

①

8-29. From Eqs. (8-156) through (8-161) on PP. 403,

$$\vec{E}_1 = \hat{a}_x (E_{i0} e^{-j\beta_0 z} + E_{r0} e^{j\beta_0 z})$$

$$\vec{H}_1 = \hat{a}_y \frac{1}{\eta_0} (E_{i0} e^{-j\beta_0 z} - E_{r0} e^{j\beta_0 z})$$

$$\vec{E}_2 = \hat{a}_x (E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z})$$

$$\vec{H}_2 = \hat{a}_y \frac{1}{\eta_2} (E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z})$$

$$\vec{E}_t = \hat{a}_x E_{t0} e^{-j\beta_0 z}, \quad \vec{H}_t = \hat{a}_y E_{t0} \frac{1}{\eta_0} e^{-j\beta_0 z}$$

Boundary conditions:

$$\text{at } z=0: \quad \vec{E}_1(0) = \vec{E}_2(0), \quad \vec{H}_1(0) = \vec{H}_2(0)$$

$$\text{at } z=d: \quad \vec{E}_2(d) = \vec{E}_t(d), \quad \vec{H}_2(d) = \vec{H}_t(d)$$

$$(a) \quad E_{r0} = - \frac{j(\eta_0^2 - \eta_2^2) \tan(\beta_2 d)}{\eta_0 \eta_2 + j(\eta_0^2 + \eta_2^2) \tan(\beta_2 d)} E_{i0}$$

$$E_2^+ = \frac{\eta_2 (\eta_0 + \eta_2) e^{j\beta_2 d}}{\eta_0 \eta_2 \cos(\beta_2 d) + j(\eta_0^2 + \eta_2^2) \sin(\beta_2 d)} E_{i0}$$

$$E_2^- = \frac{\eta_2 (\eta_0 - \eta_2) e^{-j\beta_2 d}}{\eta_0 \eta_2 \cos(\beta_2 d) + j(\eta_0^2 + \eta_2^2) \sin(\beta_2 d)} E_{i0}$$

$$E_{t0} = \frac{2\eta_0 \eta_2 e^{j\beta_2 d}}{\eta_0 \eta_2 \cos(\beta_2 d) + j(\eta_0^2 + \eta_2^2) \sin(\beta_2 d)} E_{i0}$$

$$\text{where } \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}, \quad \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}, \quad \beta_0 = \frac{\omega}{c}, \quad \beta_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$(b) \quad \text{if } d = \lambda_2/4, \quad \beta_2 d = \frac{2\pi}{\lambda_2} \cdot \frac{\lambda_2}{4} = \frac{\pi}{2}$$

$$E_{r0} = - \frac{\eta_0^2 - \eta_2^2}{\eta_0^2 + \eta_2^2} E_{i0}$$

(2)

$$\Gamma = - \frac{\eta_0^2 - \eta_2^2}{\eta_0^2 + \eta_2^2}$$

$$\eta_2 = \eta_0 : \Gamma = 0$$

$$\eta_2 \neq \eta_0 : \Gamma \neq 0$$

$$\text{if } d = n\lambda_2/2, \quad n = 1, 2, \dots$$

$$\tan \beta_2 d = \tan(\pi \cdot n) = 0, \quad \Gamma = 0$$

8-30. Recall Example 8-12,

$\eta_2 = \sqrt{\eta_1 \eta_3}$ because material ① is air, and ③ is glass, $\eta_1 \neq \eta_3$.

$$\epsilon_{r2} = \sqrt{\epsilon_{r1} \epsilon_{r3}} = \sqrt{1 \cdot 4} = 2$$

(a). The required dielectric constant $\epsilon_r = 2$.

The wavelength in dielectric coating is

$$\lambda_2 = \frac{0.75}{\sqrt{\epsilon_{r2}}} = 0.53 \text{ } (\mu\text{m})$$

The thickness of the coating is

$$d = \frac{\lambda_2}{4} = 0.133 \text{ } (\mu\text{m})$$

(b). For violet light: $\lambda'_2 = \frac{0.42}{\sqrt{2}} = 0.297 \text{ } (\mu\text{m})$.

$$\frac{d}{\lambda'_2} = 0.447, \quad \beta_2 d = \frac{2\pi}{\lambda'_2} d = 2\pi \times 0.447 = 0.894\pi$$

