1. $\omega = 10^9$ $\quad \sigma = 4$. $\quad \varepsilon = \varepsilon_r \varepsilon_0 = 81 \times \frac{1}{36\pi} \times 10^{-9}$.

$$\frac{\sigma}{\omega\varepsilon} = \frac{4}{10^9 \times 81 \times \frac{1}{36\pi} \times 10^{-9}} = 5.6 \sim 1.$$

(a). Use the general expressions of α and β,

$$\alpha = \omega \cdot \sqrt{\frac{\mu\varepsilon}{2}} \cdot \left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1 \right]^{\frac{1}{2}} = 45.8.$$

$$\beta = \omega \cdot \sqrt{\frac{\mu\varepsilon}{2}} \cdot \left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1 \right]^{\frac{1}{2}} = 54.8.$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{4\pi \times 10^{-7}}{\varepsilon_r \varepsilon_0 + \frac{\sigma}{j\omega}}} = 17.58 \times e^{j \cdot 0.695}$$

phase velocity

$$u_p = \frac{\omega}{\beta} = 0.18 \times 10^9$$

Wave-length

$$\lambda = \frac{2\pi}{\beta} = 0.115.$$

skin depth

$$\delta = \frac{1}{\alpha} = 0.0218.$$

(b). $\vec{E} = \mathrm{Re}\left\{ \hat{a}_x E_0 \cdot e^{j(10^9 t + \frac{\pi}{6})} \right\}$.

at $z = 0$.

$$\vec{E}(z) = \hat{a}_x E_o \cdot e^{-\alpha z} \cdot e^{-j\beta z + j\phi}$$

Thus, $\phi = \frac{\pi}{6}$

$$\vec{E}(z,t) = \mathrm{Re}\left\{ \hat{a}_x E_o \cdot e^{-\alpha z} \cdot e^{j(\omega t - \beta z) + j\phi} \right\}$$

$$= \hat{a}_x E_o \cdot e^{-\alpha z} \cdot \cos\left(\omega t - \beta z + \phi\right)$$

$$\vec{H}(z) = \frac{1}{\eta} \hat{a}_z \times \hat{a}_x \hat{E}(z)$$

$$= \frac{1}{\eta} \hat{a}_y E_o e^{-\alpha z} \cdot e^{-j\beta z + j\phi}$$

$$= \hat{a}_y \frac{E_o}{17.58} e^{-\alpha z} \cdot e^{-j\beta z + j\left(\frac{\pi}{6} - 0.695\right)}$$

$$\vec{H}(z,t) = \mathrm{Re}\left\{ \vec{H}(z) \cdot e^{j\omega t} \right\}$$

$$= \hat{a}_y \frac{E_o}{17.58} \overset{e^{-\alpha z}}{\vee} \cos\left(\omega t - \beta z - 0.1714\right)$$

(c) $P_{av} = \frac{1}{2}\mathrm{Re}\left\{ \vec{E} \times \vec{H}^* \right\}$

$$= \frac{1}{2}\mathrm{Re}\left\{ \hat{a}_z \frac{E_o^2}{17.58} e^{-2\alpha z} \cdot e^{j \cdot 0.695} \right\}$$

$$= \hat{a}_z \frac{E_o^2}{17.58} \cdot e^{-2\alpha z} \cdot \frac{1}{2} \times \cos(0.695)$$

$$\simeq \hat{a}_z \, 0.0218 \cdot E_o^2 \cdot e^{-2\alpha z}$$

2.

