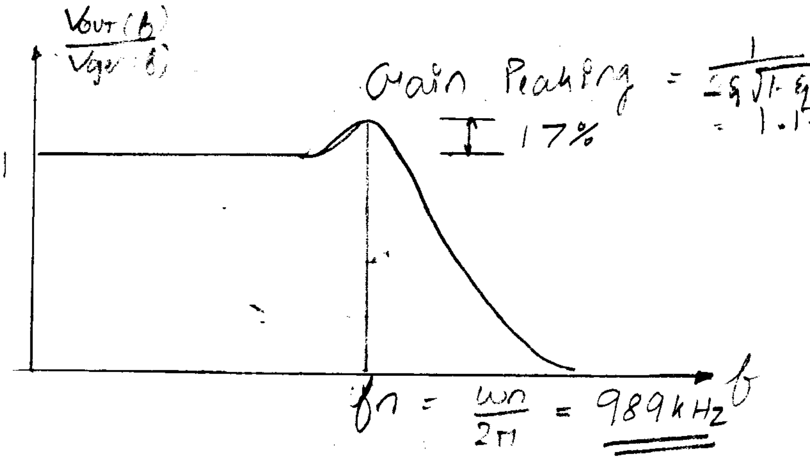
$\xi = 0.4915 \rightarrow$ Damping Factor

$\omega_d = \omega_n \sqrt{1-\xi^2} = 4.71 \times 10^6$ Rad/s $\rightarrow$ Damped freq

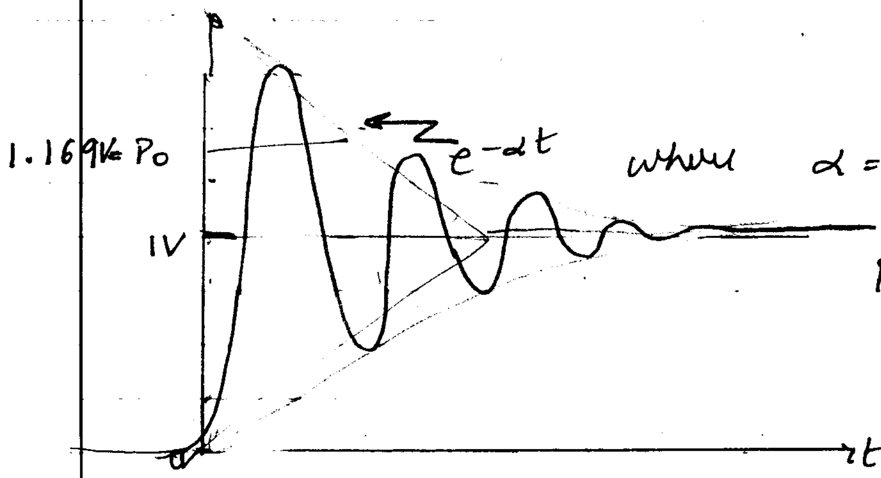
Pole Frequencies: $\omega_{P_1} = -3e6 \pm (5.416e6)i$



Location of poles on s-plane



Plot of Vout(f) / Vgn(f)



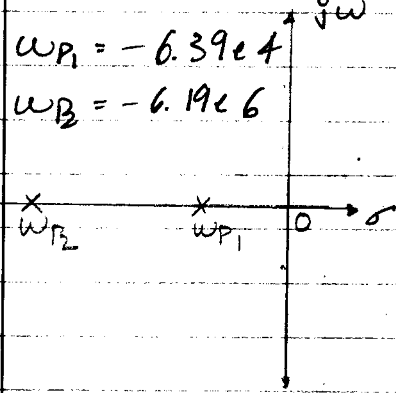
where $\alpha = \frac{1}{\xi \omega_n} = 333\, nsec$