From Eq. (8-173), and $\eta_1 = \eta_0$

$$Z_2(0) = \eta_2 \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)} = \frac{\eta_0}{\sqrt{2}} \cdot \frac{\frac{1}{2} + j\frac{1}{\sqrt{2}} \tan(\beta_2 d)}{\frac{1}{2} + j\frac{1}{2} \tan(\beta_2 d)}$$

$$\Gamma = \frac{Z_2(0) - \eta_0}{Z_2(0) + \eta_0} = 0.317 e^{j198^\circ}$$

$$|\Gamma|^2 = (0.317)^2 = 10\% \text{ Percentage of power reflected}$$

(3)

8-31

Γ_{12} , Γ_{23} denote the reflection coefficients between media 1 and 2 and between media 2 and 3 respectively. They are intrinsic parameters, depending on η_1 , η_2 , and η_3 .

$$\Gamma_{12} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \rightarrow \quad \frac{\eta_1}{\eta_2} = \frac{1 - \Gamma_{12}}{1 + \Gamma_{12}}$$

$$\Gamma_{23} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} \quad \rightarrow \quad \frac{\eta_2}{\eta_3} = \frac{1 - \Gamma_{23}}{1 + \Gamma_{23}}$$

Recall pp. 405.

The effective reflection coefficient Γ_o is:

$$\Gamma_o = \frac{Z_2(0) - \eta_1}{Z_2(0) + \eta_1}$$

$$Z_2(0) = \eta_2 \cdot \frac{\eta_3 + j \eta_2 \tan(\beta_2 d)}{\eta_2 + j \eta_3 \tan(\beta_2 d)}$$

so

$$\Gamma_o = \frac{1 + j \frac{\eta_2}{\eta_3} \tan(\beta_2 d) - \frac{\eta_1}{\eta_2} \left(\frac{\eta_2}{\eta_3} + j \tan(\beta_2 d) \right)}{1 + j \frac{\eta_2}{\eta_3} \tan(\beta_2 d) + \frac{\eta_1}{\eta_2} \left(\frac{\eta_2}{\eta_3} + j \tan(\beta_2 d) \right)}$$

$$= \frac{(\Gamma_{12} + \Gamma_{23}) + j(\Gamma_{12} - \Gamma_{23}) \cdot \tan(\beta_2 d)}{(1 + \Gamma_{12} \Gamma_{23}) + j(1 - \Gamma_{12} \Gamma_{23}) \cdot \tan(\beta_2 d)}$$

8-36. (a). From Snell's law.

$$\frac{\sin \theta_i}{\sin \theta_t} = n. \quad \text{so} \quad \theta_t = \sin^{-1} \left(\frac{1}{n} \sin \theta_i \right).$$

(4)

(b).

$$l_1 = d \cdot \tan(\theta_t) = d \cdot \frac{\sin \theta_t}{\cos \theta_t} = \frac{d \cdot \sin \theta_t}{\sqrt{1 - \sin^2 \theta_t}}$$

$$\sin \theta_t = \frac{1}{n} \cdot \sin \theta_i \quad \cos \theta_t = \sqrt{1 - \left(\frac{1}{n} \sin \theta_i\right)^2}$$

$$(c) \quad l_2 = \frac{d}{\cos \theta_t} \cdot \sin(\theta_i - \theta_t)$$

$$= \frac{d}{\cos(\theta_t)} \cdot (\sin \theta_i \cdot \cos \theta_t - \sin \theta_t \cdot \cos \theta_i)$$

8-37.

$$(a), \quad \sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\sin \theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \cdot \sin \theta_i > \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_c = 1$$

$$\cos \theta_t = \sqrt{1 - \sin^2 \theta_t} = \pm j \cdot \sqrt{\frac{\epsilon_1}{\epsilon_2} \sin^2 \theta_i - 1}$$

(From (8-200),

$$\vec{E}_t = \hat{a}_y \cdot E_{t0} \cdot e^{-j\beta_2(x \sin \theta_t + z \cos \theta_t)}$$

$$\vec{H}_t = \frac{E_{t0}}{\eta_2} (-\hat{a}_x \cos \theta_t + \hat{a}_z \sin \theta_t) e^{-j\beta_2(x \sin \theta_t + z \cos \theta_t)}$$

From (8-207).

$$E_{t0} = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} \cdot E_{i0}$$

Note: $\beta_2 \cdot \sin \theta_t$ is a real number. $\beta_2 \cos \theta_t$ is a ~~complex~~ imaginary number.

