Homework #7

9-30 (a). $V_g = 0.1\angle 0^\circ$ (V), $Z_g = R_0 = 50(\Omega)$.

$R_L = 25(\Omega)$ $\quad \Gamma = \dfrac{R_L - Z_0}{R_L + Z_0} = -\dfrac{1}{3}$.

$V_i = \dfrac{Z_i}{Z_0 + Z_i} \cdot V_g$. $\quad I_i = \dfrac{V_g}{Z_0 + Z_i}$.

where $Z_i = Z_0 \dfrac{R_L + j\cdot Z_0 \tan(\beta L)}{Z_0 + j\cdot R_L \tan(\beta l)} = Z_0 \dfrac{1 + j\cdot 2\tan(\beta l)}{2 + j\cdot \tan(\beta l)}$

$\therefore \quad V_i = \dfrac{1 + j\cdot 2\tan(\beta l)}{3(1 + j\cdot \tan(\beta l))} V_g = \dfrac{1}{30}\left(\dfrac{1 + j\cdot 2\tan(\beta l)}{1 + j\cdot \tan(\beta l)}\right)$ (V).

$I_i = \dfrac{2 + j\cdot \tan(\beta l)}{3Z_0\cdot(1 + j\cdot \tan(\beta l))} V_g = \dfrac{2}{3}\left(\dfrac{2 + j\cdot \tan(\beta l)}{1 + j\cdot \tan(\beta l)}\right)$ (mA)

$V_L = V(z'=0) = \dfrac{V_g Z_0}{Z_0 + Z_g} \cdot e^{-j\beta l}(1+\Gamma) = \dfrac{1}{30} e^{-j\cdot\beta l}$ (V).

$I_L = \dfrac{V_g(Z_0)}{Z_0\cdot(Z_0 + Z_g)} \cdot e^{-j\beta l}(1-\Gamma) = \dfrac{4}{3} e^{-j\beta l}$ (mA)

(b). $S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = 2$.

(c). $P_{av} = \dfrac{1}{2}\mathrm{Re}(V_L \cdot I_L^*) = \dfrac{1}{2}\left(\dfrac{1}{30}\right)\cdot\left(\dfrac{4}{3}\times 10^{-3}\right) = 2.22\times 10^{-5}$ (W)

if $R_L = 50(\Omega)$. $\quad V_L = \dfrac{V_g}{2} e^{-j\beta L}$, $\quad I_L = \dfrac{V_g}{2Z_0} e^{-j\beta L}$.

$P_{av} = 2.50\times 10^{-5}$ (W)

(matched impedance)

9-31. $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$

$I(z) = \frac{1}{R_0}\left(V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}\right)$.

$V_0^+ = \frac{V_g}{2}$. $I_0^+ = \frac{V_g}{2R_0}$.

(a). $P_{inc} = \frac{1}{2} Re\left[V_0^+ I_0^{+*}\right] = \frac{V_g^2}{8R_0}$

(b). $P_L = \frac{1}{2} Re\left[V(z) \cdot I^*(z)\right]$

$= \frac{1}{2R_0} \cdot (V_0^+)^2 \cdot (1 - |\Gamma|^2) = \frac{V_g^2}{8R_0}(1 - |\Gamma|^2)$.

(c). $\frac{P_L}{P_{inc}} = 1 - |\Gamma|^2 = 1 - \left(\frac{S-1}{S+1}\right)^2 = \frac{4S}{(S+1)^2}$.

(d). $P_{inc} = \frac{100^2}{8 \times 50} = 25$ (W)

$\Gamma = \frac{Z_L - R_0}{Z_L + R_0} = 0.243 \angle -76°$.

$S = \frac{1+|\Gamma|}{1-|\Gamma|} = 1.64$. $P_L = 25(1 - 0.243^2) = 23.5$ (W)

since $P_L = \frac{1}{2}|I_L|^2 \cdot R_L$.

$|I_L| = 0.97$ (A) . $|V_L| = |I_L| \cdot |Z_L| = 54.2$ (V) .

9-42. $f = 200$ MHz $= 2 \times 10^8$ Hz . $\lambda = \frac{c}{f} = 1.5$ (m) .

(a). 1 (m) long open-circuited case,

$\frac{L}{\lambda} = \frac{2}{3}$. start from the extreme right

point P_{oc} (+1, 0), rotate clockwise

$2\beta L = 2 \cdot \frac{2\pi}{\lambda} \cdot \frac{2}{3}\lambda = \frac{8}{3}\pi$.

Read $x = -j \cdot 0.575$. $r = 0$

Thus $z_i = R_0 \, (r + j \cdot x) = 75 \cdot (-j \, 0.575) = -j\,43.1$ (Ω

$Y_i = j \cdot 0.0232 \ (S)$.

(b). $l = 0.8 = 0.533\lambda$.

start from the extreme left point $P_{sc}(-1, 0)$,

rotate clockwise $2 \cdot \frac{2\pi}{\lambda} \cdot l = 2 \cdot \frac{2\pi}{\lambda} \cdot 0.533\lambda$

read $x = 0.21$, $r = 0$

$z_i = R_0 \cdot (r + j \cdot x) = 75 \cdot (j \cdot 0.21) = j \cdot 15.8 \ (\Omega)$.

$Y_i = -j \cdot 0.063 \ (S)$.

9-45. $S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2$. $|\Gamma| = \frac{1}{3}$.

$\lambda = \frac{2\pi}{\beta} = 0.5 \ (m)$. $\beta = 4\pi$.

(a) $\Gamma = \frac{z_L - R_0}{z_L + R_0}$. where $R_0 = 50 \ (\Omega)$.

$\Gamma = |\Gamma| \cdot e^{j\theta_L}$ since $\theta_L - 2\beta \cdot 0.05 = -\pi$.

$\theta_L = -\pi + 2 \times 4\pi \times 0.05 = -\pi + 0.4\pi = -0.6\pi$.

$\Gamma = \frac{1}{3} \cdot e^{-j \cdot 0.6\pi}$

$z_L = R_0 \cdot \frac{1 + \Gamma}{1 - \Gamma} = 50 \times \frac{1 + \frac{1}{3} e^{-j0.6\pi}}{1 - \frac{1}{3} e^{-j0.6\pi}} = 33.74 - j \cdot 24.07$

(b). $\Gamma = \frac{1}{3} e^{-j0.6\pi}$.

(c). if $z_L = 0$, $\Gamma = -1$.

$z' = 0$, is a minimum point.

$z' = \frac{\lambda}{2} = 25$ (cm), is the next one.

9-48. (a). $z_L = \frac{\hat{z}_L}{R_0} = \frac{25 + j\cdot 25}{50} = 0.5 + j\cdot 0.5$.

$y_L = 1 - j$.

P_1: $z_L = 0.5 + j\cdot 0.5$.

P_2: $y_L = 1 - j\cdot 1$ $\Rightarrow d_2 = 0$

P_3: $y_3 = 1 + j\cdot 1$. $\Rightarrow d_3 = 0.324\lambda$.

P_2'': $b_2 = j\cdot 1$. $\Rightarrow l_2 = 0.375\lambda$.

P_3'': $b_3 = -j\cdot 1$ $\Rightarrow l_3 = 0.125\lambda$.

(b). for $z_0' = 75$,

$d_2' = 0$, $l_2' = 0.406\lambda$.

$d_3' = 0.324\lambda$. $l_3' = 0.0936\lambda$.