Homework #4 .

8-29. From Eqs. (8-156) through (8-161) on PP. 403,

$$\vec{E}_1 = \hat{a}_x (E_{i0} e^{-j\beta_0 z} + E_{r0} e^{j\beta_0 z})$$

$$\vec{H}_1 = \hat{a}_y \cdot \frac{1}{\eta_0} (E_{i0} e^{-j\beta_0 z} - E_{r0} e^{j\beta_0 z})$$

$$\vec{E}_2 = \hat{a}_x (E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z})$$

$$\vec{H}_2 = \hat{a}_y \frac{1}{\eta_2} (E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z})$$

$$\vec{E}_t = \hat{a}_x E_{t0} e^{-j\beta_0 z} , \qquad \vec{H}_t = \hat{a}_y E_{t0} \frac{1}{\eta_0} e^{-j\beta_0 z}$$

Boundary conditions:

at $z = 0$; $\vec{E}_1(0) = \vec{E}_2(0)$, $\vec{H}_1(0) = \vec{H}_2(0)$

at $z = d$; $\vec{E}_2(d) = \vec{E}_t(d)$, $\vec{H}_2(d) = \vec{H}_t(d)$.

(a). $$E_{r0} = -\frac{j(\eta_0^2 - \eta_2^2) \cdot \tan(\beta_2 d)}{\eta_0 \eta_2 + j(\eta_0^2 + \eta_2^2) \tan(\beta_2 d)} E_{i0}$$

$$E_2^+ = \frac{\eta_2 (\eta_0 + \eta_2) \cdot e^{j\beta_2 d}}{\eta_0 \eta_2 \cos(\beta_2 d) + j(\eta_0^2 + \eta_2^2) \sin(\beta_2 d)} E_{i0}$$

$$E_2^- = \frac{\eta_2 (\eta_0 - \eta_2) e^{-j\beta_2 d}}{\eta_0 \eta_2 \cdot \cos(\beta_2 d) + j(\eta_0^2 + \eta_2^2) \sin(\beta_2 d)} E_{i0}$$

$$E_{t0} = \frac{2\eta_0 \eta_2 e^{j\beta_0 d}}{\eta_0 \eta_2 \cos(\beta_2 d) + j(\eta_0^2 + \eta_2^2) \sin(\beta_2 d)} E_{i0} .$$

where $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ $\quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$. $\quad \beta_0 = \frac{\omega}{c}$, $\quad \beta_2 = \omega \sqrt{\mu_2 \varepsilon_2}$

(b). if $d = \lambda_2/4$. $\quad \beta_2 d = \frac{2\pi}{\lambda_2} \cdot \frac{\lambda_2}{4} = \frac{\pi}{2}$.

$$E_{ro} = -\frac{\eta_0^2 - \eta_2^2}{\eta_0^2 + \eta_2^2} E_{io}.$$

$$\Gamma = -\frac{\eta_0^2 - \eta_2^2}{\eta_0^2 + \eta_2^2}.$$

$\eta_2 = \eta_0 \quad : \quad \Gamma = 0.$

$\eta_2 \neq \eta_0 \quad : \quad \Gamma \neq 0.$

if $d = n\lambda_2/2$. $n = 1, 2 \cdots$

$\tan \beta_2 d = \tan(\pi \cdot n) = 0. \quad \Gamma = 0.$

8-30. Recall Example 8-12.

$\eta_2 = \sqrt{\eta_1 \eta_3}$, because material ① is air, and ③ is glass, $\eta_1 \neq \eta_3$.

$\varepsilon_{r2} = \sqrt{\varepsilon_{r1} \varepsilon_{r3}} = \sqrt{1 \cdot 4} = 2.$

(a). The required dielectric constant $\varepsilon_r = 2$.

The wavelength in dielectric coating is

$$\lambda_2 = \frac{0.75}{\sqrt{\varepsilon_{r2}}} = 0.53 \ (\mu m).$$

The thickness of the coating is

$$d = \frac{\lambda_2}{4} = 0.133 \ (\mu m).$$

(b). For violet light: $\lambda_2' = \frac{0.42}{\sqrt{2}} = 0.297 \ (\mu m)$.

