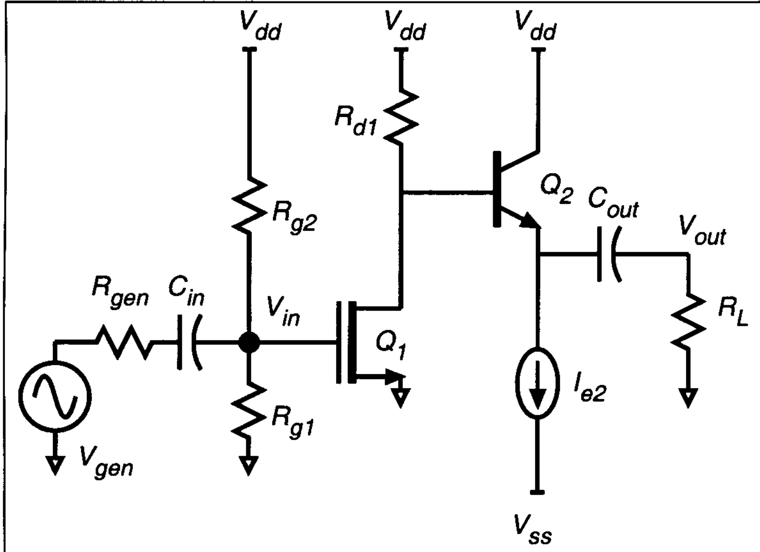
MID TERM A SOLUTIONS

Problem 1, 70 points



The supplies are +/- 2 Volts

Q1: $|V_{th}|$=0.3 Volt, $v_{sat}C_{ox}W_g$ =20 mA/V, lambda=0, C_{gs} =20 fF, C_{gd} = 0 fF

Q2: $\beta=\infty$, τ_f=1 ps, C_{je} = 5 fF, C_{cb} =10 fF, $V_A=\infty$ Volts

Rgen=100 kOhm.
RL=100 Ohm
Ie2=1 mA
Rg1= 1 MOhm

The DC drain voltage of Q1 is to be 1 volts.
The DC drain current of Q1 is to be 5 mA.
Cin and Cout are AC short-circuits at all frequencies of interest---THEREFORE DO NOT TREAT THEM AS CAPACITORS in an MOTC analysis.

$$Rd_1 = \frac{(Vdd - Vd)}{I_D} \quad \frac{V}{A}$$

$I_D = 5\,mA$

V_d = dc drain voltage = 1 V

$$Rd_1 = \frac{(2-1)V}{5\,mA} = \frac{1V}{5\,mA} = 200\,\Omega$$

2

$$Id = gm\left(\frac{A}{V}\right)(Vgs - Vth)\,V \Rightarrow \frac{Id}{gm}\frac{A}{A/V} = (Vgs - Vth)\,V$$

$$\frac{5\,mA}{20\,mA/V} = Vgs - 0.3 \Rightarrow \left(\frac{1}{4} + 0.3\right)V = Vgs$$

$$Vgs = 0.55\,V$$

See next page

a

Part a, 10 points

Find the following

Rg2= 4.45 MΩ Rd1= 200Ω

$(C_{je} + C_{diff})$ of Q2= 43.5 fF

$$I_{Rg1} = \frac{V_g\ (V)}{R_{g1}\ (A)}$$

$$V_g = V_{gs} = 0.55\ V$$

$$I_{Rg1} = \frac{0.55\ V}{1 M\Omega} = 0.55\ \mu A$$

$$R_{g2} = \frac{V_{dd} - V_g\ (V)}{I_{Rg1}\ (A)} = \frac{(2 - 0.55)V}{0.55 \mu A}$$

$$R_{g2} = \frac{1.45\ V}{0.55\ \mu A} = 2.64\ M\Omega$$

$$C_{be} = g_m \tau_f + C_{je}$$

$$C_{be} = \frac{1 mA}{26 mV} \times 1 ps + 5 fF$$

$$C_{be} = 43.5\ fF$$

$$1\ fF = 10^{-15}\ F$$

3

a

Part b, 10 points

Mid Band Analysis:
Find the mid-band small signal voltage gain of Q2 (the small signal voltage at the emitter of Q2 divided by the small signal voltage at the base of Q2)

Av2= 0.7936

Find the mid-band small signal voltage gain of Q1 (the small signal voltage at the drain of Q1 divided by the small signal voltage at the gate of Q1)

