Mid-Term Exam, ECE-137B
MONDAY May 14, 2007

Closed-Book Exam

There are 2 problems on this exam , and you have 50 minutes.
1) show all work. Full credit will not be given for correct answers if supporting work is not shown.
2) please write answers in provided blanks
3) Don't Panic !
4) 137a, 137b crib sheets, and 2 pages personal sheets permitted.

Do not turn over the cover page until requested to do so.

Name:

→ MID TERM B SOLUTIONS

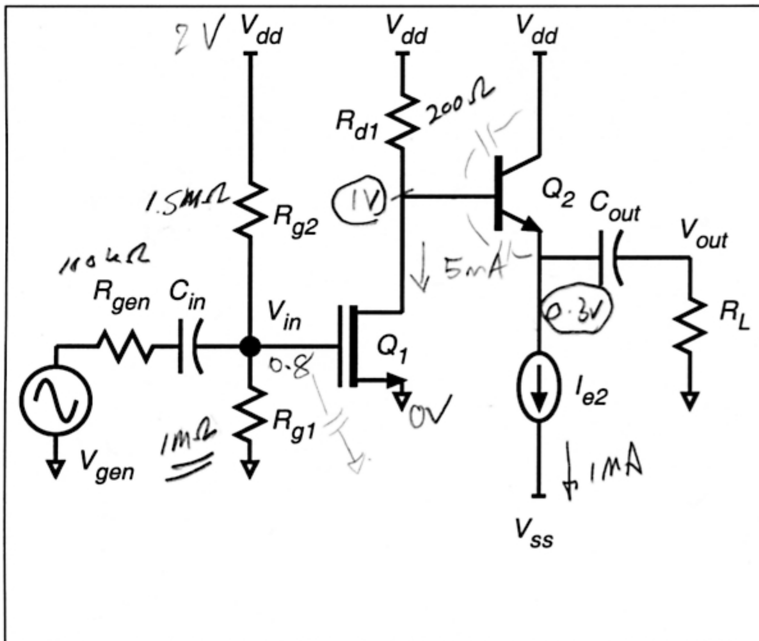
Use any and all reasonable approximations. 5% accuracy is fine if the method is correct.

Time function	LaPlace Transform
$\delta(t)$	1
$U(t)$	$1/s$
$e^{-\alpha t}U(t)$	$\frac{1}{s+\alpha}$
$e^{-\alpha t}\cos(\omega_d t)U(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)U(t)$	$\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Problem	Points Received	Points Possible
1a		10
1b		10
1c		20
1d		20
1e		10
2a		10
2b		10
2c		10
total		100

b

Problem 1, 70 points



The supplies are +/- 2 Volts

Q1: $|V_{th}|$=0.3 Volt,
$v_{sat}C_{ox}W_g$=10 mA/V,
lambda=0, C_{gs}=10 fF,
C_{gd} = 0 fF

Q2: $\beta=\infty$, τ_f=0.5 ps,
C_{je} = 5 fF, C_{cb}=10 fF,
$V_A=\infty$ Volts

Rgen=100 kOhm.
RL=100 Ohm
Ie2=1 mA
Rg1= 1 MOhm

The DC drain voltage of Q1 is to be 1 volts.
The DC drain current of Q1 is to be 5 mA.
Cin and Cout are AC short-circuits at all frequencies of interest---THEREFORE DO NOT TREAT THEM AS CAPACITORS in an MOTC analysis.

0.6

60

$$V_{dd}\left(\frac{1}{1+x}\right) = 0.8$$

$$\frac{1}{1+x} = 0.8$$

$$1+x = 1.25$$

$$x = .25$$

b

Part a, 10 points

Find the following

Rg2= 1.5 MΩ Rd1= 200Ω

$(C_{je} + C_{diff})$ of Q2= 25 fF

$I_{DQ_1} = g_{m_1}(V_{gs} - V_{thQ_1})$ $V_{thQ_1} = 0.3V$

$I_{DQ_1} = 5mA$ (given) $g_m = V_{sat} C_{ox} W_g = 10mA/V$

$V_{gsQ_1} = \frac{I_{DQ_1}}{g_{m_1}} + V_{thQ_1} = \left(\frac{5mA}{10mA/V}\right) + 0.3V$

$= \underline{0.8V}$

1

Now, $V_{gQ_1} = V_{dd} \cdot \frac{R_{g_1}}{R_{g_1} + R_{g_2}} = 0.8V$ $(V_s = 0)$ 3

