Homework 5.

9-21 $Z_{io} = Z_0 \cdot \coth(\gamma l)$

$$= Z_0 \cdot \frac{\cosh[(\alpha + j\beta)l]}{\sinh[(\alpha+j\beta)l]}$$

$$= Z_0 \cdot \frac{\cosh(\alpha l)\cdot \cos(\beta l) + j\sinh(\alpha l)\cdot \sin(\beta l)}{\sinh(\alpha l)\cdot \cos(\beta l) + j\cosh(\alpha l)\cdot \sin(\beta l)}.$$

For a low-loss line, $\alpha l \ll 1$

$\cosh(\alpha l) \cong 1$. $\sinh(\alpha l) \cong \alpha l$.

and $l = \frac{n\lambda}{2}$. $\sin(\beta l) = \sin\left(\frac{2\pi}{\lambda}\cdot\frac{n\lambda}{2}\right) = 0$.

$$Z_{io} \cong Z_0 \cdot \frac{\cos(\beta l) + j\alpha l \cdot \sin(\beta l)}{\alpha l \cdot \cos(\beta l) + j\cdot \cosh(\alpha l)\cdot \sin(\beta l)}.$$

$$(Z_{io})_{max} \cong Z_0 \cdot \frac{1}{\alpha l}.$$

When the frequency is slightly changed,

$f' = f + \Delta f$, $|\Delta f| \ll f$.

$$\beta l = \frac{2\pi}{\lambda'}\cdot\frac{n\lambda}{2} = \frac{2\pi f'}{c}\cdot\frac{nc}{2f} = n\pi\cdot\left(1+\frac{\Delta f}{f}\right).$$

$\cos(\beta l) \cong \cos(n\pi) = (-1)^n$.

$\sin(\beta l) \cong (-1)^n \cdot n\pi \cdot \frac{\Delta f}{f}$.

$$Z_{io} \cong Z_0 \cdot \frac{(-1)^n + j\cdot \alpha l \cdot (-1)^n \cdot n\pi \cdot \frac{\Delta f}{f}}{\alpha l \cdot (-1)^n + j\cdot(-1)^n \cdot n\pi\cdot\frac{\Delta f}{f}}$$

$$\cong Z_0 \frac{1}{\alpha l + j\cdot n\pi \cdot \frac{\Delta f}{f}}$$

Thus,
$$\frac{|Z_{io}|^2}{|(Z_{io})_{max}|^2} = \frac{1}{1 + \left(n\pi\cdot\frac{\Delta f}{\alpha l \cdot f}\right)^2}$$

Half-power needs:

$$n\pi \cdot \frac{\Delta f}{f \cdot \alpha l} = \pm 1 \Rightarrow \Delta f = \pm \frac{f \alpha l}{n\pi}.$$

For low-loss transmission line,

$$\alpha = \frac{1}{2}\left(R \cdot \sqrt{\frac{C}{L}} + G \cdot \sqrt{\frac{L}{C}}\right).$$

and $l = \frac{n\lambda}{2}$

$$f l = \frac{n}{2} \cdot (f\lambda) = \cancel{\frac{n}{2} c} = \frac{n}{2} \cdot \frac{1}{\sqrt{LC}}.$$

$$\Delta f = \pm \frac{\alpha \cdot f l}{n\pi} = \pm \frac{\alpha}{n\pi} \cdot \frac{n}{2} \cdot \frac{1}{\sqrt{LC}}$$

$$= \pm \frac{1}{2\pi} \cdot \frac{1}{2} \cdot \left(\frac{R}{L} + \frac{G}{C}\right).$$

Therefore, the half-power bandwidth is

$$2 \cdot |\Delta f| = \frac{1}{2\pi}\left(\frac{R}{L} + \frac{G}{C}\right).$$

$$\cancel{Q = \frac{|\Delta f|}{f} = \frac{\alpha l}{n\pi} = \frac{\alpha \cdot n\lambda}{2n\pi} = \frac{\alpha\lambda}{2\pi}}$$

$$Q = \frac{f}{2 \cdot |\Delta f|} = \frac{f \cdot 2\pi}{\frac{R}{L} + \frac{G}{C}} = \frac{\omega}{\frac{R}{L} + \frac{G}{C}}.$$

9-24. (a).
$$|\Gamma|^2 = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|^2 = \left|\frac{40 + j \cdot 30 - Z_0}{40 + j \cdot 30 + Z_0}\right|^2.$$

$$= \frac{(40 - Z_0)^2 + 30^2}{(40 + Z_0)^2 + 30^2}.$$

$$\frac{\partial |\Gamma|^2}{\partial Z_0} = 0 \Rightarrow Z_0 = 50\ (\Omega)$$

(b). $|\Gamma| = \frac{1}{3}$.

