

$$(a) \quad Z_i = Z_0 \cdot \frac{Z_L + Z_0 \tanh(\Gamma L)}{Z_0 + Z_L \tanh(\Gamma L)}$$

$$= \frac{Z_0}{\tanh(\Gamma L)} \quad (Z_L = +\infty)$$

$$\tanh(\Gamma L) = \tanh((\alpha + j\beta)L) = \frac{\sinh(\alpha L + j\beta L)}{\cosh(\alpha L + j\beta L)}$$

$$= \frac{\sinh(\alpha L) \cdot \cos(\beta L) + j \cosh(\alpha L) \cdot \sin(\beta L)}{\cosh(\alpha L) \cdot \cos(\beta L) + j \sinh(\alpha L) \cdot \sin(\beta L)}$$

Since $\alpha L \ll 1$,

$$\sinh(\alpha L) = \frac{e^{\alpha L} - e^{-\alpha L}}{2} = \frac{(1 + \alpha L) - (1 - \alpha L)}{2} = \alpha L$$

$$\cosh(\alpha L) = \frac{e^{\alpha L} + e^{-\alpha L}}{2} = \frac{(1 + \alpha L) + (1 - \alpha L)}{2} = 1$$

$$\therefore \tanh(\Gamma L) = \frac{\alpha L \cdot \cos(\beta L) + j \cdot \sin(\beta L)}{\cos(\beta L) + j \cdot (\alpha L) \cdot \sin(\beta L)}$$

When $\beta L = n\pi$, i.e. $L = \frac{\lambda}{2} \cdot n$,

$\tanh(\Gamma L)$ attains its minima αL .

$$\text{Thus } Z_i = \frac{Z_0}{\alpha L} \quad \text{at } L = \frac{\lambda}{2} n$$

Note $\tanh(\Gamma L)$ doesn't vanish as in lossless case.

$$\text{Hence } (Z_i)_{\max} = \frac{Z_0}{\alpha L}, \quad \text{where } L = \frac{\lambda}{2}, \quad n=1.$$

(b).

$$\alpha = 0.01 \text{ dB/m}$$

$$\Rightarrow -20 \log_{10} e^{-\alpha} = 0.01$$

$$\alpha = \frac{0.01}{20} \cdot \frac{1}{\log_{10} e} = 1.15 \times 10^{-3} \text{ Np/m}$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{\frac{c}{\sqrt{\epsilon_r} f}} = \frac{2\pi \cdot 2 \cdot 3 \times 10^8}{3 \times 10^8} = 4\pi$$

$$Q = \frac{f_0}{\Delta f} = \frac{\beta}{2\alpha} = \frac{4\pi}{2 \times 1.15 \times 10^{-3}} = 5463.6$$

The half-power bandwidth is

$$\Delta f = \frac{f_0}{Q} = \frac{3 \times 10^8}{5463.6} \approx 54.9 \text{ kHz}$$

2.

$$S = \frac{1 + |P|}{1 - |P|} = 2$$

$$\text{Thus, } |P| = \frac{1}{3}$$

The distance between successive voltage maxima is $\frac{\lambda}{2} = 18 \text{ (cm)}$

$$\lambda = 36 \text{ (cm)}$$

$$(a). \quad \Gamma = |\Gamma| \cdot e^{j\theta_r}$$

$$\text{Voltage maxima} \Rightarrow \theta_r - 2\beta \cdot L = 2n\pi$$

$$\therefore \theta_r - 2 \cdot \frac{2\pi}{\lambda} \cdot 6 = \theta_r - 2 \cdot \frac{2\pi}{30} \cdot 6 = 0$$

$$\theta_r = \frac{4\pi}{30} = \frac{2\pi}{3}$$

$$\Gamma = \frac{1}{3} \cdot e^{j\frac{2\pi}{3}}$$

$$(b). \quad Z_L = Z_0 \cdot \frac{1 + \Gamma}{1 - \Gamma}$$

$$= 50 \cdot \frac{1 + \frac{1}{3} \cdot e^{j\frac{2\pi}{3}}}{1 - \frac{1}{3} \cdot e^{j\frac{2\pi}{3}}}$$

$$= 30.77 + j \cdot 20$$

$$(c). \quad \text{at } z' = 15 \text{ (cm)}$$

$$\Gamma' = \frac{1}{3} \cdot e^{j(\theta_r - 2\beta \cdot z')} = -\frac{1}{3}$$

$$Z_{in} = Z_0 \cdot \frac{1 + \Gamma'}{1 - \Gamma'} = 50 \cdot \frac{\frac{2}{3}}{\frac{4}{3}} = 25 \text{ (}\Omega\text{)}$$

$$3. \quad (a). \quad P_i = \frac{1}{2} \cdot \frac{|V_o|^2}{R_o} = 10^2 \text{ (W)}$$

(b). At the load,

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 + j50 - 50}{50 + j50 + 50}$$
$$= 0.2 + j0.4.$$

(c). The reflected power flow is

$$P_r = \frac{1}{2} \cdot \frac{|V_r|^2}{R_0} = \frac{1}{2} \cdot \frac{|V_0|^2 \cdot |\Gamma|^2}{R_0} = 10^2 \times |\Gamma|^2$$
$$= 10^2 \times 0.2 = 20 \text{ (W)}$$

(d). The power dissipation is

$$P_i - P_r = 80 \text{ (W)}.$$