$V_{dd} = 2V$ $R_{g_1} = 1M\Omega \rightarrow$ given 1

$\left(\frac{V_{gQ_1}}{V_{DD}}\right) = \frac{R_{g_1}}{R_{g_1} + R_{g_2}} \Rightarrow R_{g_2} = R_{g_1}\left[\left(\frac{V_{DD}}{V_{gQ_1}}\right) - 1\right]$

$R_{g_2} = \underline{1.5 M\Omega}$

$V_{dd} - (5mA)(R_{D_1}) = 1V$

$\Rightarrow R_{D_1} = \left(\frac{2-1}{5mA}\right) = 200\Omega$

3 $C_{diff}\,Q_2 = g_m \tau_f = \left(\frac{I_c}{V_t}\right)\tau_f = \left(\frac{1mA}{26mV}\right)(0.5ps) = 0.02pF$

$= 20fF$

Part b, 10 points

Mid Band Analysis:

Find the mid-band small signal voltage gain of Q2 (the small signal voltage at the emitter of Q2 divided by the small signal voltage at the base of Q2)

Av2= 0.793

Find the mid-band small signal voltage gain of Q1 (the small signal voltage at the drain of Q1 divided by the small signal voltage at the gate of Q1)

Av1= -2

Find Vin/Vgen

Vin/Vgen= 0.8571

(3) (1)

(4) $$Av_{Q_2} = \frac{R_L}{R_L + \left(\frac{1}{g_m}\right)_2} = \frac{100}{100+26} = \underline{0.793}$$

(3) (1)

(4) $$Av_{Q_1} = -(g_{mQ_1})(R_D) = -\left(10\frac{m\Omega}{V}\right)(200\Omega) = (-2\ V/V)$$

(2) (1) (1)

$$\frac{V_{in}}{V_{gen}} = \frac{R_{g_1} \| R_{g_2}}{R_{g_1} \| R_{g_2} + R_{gen}} = \underline{0.8571}$$

b

Part c: 20 points

USING MOTC, you will find the two dominant pole frequencies of the transfer function. Give the frequencies of these in Hz:

Component of a1 due to C_{gs} of transistor Q1 = 0.857 nsec seconds.

Component of a1 due to C_{cb} of transistor Q2 = 2 psec seconds.

Component of a1 due to $(C_{je} + C_{diff})$ of transistor Q2 = 1.55 psec seconds.

a1 = 8.6×10^{-10} seconds. 2

$$O_1 \Big|_{C_{gs_1}} = (R_{g1} \| R_{g2} \| R_{gen}) C_{gs} \quad 5$$

$$= (600K \| 100K)(10fF) = 0.86\,ns \quad 1$$

6

$$\Big|_{C_{cb_2}} = (R_{d_1})(C_{cb_2}) = (200\Omega)(10fF) \quad 5 \quad 1$$

$$= 2\,psec$$

6

$$\Big|_{C_{je}+C_{diff}} = \left(R_{d_1}(1 - A_{v_2}) + \left(R_L \| \frac{1}{g_{m_2}}\right)\right) C_{be} \quad 5$$

$$= ((200)(1 - 0.793) + (100 \| 26)) C_{be}$$

$$= (41.4 + 20.63) C_{be} = 62.6\Omega \cdot C_{BE}$$

$$= 1.55\,psec \quad 1$$

6

b

Part d: 20 points

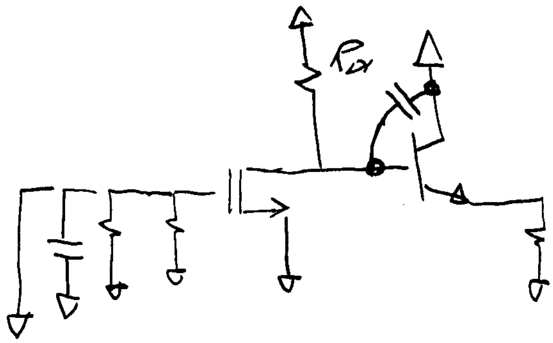
Component of a2 due to C_{gs1} and C_{cb2} = $1.71 \cdot 10^{-21}$ seconds^2.

Component of a2 due to C_{gs1} and $(C_{je}+C_{diff})$ = $1.3 \cdot 10^{-21}$ seconds^2.

Component of a2 due to C_{cb2} and $(C_{je}+C_{diff})$ = $1.03 \cdot 10^{-24}$ seconds^2.

a2 = $3 \cdot 10^{-21}$ seconds^2

f_{p1}= 185 MHz , f_{p2}= 45.6 GHz .