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = 2.$$

(c). The voltage reflection coefficient at the load is $\Gamma = \frac{1}{3} \cdot e^{j\frac{\pi}{2}} = \frac{1}{3} \cdot e^{j\theta_\Gamma}$

The location of the voltage minimum is

$$\theta_\Gamma - 2\beta z' = (2n+1)\pi.$$

$$z' = \frac{1}{2\beta}\left(\theta_\Gamma - (2n+1)\pi\right)$$

$$= \frac{\lambda}{4\pi}\left(\frac{\pi}{2} + \pi\right) = \frac{3}{8}\lambda. \quad \text{from the load.}$$

9-28.

$$R_i + j \cdot X_i = R_0 \cdot \frac{R_m + j \cdot R_0 \cdot \tan(\beta l_m)}{R_0 + j \cdot R_m \cdot \tan(\beta l_m)}.$$

Let $r_i = \frac{R_i}{R_0}$, $x_i = \frac{X_i}{R_0}$. $r_m = \frac{R_m}{R_0}$.

$$r_i + j \cdot x_i = \frac{r_m + j \cdot \tan(\beta l_m)}{1 + j \cdot r_m \tan(\beta l_m)}.$$

Hence,

$$r_m = \frac{r_i}{1 + x_i \cdot \tan(\beta l)}, \quad \tan(\beta l_m) = \frac{x_i}{1 - r_m r_i}.$$

$$r_m = \frac{1}{2r_i} \cdot \left[(1 + r_i^2 + x_i^2) \pm \sqrt{(1 + r_i^2 + x_i^2)^2 - 4r_i^2} \right]$$

$$\tan(\beta l_m) = \frac{1}{2x_i}\left[-\left(1 - (r_i^2 + x_i^2)\right) \pm \sqrt{[1 - (r_i^2 + x_i^2)]^2 + 4x_i^2} \right]$$

9-29. $Z_L = Z_0 \cdot \dfrac{1+\Gamma}{1-\Gamma}$. ④

$\Gamma = |\Gamma| \cdot e^{j\theta_\Gamma}$. $|\Gamma| = \dfrac{S-1}{S+1}$.

$\theta_\Gamma = 2 \cdot \beta \cdot \dfrac{z_m'}{\lambda} \pm \pi$. $= \dfrac{4\pi}{\lambda} \cdot z_m' \pm \pi$.

$$Z_L = Z_0 \cdot \frac{S+1 - (S-1) \cdot e^{j\left(\frac{4\pi}{\lambda} z_m'\right)}}{S+1 + (S-1) \cdot e^{j\left(\frac{4\pi}{\lambda} z_m'\right)}}$$

$$= Z_0 \cdot \frac{1 - j \cdot S \cdot \tan(2\pi z_m'/\lambda)}{S \cancel{+} j \cdot \cancel{S} \cdot \tan(2\pi z_m'/\lambda)}.$$

9-31. $V(z) = V_0^+ e^{-j\beta z} + V_0^- \cdot e^{j\beta z}$

$I(z) = \dfrac{1}{R_0}\left(V_0^+ e^{-j\beta z} - V_0^- \cdot e^{j\beta z}\right)$

$V_{inc}(z=0) = V_0^+ = \dfrac{V_g}{2}$. $I_{inc}(z=0) = I_0^+ = \dfrac{V_g}{2R_0}$.

(a). $P_{inc} = \frac{1}{2} Re\left[V_0^+ \cdot (I_0^+)^*\right] = \dfrac{V_g^2}{8R_0}$

(b). $P_L = \frac{1}{2} Re\left[V(z) \cdot I(z)^*\right]$

$= \dfrac{1}{2R_0} \cdot \left\{|V_0^+|^2 - |V_0^-|^2\right\} = \dfrac{V_g^2}{2R_0} \cdot \left(1 - |\Gamma|^2\right)$.

(c). $\dfrac{P_A}{P_{inc}} = 1 - |\Gamma|^2 = \dfrac{4S}{(S+1)^2}$.

(d). $P_{inc} = 25$ (W). $\Gamma = 0.243\, e^{-j76^\circ}$

$S = 1.64$. $P_L = 23.5$. $|V_L| = 54.2$ (V)

$|I_L| = 0.97$ (A).

9-48. (5)

(a). $z_L = \frac{25 + j\cdot 25}{50} = 0.5 + j\cdot 0.5$.

$y_L = \frac{1}{z_L} = 1 - j$.

$\Rightarrow$ position $d = 0$ from the load, $l = 0.375\lambda$

or $d = 0.324\lambda$, $l = 0.125\lambda$.

(b). $d' = 0$, $l' = 0.406\lambda$

or $d' = 0.324\lambda$, $l' = 0.0936\lambda$.

9-50. $z_L = \frac{100 + j\cdot 100}{300} = 0.33 + j\cdot 0.33$

$y_L = \frac{1}{z_L} = 1.5 - j\cdot 1.5$.

(a). short-circuited stubs.

$l_1 = 0.203\lambda$, $l_2 = 0.399\lambda$.

or

$l_1 = 0.089\lambda$, $l_2 = 0.189\lambda$.

(b). open-circuited stubs.

$l_1 = 0.453\lambda$. $l_2 = 0.149\lambda$

or

$l_1 = 0.339\lambda$ $l_2 = 0.439\lambda$.