$-j\beta_2 \cos \theta_t = \pm \beta_2 \left(\frac{\epsilon_1}{\epsilon_2} \sin^2 \theta_i - 1 \right)^{\frac{1}{2}}$, since the energy shall decrease, we have:

$$\vec{E}_t = \hat{a}_y E_{t0} e^{-j\beta_2 x \sin\theta_t} \cdot e^{-\beta_2 \left(\frac{\epsilon_1}{\epsilon_2} \sin^2\theta_i - 1\right)^{\frac{1}{2}}} \quad (5)$$

$$\vec{H}_t = \frac{E_{t0}}{\eta_2} \left(+\hat{a}_x j \left(\frac{\epsilon_1}{\epsilon_2} \sin^2\theta_i - 1\right)^{\frac{1}{2}} + \hat{a}_z \sin\theta_t \right) \cdot e^{-j\beta_2 x \sin\theta_t} \cdot e^{-\beta_2 \left(\frac{\epsilon_1}{\epsilon_2} \sin^2\theta_i - 1\right)^{\frac{1}{2}}}$$

(b).

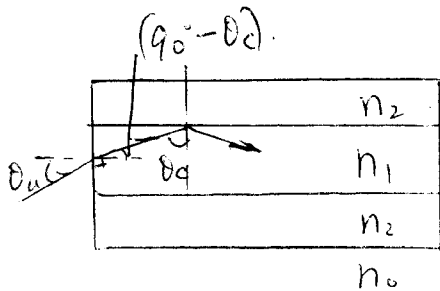
$$\vec{P}_{av} = \frac{1}{2} \text{Re}(\vec{E}_t \times \vec{H}_t^*)$$

We need to consider $\vec{P}_{av}$ in z direction.

$$\begin{aligned} (\vec{P}_{av})_z &= \frac{1}{2} \text{Re}(E_{ty} \cdot H_{tx}^*) \\ &= \frac{1}{2} \text{Re} \left(j \cdot \frac{E_{t0}^2}{\eta_2} \left(\frac{\epsilon_1}{\epsilon_2} \sin^2\theta_i - 1\right)^{\frac{1}{2}} \cdot e^{-2\beta_2 \left(\frac{\epsilon_1}{\epsilon_2} \sin^2\theta_i - 1\right)^{\frac{1}{2}}} \right) \\ &= 0 \end{aligned}$$

8-41.

(a). From Snell's law,



$$\frac{\sin\theta_a}{\sin(90^\circ - \theta_c)} = \frac{n_1}{n_0}$$

$$\frac{\sin\theta_c}{1} = \frac{n_2}{n_1}$$

$$\begin{aligned} \text{So } \sin(\theta_a) &= \frac{n_1}{n_0} \cdot \cos(\theta_c) = \frac{n_1}{n_0} \cdot (1 - \sin^2\theta_c)^{\frac{1}{2}} \\ &= \frac{n_1}{n_0} \cdot \left(1 - \left(\frac{n_2}{n_1}\right)^2\right)^{\frac{1}{2}} \\ &= \frac{1}{n_0} (n_1^2 - n_2^2)^{\frac{1}{2}} \end{aligned}$$

$$\theta_a = \sin^{-1} \left(\frac{1}{n_0} (n_1^2 - n_2^2)^{\frac{1}{2}} \right)$$

(b). $\sin\theta_a = 0.986$

$$\theta_a = 1.4 \text{ radian} = 80^\circ$$

Note, we can assume $n_0 = 1$.

(6)

8-42.

From Snell's law.

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}} = \frac{n_2}{n_1} \quad (n_2 = n_1).$$

(a). From Eqs (8-206), (8-207).

$$\Gamma_{\perp} = \frac{\left(\frac{n_2}{n_1}\right) \cos \theta_i - \cos \theta_t}{\left(\frac{n_2}{n_1}\right) \cos \theta_i + \cos \theta_t}$$

$$= \frac{\sin \theta_t \cdot \cos \theta_i - \cos \theta_t \cdot \sin \theta_i}{\sin \theta_t \cdot \cos \theta_i + \cos \theta_t \cdot \sin \theta_i} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}$$

$$\tau_{\perp} = \frac{2 \left(\frac{n_2}{n_1}\right) \cdot \cos \theta_i}{\cos \theta_t + \left(\frac{n_2}{n_1}\right) \cdot \cos \theta_i}$$

$$= \frac{2 \sin \theta_t \cdot \cos \theta_i}{\sin \theta_i \cos \theta_t + \sin \theta_t \cdot \cos \theta_i} = \frac{2 \sin \theta_t \cdot \cos \theta_i}{\sin(\theta_i + \theta_t)}$$

