(10)

8-12.

(a). $\sigma = 4\ (S/m)$.

so seawater is a conducting media.

$$\frac{\sigma}{w\varepsilon} = \frac{4}{10^{10}\pi \times 80 \times \frac{1}{36\pi} \times 10^{-9}} = 0.18$$

so the seawater is neither a perfect conductor, nor an insulator,

Choose the equations shown in Problem 8-9.

the attenuation constant $\alpha = w\sqrt{\frac{\mu\varepsilon}{2}} \cdot \left(\sqrt{1+\left(\frac{\sigma}{w\varepsilon}\right)^2} - 1\right)^{\frac{1}{2}} = 84\ (Np/m)$

the phase constant $\beta = w\sqrt{\frac{\mu\varepsilon}{2}} \cdot \left(\sqrt{1+\left(\frac{\sigma}{w\varepsilon}\right)^2} + 1\right)^{\frac{1}{2}} = 300\pi\ (rad/m)$.

the intrinsic impedance $\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon_0}} \cdot \frac{1}{\sqrt{\varepsilon_r} \cdot \left[1+\left(\frac{\sigma}{w\varepsilon}\right)^2\right]^{\frac{1}{4}}} \cdot e^{\frac{j}{2}\cdot arctg\left(\frac{\sigma}{w\varepsilon}\right)}$

$= 41.8\, e^{j\cdot 0.0283}\ (\Omega)$

the ~~grou~~ phase velocity

$$V_p = \frac{w}{\beta} = \frac{10^{10}\pi}{300\pi} = 3.3\times 10^{7}\ (m/s).$$

the wavelength

$$\lambda = \frac{2\pi}{\beta} = 0.67\ (cm)$$

the skin depth

$$\delta = \frac{1}{\alpha} = \frac{1}{84} = 1.19\times 10^{-2}\ (m).$$

(11) (b).

$$0.1 \cdot e^{-\alpha y} = 0.01 .$$

$$y = \frac{1}{\alpha} \cdot \ln 10 = 2.74 \text{ (cm)} .$$

(c). frome Maxwell's equation,

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial}{\partial t} \vec{D} = j\omega \cdot \varepsilon_c \vec{E} .$$

$$-j \vec{k} \times \vec{H} = j\omega\varepsilon_c \vec{E} .$$

$$\vec{E} = -\frac{k}{\omega\varepsilon_c} \hat{a}_y \times \hat{a}_x \Big( \underbrace{0.1 \cdot e^{j(\cancel{\ } -ky - \frac{\pi}{3} - \frac{\pi}{2})}}_{\text{phaser}} \Big) .$$

Here, $k = \omega\sqrt{\mu\varepsilon_c}$ .

$$\vec{E} = \hat{a}_z \cdot \sqrt{\frac{\mu}{\varepsilon_c}} \cdot 0.1 \cdot e^{j(-ky - \frac{\pi}{3} - \frac{\pi}{2})}$$

$$= \hat{a}_z \eta_c \cdot 0.1 \cdot e^{-j(k \cdot y + \frac{\pi}{3} + \frac{\pi}{2})}$$

$$= \hat{a}_z \cdot \eta_c \cdot 0.1 \cdot e^{-y \cdot \alpha} \cdot e^{-j(\beta y + \frac{\pi}{3} + \frac{\pi}{2})} .$$

We have: $\eta_c = 41.8\, e^{j\,0.0283\pi}$ .

$\alpha = 84$ .

$\beta = 300\pi$

$$\vec{E}(y,t) = Re\left( \vec{E} \cdot e^{j\omega t} \right)$$

so

$$\vec{E}(y=0.5, t) = \hat{a}_z\, 2.41 \times 10^{-18} \times \sin\left(10^{10}\pi t - \frac{\pi}{3} + 0.0283\pi\right)$$

8-15.



(a). $u_p = \frac{\omega}{\beta}$. $\omega = \beta \cdot u_p$.

so $u_g = \frac{d\omega}{d\beta} = \frac{d}{d\beta}(\beta \cdot u_p) = u_p + \beta \frac{du_p}{d\beta}$.