C_{gs1} & C_{cb2} – $R_{11}^0 C_1 C_2 R_{22}^1$

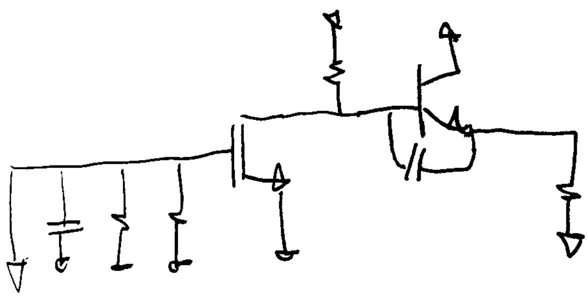
$R_{22}^1 \rightarrow$

(3) $R_{22}^1 = R_{D1}$

(2) $R_{11}^0 C_1 C_2 R_{22}^1 = (R_{g1} \| R_{g2} \| R_{gen})\, C_{gs1} \cdot C_{cb2} \cdot R_{D1}$

$= (100k\Omega \| 1M\Omega \| 1.5M\Omega) \cdot 10fF \cdot 10fF \cdot 200\Omega$

(1) $= 85.7k\Omega \cdot 10fF \cdot 10fF \cdot 200\Omega = 1.71 \cdot 10^{-21}\ sec^2$



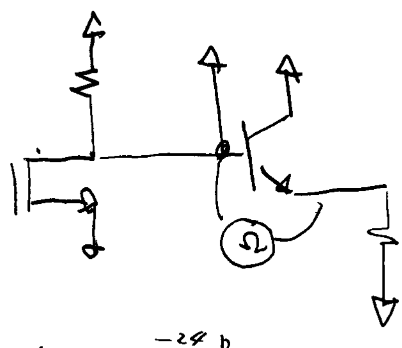
C_{gs1} & $(C_{je}+C_{diff})$ $R_{11}^0 C_1 C_3 R_{33}^1$

$R_{33}^1 \rightarrow$

(3) $R_{33}^1 = R_{33}^0$ = from part c

$= 62.6\Omega$

(2) $R_{11}^0 C_1 C_3 R_{33}^1 = 85.7k\Omega \cdot 10fF \cdot 25fF \cdot 62.6\Omega = 1.3 \cdot 10^{-21}\ sec^2$ (1)



C_{cb2} & $(C_{je}+C_{diff})$ $R_{22}^0 C_2 C_3 R_{33}^2$

$R_{33}^2 \rightarrow$ R

(3) $R_{33}^2 = r_{e2} \| R_L = 26\Omega \| 100\Omega = 20.6\Omega$

$R_{22}^0 C_2 C_3 R_{33}^2 = 200\Omega \cdot 10fF \cdot 25fF \cdot 20.6\Omega = 1.03 \cdot 10^{-24}\ sec^2$.

(2) (1)

USE SPA - check for validity

(1) $f_{p1} = \frac{1}{2\pi a_1} = 185\ MHz$

(1) $f_{p2} = \frac{1}{2\pi(a_2/a_1)} = 45.6\ GHz$

b

Part e: 10 points
The circuit has 2 zeros its transfer function. ← sorry - not true, only one because $C_{gd1} = 0$ ff
Give the frequencies of these in Hz:
f_{z1} = 244 GHz
f_{z2} =

zero due to C_{be2}

$$f_z = \frac{g_{m2}}{2\pi C_{be2}} = \frac{1}{2\pi (26\Omega) \cdot 25fF} = 244 GHz$$

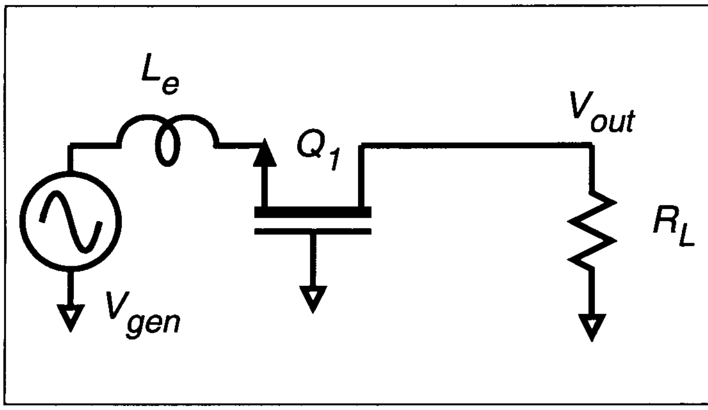
5 4 1

-2 for extra zero

b

Problem 2, 30 points

Part a 10 points

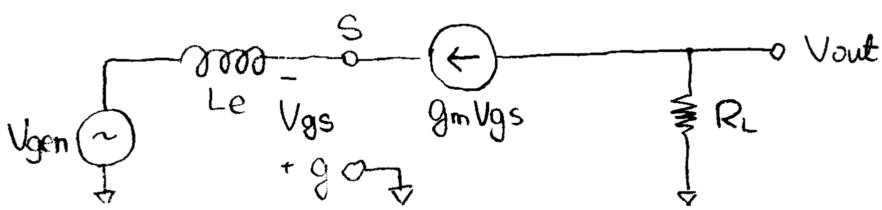


Small signal analysis.
Ignore the DC bias; you don't need it.