$$P_O = 1 + \exp\left(\frac{-\pi \xi}{\sqrt{1-\xi^2}}\right)$$

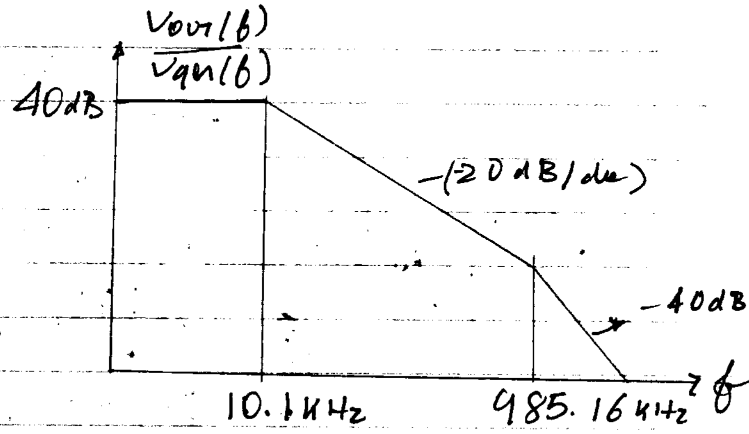
$$= 1.1698$$

10.1 kHz
985 kHz

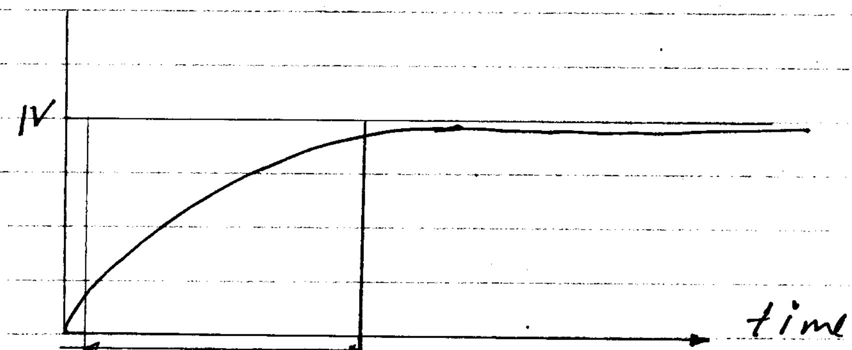
$\omega_{P_1} = -6.39e4$
$\omega_{P_2} = -6.19e6$



Location of poles in s-plane



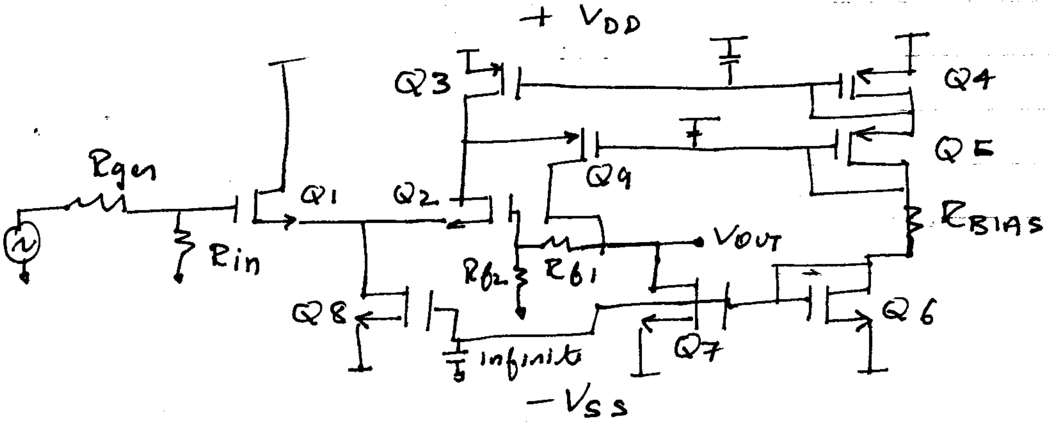
Plot of $\frac{Vout(f)}{Vgn(f)}$



Step response to a $\underline{1V \text{ step}}$

$T_{10-90\%} = 2.2 \frac{a_2}{a_1} = \underline{350 nsec}$

5)



$g_m = 1\,mS/um \times W_g$ $\qquad V_{TH} = 0.3V$

$C_{gs} = 1.5\,fF/um \times W_g$ $\qquad V_{DD} = -V_{SS} = 3V$

$C_{gd} = 0.3\,fF/um \times W_g$ $\qquad \lambda = 0$

I_D for Q1, 2, 4, 5, 6, 7, 9 = 0.3 mA

$I_{D4} = I_{D5} = I_{D6} = 0.3mA$

$W_{g4} = W_{g5} = W_{g6} = 2\,um$

$I_{D4} = -(g_{m4})(V_{gs4} - V_{th4}) = -(V_{sat} C_{ox} W_{g4})(V_{gs4} - V_{th})$

Given: $I_{D4} = 0.3mA$ $\qquad V_{sat} C_{ox} W_g = (1\,mS/um) \times 2\,um$

$V_{th} = 0.3V$ $\qquad = 2mS$

$\Rightarrow V_{gs4} = -(I_{D4}/g_{m4}) + V_{th4}$

$= -0.15 - 0.3$

$= \cancel{0.14} - 0.45$

Now, $V_s = V_{DD} = 3V \Rightarrow V_g = V_s - 0.45 = 2.55V$

$\therefore V_{g4} = 2.55$

Now, $V_{gs5} = V_{gs4} = -0.45V \Rightarrow V_{g5} = 3 - 2(0.45)$

$V_{g5} = 2.1V$

Also, $I_{D6} = 0.3mA \Rightarrow V_{g6} = (-3 + 0.45)V = -2.55V$

$R_{BIAS} = \dfrac{V_{g5} - V_{g6}}{(0.3mA)} = \dfrac{(2.1 + 2.55)}{0.3mA} = \boxed{15.5\,K\Omega}$