Note : $u_p$ is a function of $\omega$.

(b). $\lambda = \frac{2\pi}{\beta}$. $\frac{d\lambda}{d\beta} = -\frac{2\pi}{\beta^2} = -\frac{\lambda}{\beta}$.

Now,

$$u_g = \frac{d\omega}{d\beta} = u_p + \beta \cdot \frac{du_p}{d\beta}$$

$$= u_p + \beta \cdot \left(\frac{du_p}{d\lambda} \cdot \frac{d\lambda}{d\beta}\right)$$

$$= u_p + \beta \cdot \left(-\frac{\lambda}{\beta} \cdot \frac{du_p}{d\lambda}\right).$$

$$= u_p - \lambda \cdot \frac{du_p}{d\lambda}.$$

8-17. Assume

$$\vec{E} = \hat{a}_x E_0 \cos(\omega t - kz + \phi) + \hat{a}_y E_0 \sin(\omega t - kz + \phi).$$

$$\vec{H} = \hat{a}_y \frac{E_0}{\eta} \cos(\omega t - kz + \phi) - \hat{a}_x \frac{E_0}{\eta} \sin(\omega t - kz + \phi).$$

then

$$\vec{P} = \vec{E} \times \vec{H} = \hat{a}_z \frac{E_0^2}{\eta} \cdot [\cos^2(\omega t - kz + \phi) + \sin^2(\omega t - kz + \phi)]$$

$$= \hat{a}_z \cdot \frac{E_0^2}{\eta}.$$

So $\vec{P}$ is a constant, independent of time and distance.

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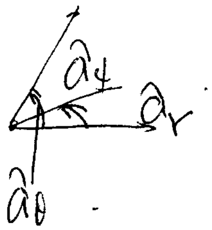
8-18.

$$\vec{E} = \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi .$$

$$\vec{H} = \frac{1}{\eta} \hat{a}_r \times \vec{E} = \frac{1}{\eta} \left( -\hat{a}_\phi E_\theta + \hat{a}_\theta E_\phi \right).$$




$$\vec{P}_{av} = \frac{1}{2} \cdot Re \left( \vec{E} \times \vec{H}^* \right)$$

$$= \frac{1}{2\eta} \cdot Re \left[ \hat{a}_r E_\theta \cdot E_\theta^* + \hat{a}_r E_\phi \cdot E_\phi^* \right]$$

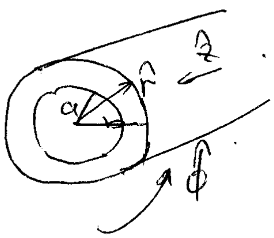
$$= \frac{1}{2\eta} \cdot \hat{a}_r \left( |E_\theta|^2 + |E_\phi|^2 \right).$$

Homework 3.



8-19. recall page. 88~89.

$\vec{E} = \hat{a}_r \cdot \frac{\rho}{2\pi\varepsilon r}$. $\rho$ is the line charge density on the inner conductor.




$V_0 = \int_a^b \vec{E} \cdot d\vec{r} = \frac{\rho}{2\pi\varepsilon} \ln\left(\frac{b}{a}\right)$

so $\frac{\rho}{2\pi\varepsilon} = \frac{V_0}{\ln\left(\frac{b}{a}\right)}$. $\vec{E} = \hat{a}_r \cdot \frac{\rho}{2\pi\varepsilon} \cdot \frac{1}{r} = \hat{a}_r \cdot \frac{V_0}{r \cdot \ln\left(\frac{b}{a}\right)}$.

From Ampère's law, $\vec{H} = \hat{a}_\phi \cdot \frac{I}{2\pi r}$.

Poynting vector, $\vec{P} = \vec{E} \times \vec{H} = \hat{a}_z \cdot \frac{V_0 I}{2\pi r^2 \cdot \ln\left(\frac{b}{a}\right)}$.

