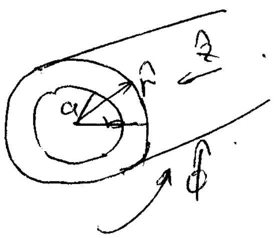
Homework 3.

8-19. recall page. 88~89.

$\vec{E} = \hat{a}_r \cdot \frac{\rho}{2\pi\varepsilon r}$. ρ is the line charge density on the inner conductor.



$V_0 = \int_a^b \vec{E} \cdot d\vec{r} = \frac{\rho}{2\pi\varepsilon} \ln\left(\frac{b}{a}\right)$

so $\frac{\rho}{2\pi\varepsilon} = \frac{V_0}{\ln\left(\frac{b}{a}\right)}$. $\vec{E} = \hat{a}_r \cdot \frac{\rho}{2\pi\varepsilon} \cdot \frac{1}{r} = \hat{a}_r \cdot \frac{V_0}{r \cdot \ln\left(\frac{b}{a}\right)}$.

From Ampère's law, $\vec{H} = \hat{a}_\phi \cdot \frac{I}{2\pi r}$.

Poynting vector, $\vec{P} = \vec{E} \times \vec{H} = \hat{a}_z \cdot \frac{V_0 I}{2\pi r^2 \cdot \ln\left(\frac{b}{a}\right)}$.

The power transmitted over cross-sectional area is:

$P = \int_s \vec{P} \cdot d\vec{s} = \int_0^{2\pi} \int_a^b \frac{V_0 I}{2\pi r^2 \cdot \ln\left(\frac{b}{a}\right)} r \cdot dr \cdot d\phi = V_0 I$.

8-21 $\vec{E}_i = \vec{E} = E_0(\hat{a}_x - j\hat{a}_y) e^{-j\beta z}$.

(a). $\vec{E}_r = -E_0(\hat{a}_x - j\hat{a}_y) \cdot e^{j\beta z}$

$= E_0(-\hat{a}_x + j \cdot \hat{a}_y) \cdot e^{j\beta z}$.

The direction of propagation is $(-\hat{a}_z)$.
So the reflected wave is left-hand circular.

(b). recall page 263.

$(-\hat{a}_z) \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$. since $\vec{H}_2$ is in perfect conductor, $\vec{H}_2 = 0$.

$\vec{H}_1 = \vec{H}_i + \vec{H}_r$.

$\vec{H}_i = \frac{E_0}{\eta_1}\cdot(j\hat{a}_x + \hat{a}_y)\cdot e^{-j\beta z}$.

$\vec{H}_r = \frac{E_0}{\eta_1}(j\hat{a}_x + \hat{a}_y)\cdot e^{j\beta z}$.

$\vec{H}_1 = \frac{2E_0}{\eta_1}(j\hat{a}_x + \hat{a}_y)\cdot \cos(\beta z)$.

$\vec{J}_s = -\hat{a}_z \times \vec{H}_1(0) = \frac{2E_0}{\eta_1}(\hat{a}_x - j\hat{a}_y)$.

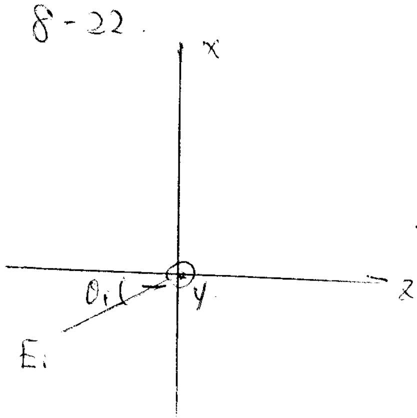
(c). $\vec{E}_1(z,t) = Re\left[(\vec{E}_i + \vec{E}_r)\,e^{j\omega t}\right]$

$= Re\left[E_0(\hat{a}_x - j\hat{a}_y)\cdot e^{j(\omega t-\beta z)} + E_0(-\hat{a}_x + j\hat{a}_y)\cdot e^{j(\omega t+\beta}\right.$

$= Re\left[E_0(-2j(\hat{a}_x - j\hat{a}_y)\cdot\sin(\beta z))\cdot e^{j\omega t}\right]$

$= 2E_0\sin(\beta z)\cdot(\sin(\omega t)\cdot\hat{a}_x - \cos(\omega t)\cdot\hat{a}_y)$

8-22.