Av1= -4

Find Vin/Vgen

Vin/Vgen= 0.8909

1

2

$$\frac{V_{in}}{V_{gen}} = \frac{2.64\,M\Omega \,||\, 1M\,\Omega}{100k + 2.64\,M \,||\, 1M\,\Omega} = \frac{0.72527M}{100k\,\Omega + 0.72527M}$$

$$\frac{V_{in}}{V_{gen}} = 0.87836$$

1

3

1

$$A_{v1} = -g_m R_{Leq} = \frac{-20\,mA}{V} \times (R_{d1})\,\Omega$$

$$A_{v1} = -0.02 \times 200\,\Omega = -4$$

4

3

1

$$A_{v2} = \frac{R_{Leq}}{R_{Leq} + r_{e2}} = \frac{100\,\Omega}{\left(100\,\Omega + \frac{26mV}{1mA}\right)} = \frac{100}{126}$$

4

$$A_{v2} = 0.7936$$

a

Part c: 20 points

USING MOTC, you will find the two dominant pole frequencies of the transfer function. Give the frequencies of these in Hz:

Component of a1 due to C_{gs} of transistor Q1 = 1.757 n seconds.

Component of a1 due to C_{cb} of transistor Q2 = 2 p seconds.

Component of a1 due to $(C_{je} + C_{diff})$ of transistor Q2 = 2.693 seconds.

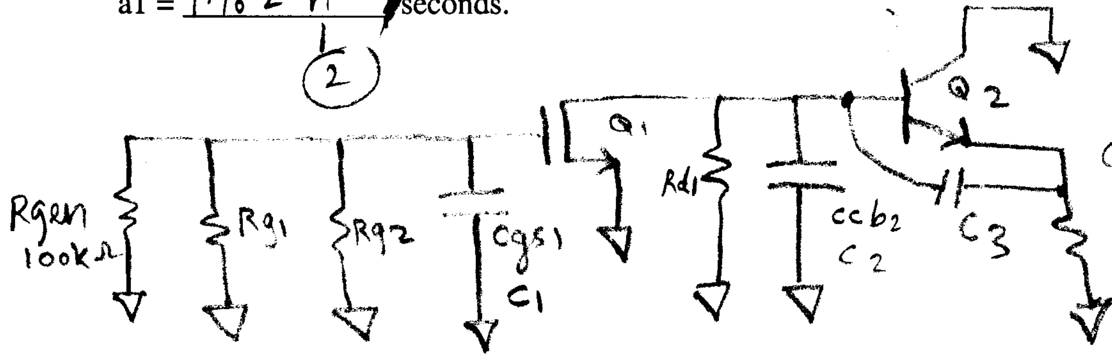
a1 = 1.762 n seconds.

2

$C_1 = C_{gs1} = 20\,fF$

$C_2 = C_{cb2} = 10\,fF$

$C_3 = C_{be2} = 43.5\,fF$



$R_{11}^{\circ} = R_{gen} \parallel R_{g1} \parallel R_{g2} = 100k\Omega \parallel 1M\Omega \parallel 2.64M\Omega$

$R_{11}^{\circ} = 87.88\ K\Omega$

$R_{22}^{\circ} = r_{d1} = 200\,\Omega$

$R_{33}^{\circ} = R_{d1}(1 - A_2) + r_e \parallel R_L$

$= 200\Omega(1 - 0.7936) + 26\Omega \parallel 100\Omega$

$= 41.28\Omega + 20.63\Omega$

$R_{33}^{\circ} = 61.9\,\Omega$

1

5

$R_{11}^{\circ} C_1 = R_{11}^{\circ} C_{gs} = (87.88\,K\Omega)\, 20\,fF = 1757.6\ p\,sec$

$R_{22}^{\circ} C_2 = (200\Omega)\, 10\,fF = 2\ p\,sec$ 1

5

$R_{33}^{\circ} C_3 = (61.915\,\Omega)\, 43.5\,fF = 2.693\ ps$ 1

5 6

$a_1 = R_{11}^{\circ}\Omega\, C_1 F + R_{22}^{\circ}\Omega\, C_2 F + R_{33}^{\circ}\Omega\, C_3 F = 1762.293\ psec$