$\frac{d}{\lambda_2'} = 0.447. \quad \beta_2 d = \frac{2\pi}{\lambda_2'} d = 2\pi \times 0.447 = 0.894\pi$

From Eq. (8-173), and $\eta_1 = \eta_0$

$$Z_2(0) = \eta_2 \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)} = \frac{\eta_0}{\sqrt{2}} \cdot \frac{\frac{1}{2} + j\frac{1}{\sqrt{2}} \tan(\beta_2 d)}{\frac{1}{\sqrt{2}} + j\frac{1}{2} \tan(\beta_2 d)}.$$

$$\Gamma = \frac{Z_2(0) - \eta_0}{Z_2(0) + \eta_0} = 0.317 e^{j198^\circ}.$$

$|\Gamma|^2 = (0.317)^2 = 10\%$. Percentage of power reflected

8-31. Γ_{12}, Γ_{23} denote the reflection coefficients between media 1 and 2 and between media 2 and 3 respectively. They are intrinsic parameters, depending on η_1, η_2, and η_3.

$$\Gamma_{12} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \longrightarrow \frac{\eta_1}{\eta_2} = \frac{1 - \Gamma_{12}}{1 + \Gamma_{12}}.$$

$$\Gamma_{23} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} \longrightarrow \frac{\eta_2}{\eta_3} = \frac{1 - \Gamma_{23}}{1 + \Gamma_{23}}.$$

Recall pp. 405.

The effective reflection coefficient Γ_0 is:

$$\Gamma_0 = \frac{Z_2(0) - \eta_1}{Z_2(0) + \eta_1}.$$

$$Z_2(0) = \eta_2 \cdot \frac{\eta_3 + j\,\eta_2 \tan(\beta_2 d)}{\eta_2 + j\,\eta_3 \tan(\beta_2 d)}$$

so

$$\Gamma_0 = \frac{1 + j\frac{\eta_2}{\eta_3}\cdot\tan(\beta_2 d) - \frac{\eta_1}{\eta_2}\left(\frac{\eta_2}{\eta_3} + j\tan(\beta_2 d)\right)}{1 + j\frac{\eta_2}{\eta_3}\tan(\beta_2 d) + \frac{\eta_1}{\eta_2}\left(\frac{\eta_2}{\eta_3} + j\tan(\beta_2 d)\right)}$$

$$= \frac{(\Gamma_{12} + \Gamma_{23}) + j(\Gamma_{12} - \Gamma_{23})\cdot\tan(\beta_2 d)}{(1 + \Gamma_{12}\Gamma_{23}) + j(1 - \Gamma_{12}\Gamma_{23})\cdot\tan(\beta_2 d)}.$$

8-36. (a). From Snell's law.

$$\frac{\sin\theta_i}{\sin\theta_t} = n. \quad \text{so} \quad \theta_t = \sin^{-1}\left(\frac{1}{n}\sin\theta_i\right).$$

(b).

$$l_1 = d \cdot \tan(\theta_t) = d \cdot \frac{\sin\theta_t}{\cos\theta_t} = \frac{d \cdot \sin\theta_t}{\sqrt{1-\sin^2\theta_t}}$$

$$\sin\theta_t = \frac{1}{n} \cdot \sin\theta_i \qquad \cos\theta_t = \sqrt{1 - (\frac{1}{n}\sin\theta_i)^2}$$

(c).

$$l_2 = \frac{d}{\cos\theta_t} \cdot \sin(\theta_i - \theta_t)$$

$$= \frac{d}{\cos(\theta_t)} \cdot [\sin\theta_i \cos\theta_t - \sin\theta_t \cdot \cos\theta_i]$$

8-37.

(a). $\sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$.

$$\sin\theta_t = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \cdot \sin\theta_i > \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sin\theta_c = 1.$$

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \pm j \sqrt{\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_i - 1}$$

From (8-200),

$$\vec{E}_t = \hat{a}_y \cdot E_{to} \cdot e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\vec{H}_t = \frac{E_{to}}{\eta_2}(-\hat{a}_x\cos\theta_t + \hat{a}_z\sin\theta_t)e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

From (8-207).

$$E_{to} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1 \cdot \cos\theta_t} \cdot E_{io}$$

Note: $\beta_2 \cdot \sin\theta_t$ is a real number.

$\beta_2 \cos\theta_t$ is a ~~complex~~ imaginery number.