$I_{DQ1} = I_{DQ2} = 0.3mA \Rightarrow I_{DQ8} = 0.6mA$

$I_{DQ8} = g_{m8}\,(V_{gs8} - V_{th})$

$V_{gs8} = V_{gs6} = 0.45V$

$\Rightarrow g_{m8} = \dfrac{I_{DQ8}}{(V_{gs8} - V_{th})} = 4mS \Rightarrow \boxed{W_{g8} = 4um}$

$I_{DQ3} = I_{DQ2} + I_{DQ9}$

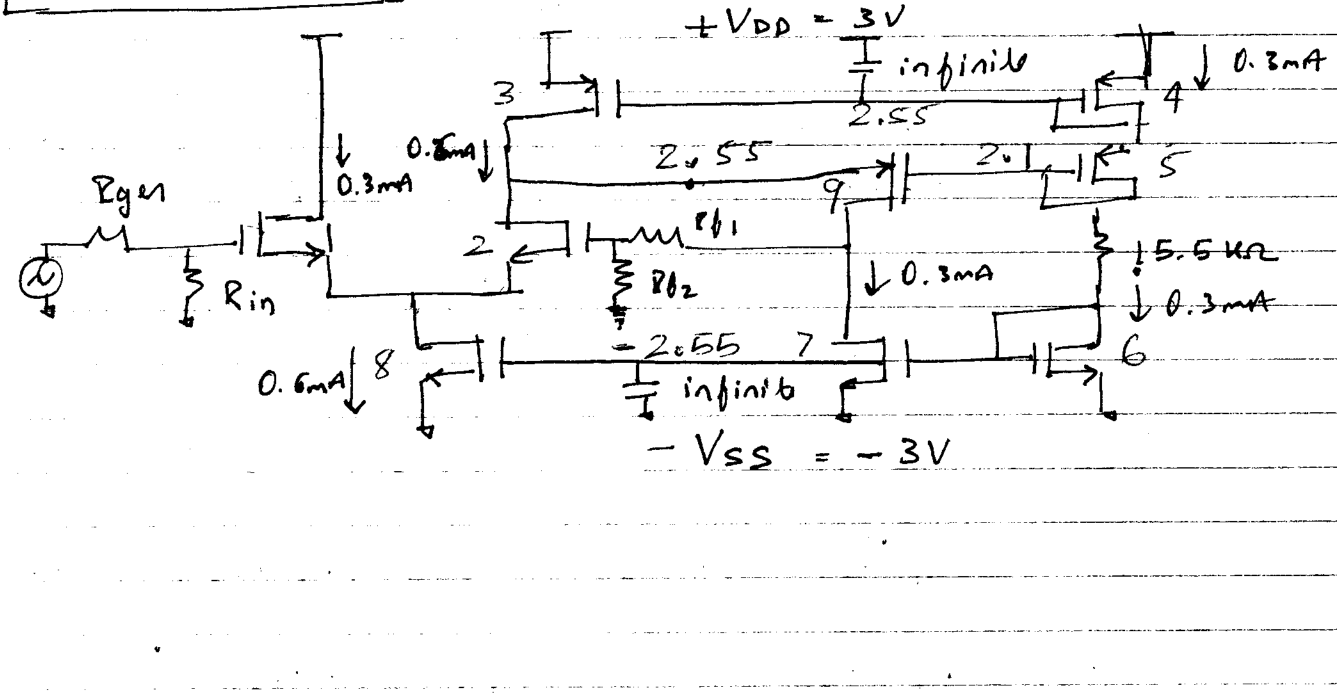
Now since R_{f1} & R_{f2} are very large we can neglect the current flowing through them