(b). From Eqs (8-221), (8-222)

$$\Gamma_{\parallel} = \frac{\left(\frac{n_2}{n_1}\right) \cdot \cos \theta_t - \cos \theta_i}{\left(\frac{n_2}{n_1}\right) \cdot \cos \theta_t + \cos \theta_i}$$

$$= \frac{\frac{\sin \theta_t}{\sin \theta_i} \cdot \cos \theta_t - \cos \theta_i}{\frac{\sin \theta_t}{\sin \theta_i} \cos \theta_t + \cos \theta_i} = \frac{\sin(2\theta_t) - \sin(2\theta_i)}{\sin(2\theta_t) + \sin(2\theta_i)}$$

$$\tau_{\parallel} = \frac{2 \left(\frac{n_2}{n_1}\right) \cdot \cos \theta_i}{\frac{n_2}{n_1} \cdot \cos \theta_t + \cos \theta_i} = \frac{2 \sin \theta_t \cdot \cos \theta_i}{\sin \theta_t \cdot \cos \theta_t + \sin \theta_i \cdot \cos \theta_i}$$

$$= \frac{4 \sin \theta_t \cdot \cos \theta_i}{\sin(2\theta_t) + \sin(2\theta_i)}$$

(7)

8-45

(a). For perpendicular polarization,

$$T_{\perp} = \frac{\sqrt{\epsilon_{r1}} \cdot \cos \theta_i - \sqrt{\epsilon_{r2}} \cdot \cos \theta_t}{\sqrt{\epsilon_{r1}} \cdot \cos \theta_i + \sqrt{\epsilon_{r2}} \cdot \cos \theta_t}$$

$$T_{\perp} = \frac{2 \sqrt{\epsilon_{r1}} \cdot \cos \theta_i}{\sqrt{\epsilon_{r2}} \cdot \cos \theta_t + \sqrt{\epsilon_{r1}} \cdot \cos \theta_i}$$

Here $\sin \theta_t = \frac{\sqrt{\epsilon_{r1}}}{\sqrt{\epsilon_{r2}}} \sin \theta_i$, $\cos \theta_t = \sqrt{1 - \left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right) \sin^2 \theta_i}$.

(b). For parallel polarization,

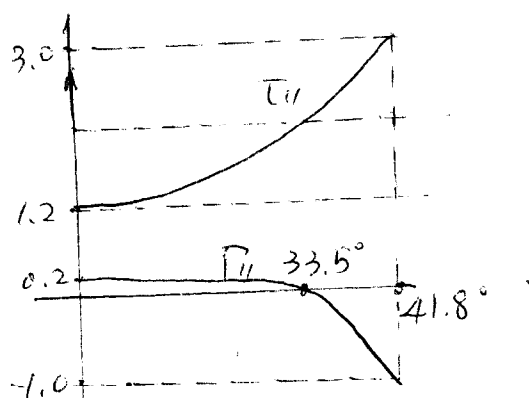
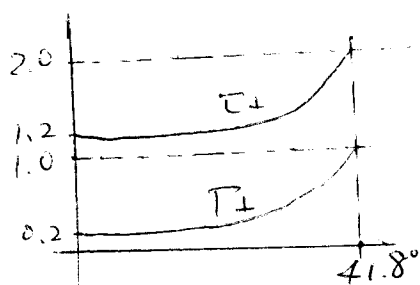
$$T_{\parallel} = \frac{\sqrt{\epsilon_{r1}} \cdot \cos \theta_t - \sqrt{\epsilon_{r2}} \cdot \cos \theta_i}{\sqrt{\epsilon_{r1}} \cdot \cos \theta_t + \sqrt{\epsilon_{r2}} \cdot \cos \theta_i}$$

$$T_{\parallel} = \frac{2 \sqrt{\epsilon_{r1}} \cos \theta_i}{\sqrt{\epsilon_{r1}} \cdot \cos \theta_t + \sqrt{\epsilon_{r2}} \cdot \cos \theta_i}$$

$$\theta_c = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8^\circ$$

(c). $\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}} = 1.5$

$$\sin \theta_c = \frac{1}{1.5}$$



8-47. (a). $T_{\parallel}' = \frac{E_{ro} \cdot \cos \theta_i}{E_{io} \cdot \cos \theta_i} = \frac{E_{ro}}{E_{io}} = T_{\parallel} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}$

$$T_{\parallel}' = \frac{E_{ro} \cdot \cos \theta_t}{E_{io} \cdot \cos \theta_i} = T_{\parallel} \cdot \left(\frac{\cos \theta_t}{\cos \theta_i}\right) = \frac{2 \eta_2 \cos \theta_t}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}$$

(b). $T_{\parallel}' + 1 = T_{\parallel}$

$$T_{\parallel} = 1 - T_{\parallel}' \left(\frac{\cos \theta_t}{\cos \theta_i}\right)$$