$-j\beta_2\cos\theta_t = \pm \beta_2\left(\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_i - 1\right)^{\frac{1}{2}}$, since the energy shall decrease, we have:

$$\vec{E}_t = \hat{a}_y E_{t0} e^{-j\beta_2 x \sin\theta_t} \cdot e^{-\beta_2 \left(\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_i - 1\right)^{\frac{1}{2}}}$$

$$\vec{H}_t = \frac{E_{t0}}{\eta_2}\left(+\hat{a}_x\, j \left(\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_i - 1\right)^{\frac{1}{2}} + \hat{a}_z \sin\theta_t\right) \cdot e^{-j\beta_2 x \sin\theta_t} \cdot e^{-\beta_2\left(\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_i -\right.}$$

(b).

$$\vec{P}_{av} = \frac{1}{2}\mathrm{Re}\left(\vec{E}_t \times \vec{H}_t^*\right)$$

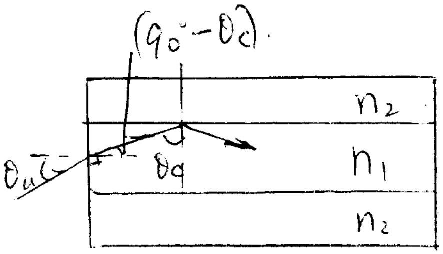
We need to consider $\vec{P}_{av}$ in z direction.

$$(\vec{P}_{av})_z = \frac{1}{2}\mathrm{Re}\left(E_{ty} \cdot H_{tx}^*\right)$$

$$= \frac{1}{2}\mathrm{Re}\left(\underline{j} \cdot \frac{E_{t0}^2}{\eta_2}\left(\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_i - 1\right)^{\frac{1}{2}} \cdot e^{-2\beta_2\left(\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_i - 1\right)^{\frac{1}{2}}}\right)$$

$$= 0.$$

8-41. (a). From Snell's law,



$$\frac{\sin\theta_a}{\sin(90^\circ - \theta_c)} = \frac{n_1}{n_0}$$

$$\frac{\sin\theta_c}{1} = \frac{n_2}{n_1}.$$

So

$$\sin(\theta_a) = \frac{n_1}{n_0}\cdot\cos(\theta_c) = \frac{n_1}{n_0}\cdot\left(1 - \sin^2\theta_c\right)^{\frac{1}{2}}$$

$$= \frac{n_1}{n_0}\cdot\left(1 - \left(\frac{n_2}{n_1}\right)^2\right)^{\frac{1}{2}}$$

$$= \frac{1}{n_0}\left(n_1^2 - n_2^2\right)^{\frac{1}{2}}.$$

$$\theta_a = \sin^{-1}\left(\frac{1}{n_0}\cdot\left(n_1^2 - n_2^2\right)^{\frac{1}{2}}\right).$$

(b). $\sin\theta_a = 0.986$.

$\theta_a = 1.4 = 80^\circ$ (radian)

Note, we can assume $n_0 = 1$.

8-42.

From Snell's law.

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} = \frac{\eta_2}{\eta_1} \qquad (\mu_2 = \mu_1).$$

(a). From Eqs (8-206), (8-207).

$$\Gamma_\perp = \frac{\left(\frac{\eta_2}{\eta_1}\right)\cos\theta_i - \cos\theta_t}{\left(\frac{\eta_2}{\eta_1}\right)\cos\theta_i + \cos\theta_t}$$

$$= \frac{\sin\theta_t\cdot\cos\theta_i - \cos\theta_t\cdot\sin\theta_i}{\sin\theta_t\cdot\cos\theta_i + \cos\theta_t\cdot\sin\theta_i} = \frac{\sin(\theta_t-\theta_i)}{\sin(\theta_t+\theta_i)}$$

$$\tau_\perp = \frac{2\left(\frac{\eta_2}{\eta_1}\right)\cdot\cos\theta_i}{\cos\theta_t + \left(\frac{\eta_2}{\eta_1}\right)\cdot\cos\theta_i}$$

$$= \frac{2\sin\theta_t\cdot\cos\theta_i}{\sin\theta_i\cos\theta_t + \sin\theta_t\cdot\cos\theta_i} = \frac{2\sin\theta_t\cdot\cos\theta_i}{\sin(\theta_i+\theta_t)}.$$