$$Z_2(0) = \eta_2 \frac{\eta_3 \cos(\beta_2 d) + j\,\eta_2 \cdot \sin(\beta_2 d)}{\eta_2 \cdot \cos(\beta_2 d) + j\,\eta_3 \cdot \sin(\beta_2 d)}$$

$$= \eta_2 \cdot \frac{j \cdot \eta_2 \sin \beta_2 d}{\eta_2 \cdot \cos(\beta_2 d)}$$

$$= j \cdot \eta_2 \cdot \tan(\beta_2 d).$$

$$\Gamma = \frac{Z_2(0) - \eta_1}{Z_2(0) + \eta_1} = \frac{-\eta_1 + j\,\eta_2 \tan(\beta_2 d)}{\eta_1 + j\,\eta_2 \tan(\beta_2 d)}$$

Define

$$\theta = \arctan\left(\frac{\eta_2 \tan(\beta_2 d)}{\eta_1}\right)$$

$$\Gamma = -e^{-j2\theta}.$$

(a).

$$\vec{E}_i(z) = \hat{a}_x \cdot E_{io} \cdot e^{-j\beta z}.$$

$$\vec{E}_r(z) = \hat{a}_x\, E_{io} \cdot \Gamma \cdot e^{+j\beta z}.$$

$$= \hat{a}_x\,(-E_{io}) \cdot e^{+j(\beta z \pm 2\theta)}$$

Therefore,

$$\vec{E}(t,z) = \hat{a}_x(-E_{io}) \cdot \cos(\omega t + \beta z - 2\theta).$$

Note, $\theta = \arctan\left(\frac{\eta_2 \tan(\beta_2 d)}{\eta_1}\right) \neq 0$.

in general, $P_{av} = 0$.

(b). If the dielectric layer were absent,

$$\vec{E}_i(z) = \hat{a}_x E_{i0}\, e^{-j\beta z},$$

$$\vec{E}_r(z) = \hat{a}_x E_{i0} \cdot (-1) \cdot e^{j\beta z}.$$

i.e. $\theta = 0$.

$$\arctan\left(\frac{\eta_2 \tan(\beta_2 d)}{\eta_1}\right) = 0.$$

$$\tan(\beta_2 d) = 0.$$

$$d = n\pi / \beta_2 = \frac{n\pi}{\beta_2} = \frac{n\pi}{\frac{2\pi}{\lambda}} = \frac{n}{2} \cdot \lambda.$$

$n = 1, 2, \cdots.$

3.

(a). $$\theta_B = \arctan\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{1/2} = 1.46.$$

$$\theta_t = \arcsin\left(\sin\theta_B \cdot \frac{\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_2}}\right) = 0.11.$$

(b) $$\vec{E}_i = E_{i0}\left(\hat{a}_x \cos(\theta_B) + \hat{a}_y - \hat{a}_z \sin(\theta_B)\right) \cdot e^{-j\beta_1(x\sin\theta_B + z\cos\theta_B)}$$

$$= E_{i0}\left(\hat{a}_x \cos(\theta_B) - \hat{a}_z \sin(\theta_B)\right) \cdot e^{-j\beta_1(x\sin\theta_B + z\cos\theta_B)}$$

$$+ E_{i0} \cdot \hat{a}_y \cdot e^{-j\beta_1(x\sin\theta_B + z\cos\theta_B)}$$

$= E_{\parallel} + E_{\perp}$.

Since the incident angle is Brewster angle θ_B,

$\Gamma_{\parallel} = 0$.

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_B - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_B + \eta_1 \cos\theta_t} = \frac{\sqrt{\varepsilon_1}\cos\theta_B - \sqrt{\varepsilon_2}\cos\theta_t}{\sqrt{\varepsilon_1}\cos\theta_B + \sqrt{\varepsilon_2}\cos\theta_t}$$

$$= \frac{0.0124 - 1}{0.0124 + 1} = -0.9755 .$$

$$\tau_{\parallel} = \left(\frac{\cos\theta_B}{\cos\theta_t}\right) \cdot \left(1 + \Gamma_{\parallel}\right) = \left(\frac{\cos\theta_B}{\cos\theta_t}\right) = 0.11 .$$

$$\tau_{\perp} = 1 + \Gamma_{\perp} = 0.0245$$