Homework #6.



9-16. (a). For a short-circuit termination,

$$Z_{is} = Z_0 \tanh(\gamma l) = Z_0 \cdot \frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} \approx Z_0 \cdot \frac{1+\gamma l - (1-\gamma l)}{1+1+\gamma l-\gamma l}$$

$$= Z_0 \cdot \frac{2\gamma l}{2} = Z_0 \cdot \gamma l.$$

since

$$\gamma = \sqrt{(R+j\omega L)\cdot(G+j\omega C)} \qquad Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}.$$

plug into $Z_{is}$

$$Z_{is} = Z_0 \cdot \gamma l = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \cdot \sqrt{(R+j\omega L)(G+j\omega C)} \cdot l$$

$$= (R+j\omega L)\cdot l$$

(b). For an open-circuit termination,

$$Z_{io} = Z_0 \coth(\gamma l) \approx Z_0 \cdot \frac{e^{\gamma l} + e^{-\gamma l}}{e^{\gamma l} - e^{-\gamma l}} \approx Z_0 \cdot \frac{1+1}{1+\gamma l-(1-\gamma l)}$$

$$= Z_0 \cdot \frac{1}{\gamma l} = \frac{1}{l\cdot(G+j\omega C)}$$

9-21. For an open-circuit lossy transmission line, the input impedance is

$$Z_{io} = Z_0 \cdot \coth(\gamma l) = Z_0 \cdot \frac{e^{\alpha l+j\beta l} + e^{-\alpha l-j\beta l}}{e^{\alpha l+j\beta l} - e^{-\alpha l-j\beta l}}$$

$\alpha l \ll 1$ (low-loss).

$$e^{\alpha l+j\beta l} = (1+\alpha l)\cdot e^{j\beta l}, \quad e^{-\alpha l-j\beta l} = (1-\alpha l)\cdot e^{-j\beta l}$$

plug into $Z_{i0}$,

$$Z_{i0} = Z_0 \cdot \frac{(1+\alpha l)\cdot e^{j\beta l} + (1-\alpha l)\cdot e^{-j\beta l}}{(1+\alpha l)\cdot e^{j\beta l} - (1-\alpha l)\cdot e^{-j\beta l}}$$

$$= Z_0 \cdot \frac{2\cos(\beta l) + 2\cdot \alpha l \cdot j \sin(\beta l)}{2j\cdot \sin(\beta l) + 2\cdot\alpha l\cdot \cos(\beta l)}$$

$$= Z_0 \cdot \frac{\cos(\beta l) + j\,\alpha l\cdot \sin(\beta l)}{\alpha l\cdot\cos(\beta l) + j\sin(\beta l)}.$$

At $l = \frac{n\lambda}{2}$, $\beta\cdot l = \frac{2\pi}{\lambda}\cdot\frac{n\lambda}{2} = n\cdot\pi$.

$$Z_{i0} = Z_0\cdot\frac{1}{\alpha l} = Z_{max}.$$

Now consider $f = f_0 + \Delta f$.

$$\beta\cdot l = n\pi\cdot\left(1+\frac{\Delta f}{f_0}\right).$$

$$Z_{i0} = Z_0\cdot\frac{1}{\alpha l + j\cdot n\pi\cdot\frac{\Delta f}{f_0}}.$$

So that,

$$\frac{|Z_{i0}|^2}{|Z_{max}|^2} = \frac{1}{1+[n\pi\cdot\alpha l\cdot \Delta f/f_0]^2}.$$

To obtain half-power,

$$[n\pi\,\alpha l\cdot\frac{\Delta f}{f_0}]^2 = 1.$$

$$\Delta f = \pm\frac{\alpha f_0\cdot l}{n\pi} = \pm\frac{\alpha\cdot f_0\, n\lambda}{2n\pi} = \pm\frac{\alpha}{2\pi\sqrt{LC}}.$$

From eq. 55 $\quad \alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right).$

$$\Delta f = \pm\frac{1}{4\pi}\cdot\left(\frac{R}{L}+\frac{G}{C}\right).$$

So the half-power bandwidth is

$$2\Delta f = \frac{1}{2\pi}\left(\frac{R}{L} + \frac{G}{C}\right).$$

$$Q = \frac{f_0}{2\Delta f} = \frac{2\pi f_0}{\frac{R}{L} + \frac{G}{C}}$$

9-24. (a) $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$. for a lossless transmission line, $Z_0 = R_0$ is real value.