The power transmitted over cross-sectional area is:

$P = \int_s \vec{P} \cdot d\vec{s} = \int_0^{2\pi} \int_a^b \frac{V_0 I}{2\pi r^2 \cdot \ln\left(\frac{b}{a}\right)} r \cdot dr \cdot d\phi = V_0 I$.

8-21 $\vec{E}_i = \vec{E} = E_0(\hat{a}_x - j\hat{a}_y) e^{-j\beta z}$.

(a). $\vec{E}_r = -E_0(\hat{a}_x - j\hat{a}_y) \cdot e^{j\beta z}$

$= E_0(-\hat{a}_x + j \cdot \hat{a}_y) \cdot e^{j\beta z}$.

The direction of propagation is $(-\hat{a}_z)$.
So the reflected wave is left-hand circular.

(b). recall page 263.

$(-\hat{a}_z) \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$. since $\vec{H}_2$ is in perfect conductor, $\vec{H}_2 = 0$.

$\vec{H}_1 = \vec{H}_i + \vec{H}_r$.

$\vec{H}_i = \frac{E_0}{\eta_1} \cdot (j\hat{a}_x + \hat{a}_y) \cdot e^{-j\beta z}$.

$\vec{H}_r = \frac{E_0}{\eta_1} (j\hat{a}_x + \hat{a}_y) \cdot e^{j\beta z}$.

$\vec{H}_1 = \frac{2E_0}{\eta_1} (j\hat{a}_x + \hat{a}_y) \cdot \cos(\beta z)$.

$\vec{J}_s = -\hat{a}_z \times \vec{H}_1(0) = \frac{2E_0}{\eta_1} (\hat{a}_x - j\hat{a}_y)$.

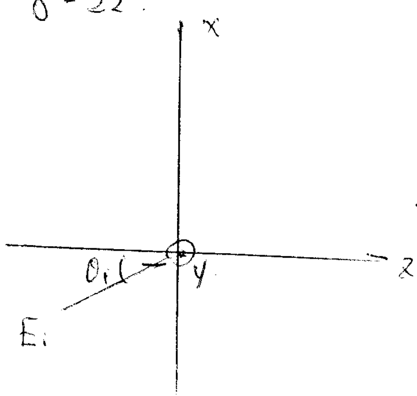
(c). $\vec{E}_1(z,t) = Re\left[(\vec{E}_i + \vec{E}_r) e^{j\omega t}\right]$

$= Re\left[ E_0 (\hat{a}_x - j\hat{a}_y) \cdot e^{j(\omega t - \beta z)} + E_0(-\hat{a}_x + j\hat{a}_y) \cdot e^{j(\omega t + \beta z)}\right.$

$= Re\left[ E_0 (-2j(\hat{a}_x - j\hat{a}_y) \cdot \sin(\beta z)) \cdot e^{j\omega t}\right]$

$= 2E_0 \sin(\beta z) \cdot (\sin(\omega t) \cdot \hat{a}_x - \cos(\omega t) \cdot \hat{a}_y)$

8-22.




(a) $\hat{k}_i = 6\hat{a}_x + 8\hat{a}_z$.

$k_i = (6^2 + 8^2)^{1/2} = 10 = 2\pi f \sqrt{\mu_0 \varepsilon_0} = \frac{2\pi f}{c}$

frequency: $f = \frac{k_i \cdot c}{2\pi} = \frac{3 \times 10^9}{2\pi} = 4.77 \times 10^8$ (m/s)

wavelength: $\lambda = \frac{2\pi}{k} = 0.628$ (m).

(b). $\vec{E}_i = Re\left(\hat{a}_y 10 e^{-j(6x+8z)} \cdot e^{j\omega t}\right)$.

$= \hat{a}_y \cdot 10 \cdot \cos(\omega t - 6x - 8z)$.

$= \hat{a}_y \cdot 10 \cdot \cos(3 \times 10^9 t - 6x - 8z)$