Part d: 20 points

Component of a2 due to C_{gs1} and C_{cb2} = 3.515×10^{-21} seconds^2.
Component of a2 due to C_{gs1} and $(C_{je} + C_{diff})$ = 4.733×10^{-21} seconds^2.
Component of a2 due to C_{cb2} and $(C_{je} + C_{diff})$ = 1.795×10^{-24} seconds^2.
a2 = 8.25×10^{-21} seconds^2

f_{p1} = 33.87 GHz, f_{p2} = 90.5 MHz

1 1

$$a_2 = \left(R_{11}^{\circ} C_1 C_2 R_{22}^{1} + R_{11}^{\circ} C_1 C_3 R_{33}^{1} + R_{22}^{\circ} C_2 C_3 R_{33}^{2}\right) sec^2 \quad 1$$

3 $R_{22}^{1} = R_{22}^{\circ} = 200\Omega$; $R_{33}^{2} = (r_e \| R_L)\Omega = (26\Omega)\|100\Omega$

$= 20.635\Omega$

3 $R_{33}^{1} = R_{33}^{\circ} = 61.9\ \Omega$

2 $R_{11}^{\circ} C_1 C_2 R_{22}^{1} = (87.88k\Omega)(200\Omega)(20fF)(10fF)$

$$= (7.030 \times 10^{9} \times 10^{-15} \times 10^{-15})\, sec^2$$

$R_{11}^{\circ} C_1 C_2 R_{22}^{1} = 3.515 \times 10^{-21}\, sec^2$

$R_{11}^{\circ} C_1 C_3 R_{33}^{1} = (87.88\ k\Omega)(61.9\ \Omega)\ 20fF \times 43.5\ fF$ 2

$R_{11}^{\circ} C_2 C_3 R_{33}^{2} = 4.733 \times 10^{-21}\, sec^2$ 1

$R_{22}^{\circ} C_2 C_3 R_{33}^{2}$: $200\Omega\ (20.635\Omega)(10fF)(43.5\ fF)$

$R_{22}^{\circ} C_2 C_3 R_{33}^{2} = 1.795 \times 10^{-24}\, sec^2$ 1

$a_2 = 8.249 \times 10^{-21}\, sec^2$ - from 1

$[a_2 = (3.515 \times 10^{-21} + 4.733 \times 10^{-21} + 1.795 \times 10^{-24})\, sec^2]$ See next page for f_p ...

$1 + a_1 s + a_2 s^2 = 0$

$a_1 = 1762.3 \text{ psec}$

$a_2 = 8.249 \times 10^{-21} \text{ sec}^2$

$s_2 = -2.130 \times 10^{11}$

$s_1 = -1.781 \times 10^{-9} = -5.6915 \times 10^{8}$

$$f_{p2} = \frac{|s_1|}{2\pi} = \left(\frac{2.13 \times 10^{11}}{2\pi}\right) \text{Hz} = 33.867 \text{ GHz}$$

$$f_{p1} = \frac{|s_2|}{2\pi} = \left(5.6915 \times 10^{8} \times 0.159\right) \text{Hz}$$

$= 90.5 \times 10^{6} \text{ Hz}$

$= 90.5 \text{ MHz}$

a

Part e: 10 points

The circuit has 2 zeros its transfer function.

Give the frequencies of these in Hz:

$f_{z1} =$ —

$f_{z2} =$ 140.58 GHz

$$b_1 = -\frac{C_{gd}}{g_m} = \frac{0}{g_m} = 0 \text{ sec}$$

$$f_{z1} = \frac{1}{2\pi b_1} = \infty .$$

5 $$b_2 = \frac{C_{be}}{g_m} = \frac{43.5 \text{ fF}}{1/26} = 1131 \times 10^{-} = 1.131 \text{ psec}$$

4 $$f_{z2} = \frac{1}{2\pi b_2} = \frac{1}{2\pi \times 1.131 \text{ psec}}$$

$$= 0.159 \times \left(\frac{10^{12}}{1.131}\right)$$

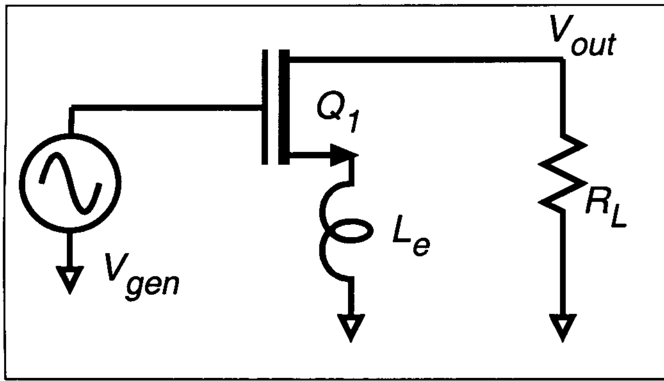
$$f_{z2} = 140.58 \times 10^{11} \text{ Hz}$$