So, $I_{DQ9} = I_{DQ7} = 0.3mA$

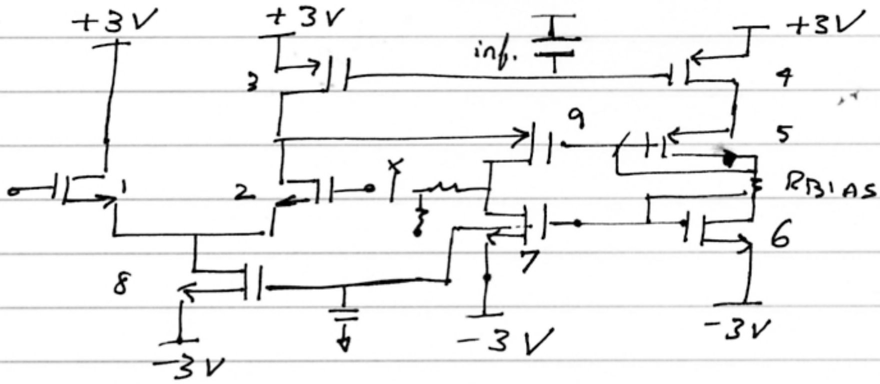
$\Rightarrow I_{DQ3} = 0.6mA = g_{m3} \cdot (V_{gs3} - V_{th})$

$\Rightarrow g_{m3} = \dfrac{0.6mA}{(0.45 - 0.3)} = 4mS \Rightarrow \boxed{W_{g3} = 4um}$

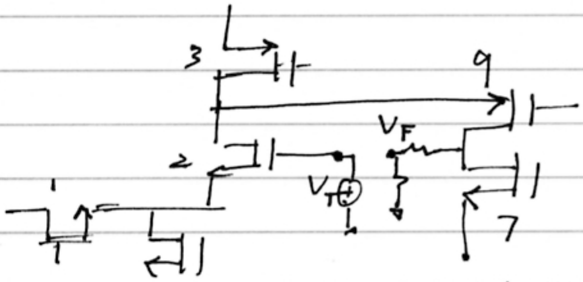
$\boxed{W_{g7} = W_{g6} = 2um}$



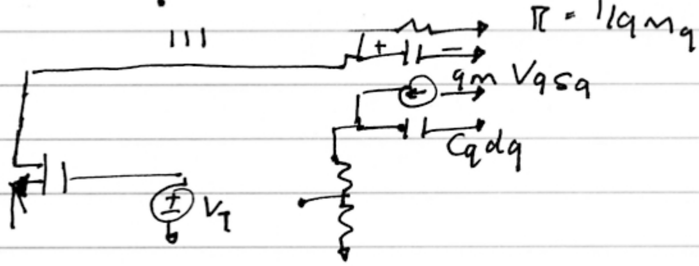
b.) Poles on loop transmission:



Breaking the Feed Back Loop at point X we get:



$\frac{V_F}{V_T}$ is the reqd. loop tx



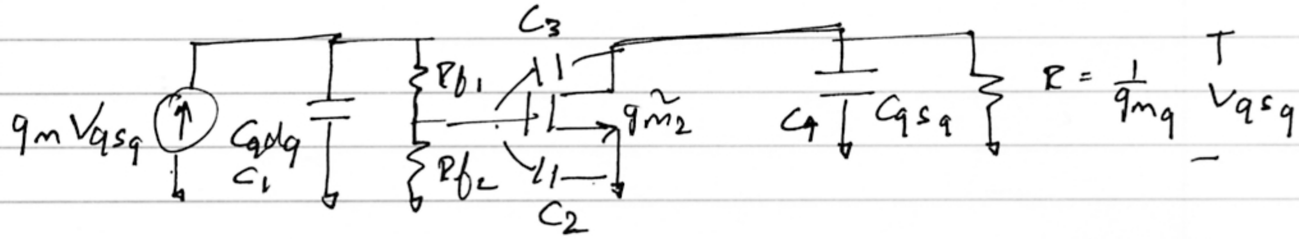
Degenerate 2 → 2

$\tilde{g}_{m_2} = \dfrac{g_{m_2}}{1 + g_{m_2} \cdot \frac{1}{g_{m_1}}}$

$= (g_{m2}/2)$

|||

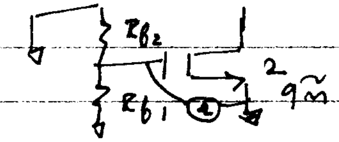
Using the small signal Equivalent Circuits



$C_1 = C_{gd9} \rightarrow R_1^0 = \cancel{\left(\frac{1}{g_{m5}}\right)} \parallel \; (R_{f1} + R_{f2}) + (1 + A_{v_9})\left(\frac{1}{g_{m5}}\right)$

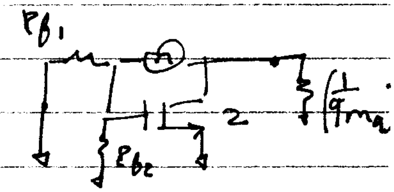
$=$

Now.



