Homework 9

10-13. Eqs (10-134) ~ (10-137) for TM_{11} mode:

$$E_x^\circ(x,y) = \frac{-j\beta}{h^2}\cdot\left(\frac{\pi}{a}\right)\cdot E_0\cdot\cos\left(\frac{\pi x}{a}\right)\cdot\sin\left(\frac{\pi y}{b}\right).$$

$$E_y^\circ(x,y) = \frac{-j\cdot\beta}{h^2}\left(\frac{\pi}{b}\right)\cdot E_0\cdot\sin\left(\frac{\pi x}{a}\right)\cdot\cos\left(\frac{\pi y}{b}\right).$$

$$E_z^\circ(x,y) = E_0\cdot\sin\left(\frac{\pi x}{a}\right)\cdot\sin\left(\frac{\pi y}{b}\right).$$

$$H_x^\circ(x,y) = \frac{j\omega\epsilon}{h^2}\cdot\left(\frac{\pi}{b}\right)\cdot E_0\cdot\sin\left(\frac{\pi x}{a}\right)\cdot\cos\left(\frac{\pi y}{b}\right)$$

$$H_y^\circ(x,y) = \frac{-j\omega\epsilon}{h^2}\cdot\left(\frac{\pi}{a}\right)\cdot E_0\cdot\cos\left(\frac{\pi x}{a}\right)\cdot\sin\left(\frac{\pi y}{b}\right).$$

surface current density.

$$\vec{J}_s(y=0) = \hat{a}_n\times\vec{H}\big|_{y=0} = \hat{a}_y\times\left(\hat{a}_x H_x^\circ(x,0) + \hat{a}_y H_y^\circ(x,0)\right)$$

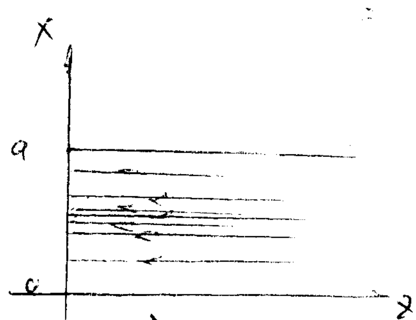
$$= -\hat{a}_z H_x^\circ(x,0) = -\hat{a}_z\frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{b}\right)E_0\sin\left(\frac{\pi x}{a}\right)\cdot e^{-j\beta z}$$

$$= \vec{J}_s(y=b).$$

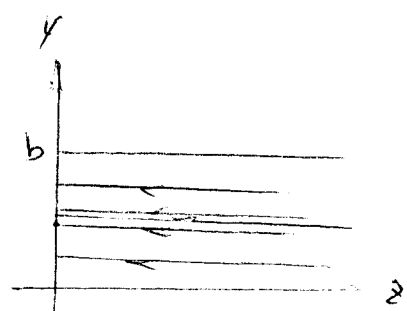
$$\vec{J}_s(x=0) = \hat{a}_n\times\vec{H}\big|_{x=0} = \hat{a}_x\times\left[\hat{a}_x H_x^\circ(0,y) + \hat{a}_y H_y^\circ(0,y)\right]$$

$$= \hat{a}_z H_y^\circ(0,y) = -\hat{a}_z\frac{j\omega\epsilon}{h^2}\cdot\left(\frac{\pi}{a}\right)\cdot E_0\sin\left(\frac{\pi y}{b}\right)e^{-j\beta z}$$

$$= \vec{J}_s(x=a).$$



$\vec{J}_s$ at $y=b$



$\vec{J}_s$ at $x=0$.

10-15 $u_{en} = u \cdot \sqrt{1-\left(\frac{f_c}{f}\right)^2}$

For the TE_{10} mode, $f_{10} = \frac{u}{2a}$.

$$u_{en} = u \cdot \sqrt{1-\left(\frac{u}{f\cdot 2a}\right)^2} = \frac{1}{\sqrt{\mu\varepsilon}} \cdot \sqrt{1-\frac{1}{\mu\varepsilon(2af)^2}}.$$

10-16. $f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$.

(a). $a=2b$. $f_c = \frac{1}{2\sqrt{\mu\varepsilon}} \cdot \frac{1}{a} \cdot \sqrt{m^2+4n^2}$.

TE_{10}	$\frac{1}{2a\sqrt{\mu\varepsilon}}$.
TE_{01}, TE_{20}	$\frac{1}{a\sqrt{\mu\varepsilon}}$
TE_{11}. TM_{11}	$\sqrt{5}/2a\sqrt{\mu\varepsilon}$.
TE_{02}	$2/a\sqrt{\mu\varepsilon}$.
TM_{12}.	$\sqrt{17}/2\sqrt{\mu\varepsilon}a$
TM_{22}.	$\sqrt{5}/a\sqrt{\mu\varepsilon}$.

(b). $a=b$. $f_c = \frac{1}{2\sqrt{\mu\varepsilon}} \cdot \frac{1}{a} \cdot \sqrt{m^2+n^2}$.