The FET has lambda=0 hence Gds=0. Also, Cgs =Cgd=0 fF

Replacing the transistor with its high frequency small-signal model, draw a small-signal equivalent circuit diagram.

Small – Signal Model



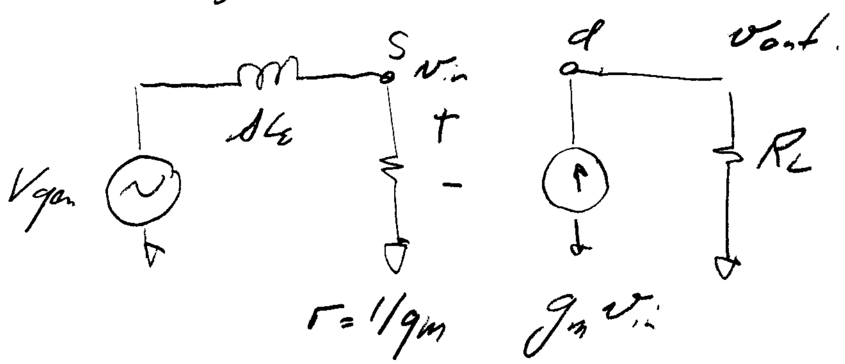
hard to give partial credit.

(Grading Rules) 1. No labels for current source −5

2. wrong sign of voltage expression in current source −5

3. wrong voltage ~~tables~~ labels in schematic −5

— or —



b

Part b, 10 points

USING NODAL ANALYSIS, compute Vout(s)/Vgen(s) in ratio-of-polynomials form:

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} \times \frac{1+b_1 s+b_2 s^2+\dots}{1+a_1 s+a_2 s^2+\dots} = g_m R_L \cdot \frac{1}{1+g_m L_e S}$$

• at S node

(3)

$$\left[\ \frac{V_{gen}-V_s}{S L_e} + g_m V_{gs} = 0 \quad , \text{ where } V_{gs} = V_g - V_s = -V_s .\right.$$

$$\Rightarrow \frac{V_s}{V_{gen}} = \frac{1}{1+S\, g_m L_e} \quad — ①$$

• at output node

(3)

$$\left[\ \frac{V_{out}}{R_L} + g_m \cdot V_{gs} = 0\right.$$

$$\Rightarrow \frac{V_{out}}{V_s} = g_m R_L \quad — ②$$

• From ① and ②

$$\frac{V_{out}(S)}{V_{gen}(S)} = \left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} \times \frac{1+b_1 S+b_2 S^2+\dots}{1+a_1 S+a_2 S^2+\dots}$$

$$= \frac{V_{out}}{V_s} \cdot \frac{V_s}{V_{gen}}$$

$$\left.= g_m R_L \cdot \frac{1}{1+g_m L_e \cdot S}\ \right]$$

(4)

b

Part c, 10 points

gm= 100 mS. RL=1 kOhm. Le= 1 nH

Find the frequencies of any zeros (there may be zero, one or two present) in the transfer function:

f_{z1} =________, f_{z2} =________, ...

There may be either 1 or 2 poles of the transfer function.

If the poles are real, give the 1 or 2 pole frequencies in Hz:

f_{p1} = 1.59 GHz, f_{p2} =________

If there are 2 poles, and they are complex, give $f_n = \omega_n / 2\pi$ and the damping factor ζ :

$f_n = \omega_n / 2\pi$ =________, ζ =________

From part b

$$\frac{V_{out}(s)}{V_{gen}(s)} = g_m R_L \cdot \frac{1}{1 + j\frac{f}{f_p}}, \text{ where } f_p = \frac{0.159}{g_m L_e}$$

9

$$= 100 \cdot \frac{1}{1 + j\frac{f}{1.59(GHz)}}$$

$$f_p = \frac{0.159}{100(mS) \cdot 1(nH)} = 1.59 \times 10^9 (Hz) = 1.59 (GHz)$$

$$g_m R_L = 100(mS) \cdot 1(k\Omega) = 100$$

1

∴ There is a real pole at f_p = 1.59 (GHz)

[deduct points 2 each for additional poles & zeros.

b