(a) $\hat{k}_i = 6\hat{a}_x + 8\hat{a}_z$.

$k_i = (6^2 + 8^2)^{1/2} = 10 = 2\pi f\sqrt{\mu_0\epsilon_0} = \frac{2\pi f}{c}$

frequency: $f = \frac{k_i\cdot c}{2\pi} = \frac{3\times10^9}{2\pi} = 4.77\times10^8\,(m/s)$

wavelength: $\lambda = \frac{2\pi}{k} = 0.628$ (m).

(b). $\vec{E}_i = Re(\hat{a}_y 10\, e^{-j(6x+8z)}\, e^{j\omega t})$.

$= \hat{a}_y\cdot 10\cdot\cos(\omega t - 6x - 8z)$.

$= \hat{a}_y\cdot 10\cdot\cos(3\times10^9 t - 6x - 8z)$

unit vector of $\vec{k}$ is $\hat{k} = \frac{1}{10}(6\hat{a}_x + 8\hat{a}_z)$ (3)

phasor: $\vec{H}_i = \frac{1}{\eta_0} \hat{k} \times \vec{E}$

$$= \frac{1}{120\pi} \times \frac{1}{10} (6\hat{a}_x + 8\hat{a}_z) \times (\hat{a}_y \cdot 10 e^{-j(6x+8z)})$$

$$= \left(-\frac{1}{15\pi} \cdot \hat{a}_x + \frac{1}{20\pi} \cdot \hat{a}_z\right) \cdot e^{-j(6x+8z)}$$

$$\vec{H}_i = Re[\vec{H}_i \cdot e^{j\omega t}]$$

$$= \left(-\frac{1}{15\pi} \hat{a}_x + \frac{1}{20\pi} \hat{a}_z\right) \cdot \cos(\omega t - 6x - 8z).$$

(c). $\cos\theta_i = 0.8$

(d). $\vec{E}_r(x, z) = -\hat{a}_y \cdot 10 \cdot e^{-j(6x-8z)}$

Because $\vec{E}_r(x, 0) + \vec{E}_i(x, 0) = 0$. for all x.

$$\vec{H}_r(x, z) = \frac{1}{\eta_0} \times \frac{1}{10} (6\hat{a}_x - 8\hat{a}_z) \times (-\hat{a}_y \cdot 10 \cdot e^{-j(6x-8z)}).$$

$$= \frac{1}{120\pi} (-6\hat{a}_z - 8\hat{a}_x) \cdot e^{-j(6x-8z)}$$

$$= -\left(\frac{1}{15\pi} \hat{a}_x + \frac{1}{20\pi} \hat{a}_z\right) \cdot e^{-j(6x-8z)}$$

(e). $\vec{E}_1(z, x) = \vec{E}_i + \vec{E}_r = \hat{a}_y \left(10 e^{-j(6x+8z)} - 10 \cdot e^{-j(6x-8z)}\right)$

$$= \hat{a}_y \cdot 10 \cdot e^{-j6x} \cdot (-2j) \cdot \sin(8z)$$

$$= -j\, 20 \cdot \sin(8z) \cdot e^{-j6x} \cdot \hat{a}_y$$

$$\vec{H}_1(z, x) = \vec{H}_i + \vec{H}_r = \hat{a}_x \frac{2}{15\pi} \cos(8z)\, e^{-j6x}$$
$$+ \hat{a}_z \cdot \frac{2}{10\pi} \cdot \sin(8z) \cdot e^{-j6x}.$$

8-23. $\vec{E}_i(y,z) = 5(\hat{a}_y + \hat{a}_z \cdot \sqrt{3}) \cdot e^{j6(\sqrt{3}y - z)}$ (4)

(a). frequency. $k = \frac{w}{c} = 6 \times 2 = 12$.