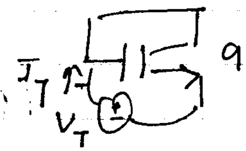
$$R_{22}^{1} = \left(\frac{1}{g_{m_2}} \| \frac{1}{g_{m_1}}\right)\left(\frac{1}{2}\right) + \left(R_{f_2} \| R_{f_1}\right)$$

$$= \underline{91.5\,k\Omega}$$



$$R_{33}^{1} = (1 - A_{v_2})(R_{f_1} \| R_{f_2}) + \left(\frac{1}{g_{m_4}}\right)$$

$$= \underline{180.5\,k\Omega}$$



$$R_{44}^{1} = \left(\frac{1}{g_{m_4}}\right) = \underline{500\,\Omega}$$

$$R_{33}^{2} = (1 - A_{v_2})(0) + \frac{1}{g_{m_4}} = \underline{500\,\Omega}$$

$$R_{44}^{2} = R_{44}^{0} = \underline{500\,\Omega}$$

$$R_{44}^{3} = \left(\frac{1}{g_{m_4}} \| \frac{1}{g_{m_2}}\right) = \underline{250\,\Omega}$$

$$a_1 = R_{11}^{0} C_1 + R_{22}^{0} C_2 + R_{33}^{0} C_3 + R_{44}^{0} C_4$$

$$= 1.07\ \text{nsec}$$

$$a_2 = R_{11}^{0} C_1 C_2 R_{22}^{1} + R_{11}^{0} C_1 C_3 R_{33}^{1} + R_{11}^{0} C_1 C_4 R_{44}^{1} + R_{22}^{0} C_2 C_3 R_{33}^{2} + R_{22}^{0} C_2 C_4 R_{44}^{2} + R_{33}^{0} C_3 C_4 R_{44}^{3}$$

$$= 2.54 \times 10^{-19}\ \text{sec}^2$$

$a_1 \not\gg a_2/a_1 \Rightarrow$ SPA doesn't hold.

Roots of $1 + a_1 s + a_2 s^2$: $s_1 = -13995542l\ldots$
$s_2 = -2813044215$

$$\Rightarrow \left.\begin{array}{l} f_{P_1} = 222\ \text{MHz} \\ f_{P_2} = 447\ \text{MHz} \end{array}\right\}$$

$$C_1 = C_{gd9} = \underline{0.6\,fF}$$

$$R_{11}^{0} = \left(\frac{1}{g_{m5}}\right)(1 - A_{v9}) + (R_{f1} + R_{f2})$$

$$= \left(\frac{1}{2mS}\right)\left(1 - \frac{(R_{f1}+R_{f2})}{\left(\frac{1}{g_{m9}} + R_{DS1} \| R_{DS2}\right)}\right) + (1.1M\Omega)$$

$$\approx \left(\frac{1}{2mS}\right) + 1.1M\Omega$$

$$\approx \underline{1.1M\Omega}$$

$$C_2 = C_{gs2} = 3\,fF$$

$$R_{22}^{0} = \left(\frac{1}{g_{m2}} \,\Big\|\, \frac{1}{g_{m1}}\right)\left(1 - \frac{1}{2}\right) + (\cancel{R_{f1}+}\,R_{f2})$$

$$= (1mS)^{-1}\left(\frac{1}{2}\right) + 100\,k\Omega$$

$$= \underline{101.5\,k\Omega}$$

$$C_3 = C_{gd2} = 0.6\,fF$$

$$R_{33}^{0} = (1 - A_{v2})(\cancel{R_{f1}+}\,R_{f2}) + \left(\frac{1}{g_{m9}}\right)$$

$$= \frac{\frac{1}{g_{m9}}}{\frac{1}{2}\left(\frac{1}{g_{m2}}\right)} \cdot (\cancel{R_{f1}+}\,R_{f2}) + \left(\frac{1}{g_{m9}}\right)$$

$$= 2(90k\Omega) + 500$$

$$= \underline{180.5\,k\Omega}$$

$$C_4 = C_{gs9} = 3\,fF$$

$$R_{44}^{0} = (1 - A_{v9})\left(\frac{2}{g_{m5}} \,\Big\|\, R_{BIAS} + \frac{1}{g_{m6}}\right) + \frac{1}{g_{m9}}$$

$$\approx \frac{1}{g_{m9}} \quad \text{as} \quad A_{v9} \approx 1$$

$$= \underline{500\,\Omega}$$

$T(j\omega) = g_{m_2} \cdot (R_{f_1} + R_{f_2}) = 90.9 \qquad \frac{1}{\beta} = 11$