$$\Gamma = \frac{(R_L - R_0) + jX_L}{(R_L + R_0) + jX_L}.$$

since $S = \frac{1+|\Gamma|}{1-|\Gamma|}$, to minimize the standing wave ratio is to minimize $|\Gamma|$.

$$|\Gamma|^2 = \frac{(R_L - R_0)^2 + X_L^2}{(R_L + R_0)^2 + X_L^2} \qquad \frac{\partial |\Gamma|^2}{\partial R_0} = 0 \Rightarrow R_0 = \sqrt{R_L^2 + X_L^2}.$$

$$= 50\,(\Omega)$$

(b). $|\Gamma| = \left|\frac{-10 + j30}{90 + j30}\right| = \sqrt{\frac{1000}{9000}} = \frac{1}{3}$.

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = 2.$$

(c). The location of minimum voltage is at $z' = l$, where $-|\Gamma| = \Gamma \cdot e^{-2\gamma l} = \Gamma \cdot e^{-2j\beta l}$.

since $\Gamma = \frac{1}{3} e^{j\frac{\pi}{2}}$,

we need $-2\beta l + \frac{\pi}{2} = -\pi$, $-\frac{4\pi}{\lambda} l = -\frac{3}{2}\pi$.

$l = \frac{3}{8}\lambda$ . is the nearest minimum voltage point.

9-28. Eq. (9-147) is

$$R_i + jX_i = R_o \frac{R_m + jR_o \tan(\beta l_m)}{R_o + jR_m \tan(\beta l_m)}.$$

$$= R_o \cdot \frac{R_m(R_o - jR_m \tan(\beta l_m)) + jR_o \tan(\beta l_m)\cdot(R_o - jR_m \tan(\beta l_m))}{R_o^2 + R_m^2 \cdot \tan^2(\beta l_m)}$$

$$\begin{cases} R_i = \dfrac{R_o}{R_o^2 + R_m^2 \tan^2(\beta l_m)}\left[R_m R_o + R_o R_m \tan^2(\beta l_m)\right]. \\ X_i = \dfrac{R_o}{R_o^2 + R_m^2 \tan^2(\beta l_m)}\cdot\left[R_o^2 \tan(\beta l_m) - R_m^2 \cdot \tan(\beta l_m)\right]. \end{cases}$$

Solve these equations:

$$R_m = \frac{R_o^2}{2R_i}\left[\left(1 + \left(\frac{R_i}{R_o}\right)^2 + \left(\frac{X_i}{R_o}\right)^2\right) \pm \sqrt{\left(1 + \left(\frac{R_i}{R_o}\right)^2 + \left(\frac{X_i}{R_o}\right)^2\right)^2 - 4\left(\frac{R_i}{R_o}\right)^2}\right].$$

$$l_m = \frac{1}{\beta}\cdot\tan^{-1}\left[\frac{R_o}{2X_i}\cdot\left\{-\left[1 - \left(\frac{R_i}{R_o}\right)^2 - \left(\frac{X_i}{R_o}\right)^2\right] \pm \sqrt{\left(1 - \left(\frac{R_i}{R_o}\right)^2 - \left(\frac{X_i}{R_o}\right)^2\right)^2 + 4\left(\frac{X_i}{R_o}\right)^2}\right\}\right]$$

9-29.

$$Z_L = Z_o \frac{1+\Gamma}{1-\Gamma}.$$

$$\Gamma = |\Gamma| \cdot e^{j\theta_i}$$

Here $|\Gamma| = \frac{S-1}{S+1}$, $\theta_i = \frac{4\pi}{\lambda} z_m' \pm \pi$.

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So $\Gamma = \frac{S-1}{S+1} \cdot e^{j\left(\frac{4\pi}{\lambda} z_m' \pm \pi\right)}$.

$$Z_L = Z_0 \cdot \frac{1+\Gamma}{1-\Gamma} = Z_0 \cdot \frac{S+1-(S-1)\cdot e^{j\frac{4\pi}{\lambda}\cdot z_m'}}{S+1+(S-1)\cdot e^{j\frac{4\pi}{\lambda} z_m'}}.$$

$$= Z_0 \cdot \frac{1 - j\, S\cdot \tan\left(\frac{2\pi z_m'}{\lambda}\right)}{S - j\cdot \tan\left(\frac{2\pi z_m'}{\lambda}\right)}$$