$w = 36 \times 10^8 = 3.6 \times 10^9$ (rad/s)

$f = \frac{w}{2\pi} = 5.73 \times 10^8$ Hz.

wave length. $\lambda = \frac{2\pi}{k} = \frac{\pi}{6} = 0.524$ (m).

(b).

$\vec{E}_i(y,z,t) = 5 \cdot \cos(wt + 6\sqrt{3}y - 6z) \cdot \hat{a}_y + 5\sqrt{3}\cos(wt + 6\sqrt{3}y - 6z) \cdot \hat{a}_z$

phasor:

$\vec{H}_i(y,z) = \frac{1}{\eta_0} \cdot \hat{k} \times \vec{E}_i$

$= \frac{-1}{120\pi} \cdot \frac{1}{2} \cdot (\sqrt{3}\hat{a}_y - \hat{a}_z) \times 5(\hat{a}_y + \sqrt{3}\hat{a}_z) e^{j6(\sqrt{3}y - z)}$

$= \frac{-1}{240\pi} \cdot 5 \cdot (3\hat{a}_x + \hat{a}_x) \cdot e^{j6(\sqrt{3}y - z)}$

$= \frac{-1}{12\pi} \hat{a}_x \cdot e^{j \cdot 6(\sqrt{3}y - z)}$.

$\vec{H}_i(y,z,t) = \frac{-1}{12\pi} \hat{a}_x \cdot \cos(wt + 6\sqrt{3}y - 6z)$.

(c). θ_i. $\cos\theta_i = \frac{1}{2}$

(d). $\vec{E}_r(y,z)$: $\begin{cases} \vec{k}_r \cdot \vec{E}_r = 0 \\ \vec{E}_{iy}(y,0) + \vec{E}_{ry}(y,0) = 0 \end{cases}$ for all y.

$\vec{E}_r = -5(\hat{a}_y \pm \hat{a}_z \sqrt{3}) \cdot e^{j6(\sqrt{3}y + z)}$.

5)

$$\vec{H}_r(y,z) = \frac{1}{\eta_0}\hat{k}_r \times \vec{E}_r = \frac{1}{120\pi}\cdot\left(-\frac{\sqrt{3}}{2}\hat{a}_y - \frac{1}{2}\hat{a}_z\right)\times 5(-\hat{a}_y+\sqrt{3}\hat{a}_z)\, e^{j(6\sqrt{3}y+z)}$$

$$= \hat{a}_x\cdot\left(-\frac{1}{12\pi}\right)\cdot e^{j6(\sqrt{3}y+z)}.$$

(e).

$$\vec{E}_1(y,z) = \vec{E}_i + \vec{E}_r = (-j10\sin(6z)\hat{a}_y + 10\sqrt{3}\cos(6z)\cdot\hat{a}_z)\, e^{j6\sqrt{3}y}.$$

$$\vec{H}_1(y,z) = \vec{H}_i + \vec{H}_r = \hat{a}_x\left(-\frac{1}{6\pi}\right)\cdot\cos(6z)\cdot e^{j6\sqrt{3}y}.$$

8-24. from equation (8-113).

$$\vec{E}_1(x,z,t) = \hat{a}_y\cdot 2E_{io}\cdot\sin(\beta_1 z\cdot\cos\theta_i)\cdot\sin(\omega t-\beta_1 x\sin\theta_i)$$

from equation (8-114)

$$\vec{H}_1(x,z,t) = \frac{2E_{io}}{\eta_1}\Big[-\hat{a}_x\cos\theta_i\cos(\beta_1 z\cdot\cos\theta_i)\cdot\cos(\omega t-\beta_1 x\sin\theta_i)$$

$$+\hat{a}_z\sin\theta_i\cdot\sin(\beta_1 z\cos\theta_i)\cdot\sin(\omega t-\beta_1 x\sin\theta_i)\Big]$$