TE_{10}. TE_{01}.	$\frac{1}{2a\sqrt{\mu\varepsilon}}$.
TE_{11}. TM_{11}.	$\frac{\sqrt{2}}{2a\sqrt{\mu\varepsilon}}$.
TE_{02}. TE_{20}	$\frac{1}{a\sqrt{\mu\varepsilon}}$.
TM_{12}.	$\frac{\sqrt{5}}{2a\sqrt{\mu\varepsilon}}$.
TM_{22}.	$\frac{\sqrt{2}}{a\sqrt{\mu\varepsilon}}$.

10-17. $f = 3\times10^9$ (Hz). $\lambda = 0.1$ m.

let $a = k\cdot b$. $1<k<2$. $f_c = \frac{3\times10^8}{2a}\cdot\sqrt{m^2+k^2n^2}$

(a). $(f_c)_{10} = \frac{1.5\times10^8}{a}$ $f > 1.2\,(f_c)_{10}$; $a > 0.06$ (m)

$$(f_c)_{01} = \frac{1.5\times 10^8}{b}, \qquad f < 0.8 (f_c)_{01}$$

$b < 0.04$ (m).

so that,

$$0.06 < a < 0.08.$$

(b).
$$u_p = \frac{c}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}}.$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}}.$$

$$\beta = \frac{2\pi}{\lambda_g} \qquad (Z_{TE})_{10} = \frac{\eta_0}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}} = \frac{120}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}}.$$

10-33. From Eqs. (10-25), (10-28)

$|y| \le \frac{d}{2}$.
$$E_z(y,z;t) = E_e \cdot \cos(k_y y)\cos(wt-\beta z).$$
$$E_y(y,z;t) = -\frac{\beta}{k_y}\cdot E_e \cdot \sin(k_y y)\cdot \sin(wt-\beta z).$$
$$H_x(y,z;t) = \frac{w\varepsilon_d}{k_y}\cdot E_e \cdot \sin(k_y y)\cdot \sin(wt-\beta z).$$

$y > \frac{d}{2}$.
$$E_z(y,z;t) = E_e \cdot \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y-\frac{d}{2})}\cdot \cos(wt-\beta z),$$
$$E_y(y,z;t) = \frac{\beta}{\alpha} E_e \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y-\frac{d}{2})} \sin(wt-\beta z),$$
$$H_x(y,z;t) = -\frac{w\varepsilon_0}{\alpha}\cdot E_e \cos\left(\frac{k_y d}{2}\right)\cdot e^{-\alpha(y-\frac{d}{2})}\sin(wt-\beta z)$$

$y < -\frac{d}{2}$.
$$E_z(y,z;t) = E_e \cdot \cos\left(\frac{k_y d}{2}\right)\cdot e^{\alpha(y+\frac{d}{2})}\cdot \cos(wt-\beta z)$$
$$E_y(y,z;t) = -\frac{\beta}{\alpha}\cdot E_e \cos\left(\frac{k_y d}{2}\right)\cdot e^{\alpha(y+\frac{d}{2})} \sin(wt-\beta z)$$
$$H_x(y,z;t) = \frac{w\varepsilon_0}{\alpha}\cdot E_e \cos\left(\frac{k_y d}{2}\right)\cdot e^{\alpha(y+\frac{d}{2})} \sin(wt-\beta z)$$

10-34. (a). For TE mode, $f_c = 0$

The dominant mode is TE_1.

From Eq. (10-278):

$$\alpha = \frac{\mu_0}{\mu_d} \cdot k_y \tan\left(\frac{k_y d}{2}\right) \approx \frac{\mu_0}{\mu_d} k_y \frac{k_y d}{2}$$

$$= \frac{\mu_0 d}{2\mu_d} \cdot k_y^2 \qquad (k_y d << 1)$$

From Eq. (10-247):

$$\beta^2 - \omega^2 \mu_0 \varepsilon_0 = \alpha^2 \approx 0$$

$$\beta = \omega\sqrt{\mu_0 \varepsilon_0} = k_0$$

From Eq. (10-245). $k_y^2 = \omega^2 \varepsilon_d \mu_d - \beta^2 \approx k_d^2 - k_0^2$.

so that $\alpha \approx \frac{\mu_0 d}{2\mu_d} \cdot (k_d^2 - k_0^2)$.