1 $$f_{z2} = 140.58 \text{ GHz} .$$

a

Problem 2, 30 points

Part a 10 points

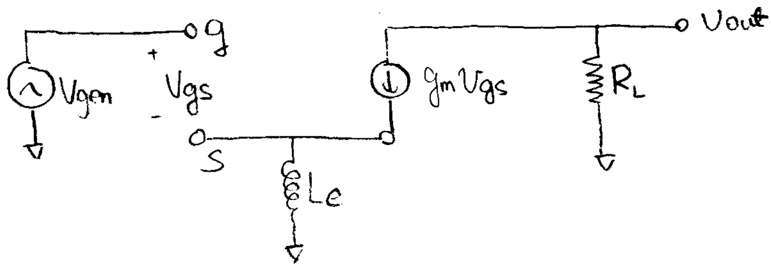


Small signal analysis.
Ignore the DC bias; you don't need it.

The FET has lambda=0 hence Gds=0. Also, Cgs =Cgd=0 fF

Replacing the transistor with its high frequency small-signal model, draw a small-signal equivalent circuit diagram.

Small-Signal Model



hard to give partial credit.

Grading Rules as following

1. No labels for current source −5
2. wrong sign of voltage expression in current source
3. wrong voltage labels in schematic −5

a

Part b, 10 points

USING NODAL ANALYSIS, compute Vout(s)/Vgen(s) in ratio-of-polynomials form:

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} \times \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots} = -g_m R_L \cdot \frac{1}{1+g_m L_e S}$$

(i) at S node

(3) $$\frac{V_s}{sL_e} = g_m V_{gs} \quad \text{, where } V_{gs} = V_g - V_s = V_{gen} - V_s$$

$$\Rightarrow V_s = \frac{s g_m L_e}{1+s g_m L_e} V_{gen} \quad — ①$$

(ii) at output node

(3) $$\frac{V_{out}}{R_L} + g_m V_{gs} = 0$$

$$\Rightarrow V_{out} = -g_m R_L (V_{gen} - V_s) \quad — ②$$

(iii) From ① and ②

$$V_{out} = -g_m R_L \left(V_{gen} - \frac{s g_m L_e}{1+s g_m L_e} V_{gen} \right)$$

$$\Rightarrow \frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} \times \frac{1+b_1 S + b_2 S^2 + \ldots}{1+a_1 S + a_2 S^2 + \ldots}$$

$$= -g_m R_L \times \frac{1}{1+g_m L_e \cdot S}$$ (4)

Part c, 10 points

gm= 100 mS. RL=1 kOhm. Le= 1 nH

Find the frequencies of any zeros (there may be zero, one or two present) in the transfer function:

f_{z1} =__________, f_{z2} =__________, ...

There may be either 1 or 2 poles of the transfer function.

If the poles are real, give the 1 or 2 pole frequencies in Hz:

f_{p1} = 1.59 GHz, f_{p2} =__________

If there are 2 poles, and they are complex, give $f_n = \omega_n / 2\pi$ and the damping factor ζ :

$f_n = \omega_n / 2\pi =$__________, ζ =__________

(10) From part b

$$\frac{V_{out}(s)}{V_{gen}(s)} = -g_m R_L \times \frac{1}{1 + j\frac{f}{f_p}}, \quad \text{where } f_p = \frac{0.159}{g_m \cdot L_e}$$

$$= \frac{0.159}{100(mS) \cdot 1(nH)}$$

9 $$= -100 \times \frac{1}{1 + j\frac{f}{1.59(GHz)}}$$

$$= 1.59 \times 10^9 (Hz)$$

$$= 1.59 (GHz)$$

$$g_m R_L = 100(mS) \cdot 1(K\Omega)$$

$$= 100$$

1 ∴ There is a real pole at f_p = 1.59 (GHz).

deduct points 2 each for additional poles and zeros

a