Time-average Poynting vector,

$$\vec{P}_{av} = \frac{1}{2}\mathrm{Re}(\vec{E}_1\times\vec{H}_1^*).$$

$$= \frac{1}{2}\mathrm{Re}\Big[\left(-\hat{a}_y\, j\,2E_{io}\sin(\beta_1 z\cos\theta_i)\cdot e^{-j\beta_1 x\sin\theta_i}\right)$$

$$\times\left(-2\frac{E_{io}}{\eta_1}\cdot\left[\hat{a}_x\cos\theta_i\cos(\beta_1 z\cdot\cos\theta_i)\cdot e^{j\beta_1 x\sin\theta_i}\right.\right.$$

$$\left.\left.-\hat{a}_z j\cdot\sin\theta_i\sin(\beta_1 z\cos\theta_i)\cdot e^{j\beta_1 x\sin\theta_i}\right]\right)\Big].$$

$$= \hat{a}_x\frac{2E_{io}^2}{\eta_1}\cdot\sin\theta_i\sin^2(\beta_1 z\cos\theta_i)$$

(6)

8-25.

(a). From Equation (8-128).

$$\vec{E}_1(x,z) = -2E_{i0}\left[\hat{a}_x \cdot j \cdot \cos\theta_i \cdot \sin(\beta_1 z\cos\theta_i) + \hat{a}_z \cdot \sin\theta_i \cdot \cos(\beta_1 z\cos\theta_i)\right] e^{-j\beta_1 x\sin\theta_i}$$

$$\vec{E}(x,z,t) = Re\left(\vec{E}_1(x,z)\, e^{j\omega t}\right).$$

$$= -2E_{i0}\left[-\hat{a}_x \cos\theta_i \cdot \sin(\beta_1 z\cdot\cos\theta_i) \cdot \sin(\omega t - \beta_1 x\sin\theta_i) + \hat{a}_z \sin\theta_i \cdot \cos(\beta_1 z\cdot\cos\theta_1) \cdot \cos(\omega t - \beta_1 x\sin\theta_i)\right].$$

$$\vec{H}_1(x,z) = \hat{a}_y \cdot 2\frac{E_{i0}}{\eta_1}\cos(\beta_1 z\cdot\cos\theta_1)\cdot e^{-j\beta_1 x\cdot\sin\theta_i}.$$

$$\vec{H}_1(x,z,t) = Re\left(\vec{H}_1(x,z)\, e^{j\omega t}\right).$$

$$= \hat{a}_y \cdot 2\frac{E_{i0}}{\eta_1}\cos(\beta_1 z\cdot\cos\theta_1)\cdot\cos(\omega t - \beta_1 x\cdot\sin\theta_i)$$

(b). Time-average Poynting vector.

$$\vec{P}_{av} = \frac{1}{2}Re\left(\vec{E}_1 \times \vec{H}_1^*\right)$$

$$= \frac{1}{2}Re\left[\left(-2E_{i0}\left(\hat{a}_x\, j\cos\theta_i\cdot\sin(\beta_1 z\cos\theta_i) + \hat{a}_z\sin\theta_i\cdot\cos(\beta_1 z\cos\theta_i)\right)\cdot e^{-j\beta_1 x\sin\theta_i}\right) \times \left(\hat{a}_y\cdot 2\frac{E_{i0}}{\eta_1}\cos(\beta_1 z\cos\theta_i)\cdot\cancel{\cos(\omega t - \beta_1 x\sin\theta_i)}\, e^{j\beta_1 x\sin\theta_i}\right)\right]$$

$$= \frac{2E_{i0}^2}{\eta_1}\hat{a}_x\cdot\cos^2(\beta_1 z\cos\theta_i)\cdot\sin\theta_i.$$