(b). From Eqs (8-221), (8-222)

$$\Gamma_\parallel = \frac{\left(\frac{\eta_2}{\eta_1}\right)\cdot\cos\theta_t - \cos\theta_i}{\left(\frac{\eta_2}{\eta_1}\right)\cdot\cos\theta_t + \cos\theta_i}$$

$$= \frac{\frac{\sin\theta_t}{\sin\theta_i}\cdot\cos\theta_t - \cos\theta_i}{\frac{\sin\theta_t}{\sin\theta_i}\cos\theta_t + \cos\theta_i} = \frac{\sin(2\theta_t) - \sin(2\theta_i)}{\sin(2\theta_t) + \sin(2\theta_i)}$$

$$\tau_\parallel = \frac{2\left(\frac{\eta_2}{\eta_1}\right)\cdot\cos\theta_i}{\frac{\eta_2}{\eta_1}\cdot\cos\theta_t + \cos\theta_i} = \frac{2\sin\theta_t\cdot\cos\theta_i}{\sin\theta_t\cdot\cos\theta_t + \sin\theta_i\cdot\cos\theta_i}$$

$$= \frac{4\sin\theta_t\cdot\cos\theta_i}{\sin(2\theta_t) + \sin(2\theta_i)}$$

(7)

8-45. (a). For perpendicular polarization,

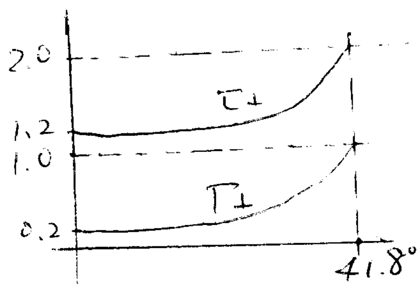
$$\Gamma_\perp = \frac{\sqrt{\varepsilon_{r1}}\cdot\cos\theta_i - \sqrt{\varepsilon_{r2}}\cdot\cos\theta_t}{\sqrt{\varepsilon_{r1}}\cdot\cos\theta_i + \sqrt{\varepsilon_{r2}}\cdot\cos\theta_t} \qquad \tau_\perp = \frac{2\sqrt{\varepsilon_{r1}}\cdot\cos\theta_i}{\sqrt{\varepsilon_{r2}}\cdot\cos\theta_t + \sqrt{\varepsilon_{r1}}\cdot\cos\theta_i}$$

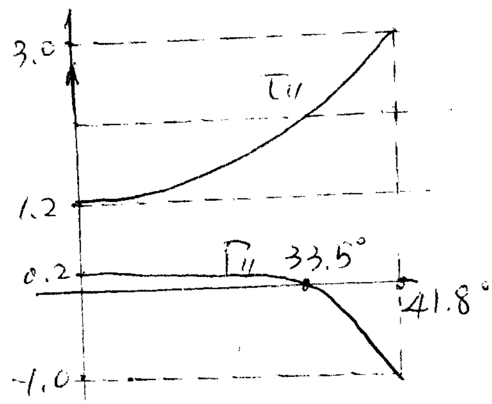
Here $\sin\theta_t = \frac{\sqrt{\varepsilon_{r1}}}{\sqrt{\varepsilon_{r2}}}\sin\theta_i$. $\cos\theta_t = \sqrt{1-\left(\frac{\varepsilon_{r1}}{\varepsilon_{r2}}\right)\sin^2\theta_i}$.

(b). For parallel polarization,

$$\Gamma_{\parallel} = \frac{\sqrt{\varepsilon_{r1}}\cdot\cos\theta_t - \sqrt{\varepsilon_{r2}}\cdot\cos\theta_i}{\sqrt{\varepsilon_{r1}}\cdot\cos\theta_t + \sqrt{\varepsilon_{r2}}\cdot\cos\theta_i} \qquad \tau_{\parallel} = \frac{2\sqrt{\varepsilon_{r1}}\cos\theta_i}{\sqrt{\varepsilon_{r1}}\cdot\cos\theta_t + \sqrt{\varepsilon_{r2}}\cdot\cos\theta_i}$$

(c). $\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} = 1.5$. $\sin\theta_c = \frac{1}{1.5}$. $\theta_c = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8°$.





8-47. (a). $\Gamma_{\parallel}' = \frac{E_{ro}\cdot\cos\theta_i}{E_{io}\cdot\cos\theta_i} = \frac{E_{ro}}{E_{io}} = \Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$

$\tau_{\parallel}' = \frac{E_{to}\cdot\cos\theta_t}{E_{io}\cdot\cos\theta_i} = \tau_{\parallel}\cdot\left(\frac{\cos\theta_t}{\cos\theta_i}\right) = \frac{2\eta_2\cos\theta_t}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$.

(b). $\Gamma_{\parallel}' + 1 = \tau_{\parallel}'$.

$\Gamma_{\parallel} = 1 - \tau_{\parallel}'\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$