(b). $d = 5 \times 10^{-3}$ (m). $\varepsilon_d = 3\varepsilon_0$ $\mu_d = \mu_0$

$f = 3 \times 10^8$ Hz. $k_0 = 2\pi$.

$$\alpha = \frac{d}{2} \cdot k_0^2 (\varepsilon_r - 1) = 0.197 \; (Np/m)$$

$$e^{-\alpha(y - \frac{d}{2})} = 0.368$$

so that,

$$y - \frac{d}{2} \approx 5.066 \; (m)$$

10-35. From Eqs. (10-50) - (10-51):

$$H_y^0 = -\frac{j\beta}{h^2}\cdot\frac{\partial H_z^0}{\partial y}, \quad E_x^0 = -\frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z^0}{\partial y}.$$

$|y| \le \frac{d}{2}$.

$$H_z(y,z;t) = H_e\cdot\cos(k_y\cdot y)\cdot\cos(\omega t-\beta z).$$

$$H_y(y,z;t) = -\frac{\beta}{k_y}\cdot H_e\sin(k_y\cdot y)\cdot\sin(\omega t-\beta z).$$

$$E_x(y,z;t) = -\frac{\omega\mu_d}{k_y}\cdot H_e\sin(k_y\cdot y)\cdot\sin(\omega t-\beta z).$$

$y \ge \frac{d}{2}$

$$H_z(y,z;t) = H_e\cdot\cos\left(\frac{k_y d}{2}\right)\cdot e^{-\alpha(y-\frac{d}{2})}\cos(\omega t-\beta z).$$

$$H_y(y,z;t) = \frac{\beta}{\alpha}\cdot H_e\cdot\cos\left(\frac{k_y d}{2}\right)\cdot e^{-\alpha(y-\frac{d}{2})}\cdot\sin(\omega t-\beta z).$$

$$E_x(y,z;t) = \frac{\omega\mu_0}{\alpha}\cdot H_e\cdot\cos\left(\frac{k_y d}{2}\right)\cdot e^{-\alpha(y-\frac{d}{2})}\cdot\sin(\omega t-\beta z).$$

$y \le -\frac{d}{2}$.

$$H_z(y,z;t) = H_e\cdot\cos\left(\frac{k_y d}{2}\right)\cdot e^{\alpha(y+\frac{d}{2})}\cdot\cos(\omega t-\beta z).$$

$$H_y(y,z;t) = -\frac{\beta}{\alpha}\cdot H_e\cdot\cos\left(\frac{k_y d}{2}\right)\cdot e^{\alpha(y+\frac{d}{2})}\cdot\sin(\omega t-\beta z).$$

$$E_x(y,z;t) = -\frac{\omega\mu_0}{\alpha}\cdot H_e\cos\left(\frac{k_y d}{2}\right)\cdot e^{\alpha(y+\frac{d}{2})}\cdot\sin(\omega t-\beta z)$$

Consider the boundary $y = \frac{d}{2}$.

$$E_x(y) = \frac{j\omega\mu_d}{k_y}\cdot H_e\sin(k_y y) = -\frac{j\omega\mu_0}{\alpha}H_e\cdot\cos\left(\frac{k_y d}{2}\right)\cdot e^{-\alpha y}$$

$y = \frac{d}{2}$.

$$\rightarrow \frac{\alpha}{k_y} = -\frac{\mu_0}{\mu_d}\cdot\cot\left(\frac{k_y d}{2}\right).$$

10-36

(a) Odd TM and Even TE are the propagating modes.

Using (2d) for (d) in the formulas in Table 10-4,

we have,

$$f_{co} = \frac{n-1}{2d\sqrt{\mu_d \varepsilon_d - \mu_0 \varepsilon_0}}$$ for odd TM modes.

$$f_{co} = \frac{n-\frac{1}{2}}{2d\sqrt{\mu_d \varepsilon_d - \mu_0 \varepsilon_0}}$$ for even TE modes.

(b). For odd TM modes From Eqs. (10-127).

$|y| \le \frac{d}{2}$. $E_y^0(y) = -\frac{j\beta}{k_y} \cdot E_0 \cos(k_y y)$.

$H_z^0(y) = \frac{j\omega\varepsilon_d}{k_y} \cdot E_0 \cdot \cos(k_y y)$.

surface current density on conductor

$$\vec{J}_s = \hat{a}_n \times \vec{H} \Big|_{y=0}$$

$$= -\hat{a}_z H_x^0(0) = -\hat{a}_z \frac{j\omega\varepsilon_d}{k_y} \cdot E_0 .$$

surface charge density on conductor

$$\rho_s = \hat{a}_n \cdot \vec{D} \Big|_{y=0}$$

$$= \varepsilon_d E^0(0) = -\frac{j\beta\varepsilon_d}{k_y} \cdot E_0 .$$

For even TE modes. From Eqs in Table 10-4.

$$H_y^0(y) = \frac{j\beta}{k_y} \cdot H_0 \cdot \sin(k_y y)$$

$$E_x^0(y) = \frac{j\omega\mu_d}{k_y} H_0 \sin(k_y y) .$$

$$H_z^0(y) = H_0 \cos(k_y y) .$$

$$\vec{J}_s = \hat{a}_y \times [\hat{a}_y H_y(0) + \hat{a}_z H_z(0)]$$

$$= \hat{a}_x H_e$$

$$\rho_s = \hat{a}_y \cdot \varepsilon_d \vec{E}(0) = 0$$