ECE 145A/218A

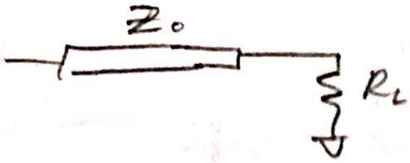
Problem Set #1 Solutions

J.R

10/12/18

Problem #1

$Z_0 = 50\Omega$



$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

a.) $R_L = 10\Omega$ — $\Gamma_L = -0.667$

b.) $R_L = 50\Omega$ — $\Gamma_L = 0$

c.) $R_L = 100\Omega$ — $\Gamma_L = 0.333$

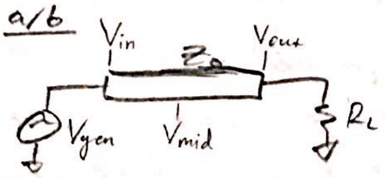
Problem #2

$Z_0 = 50\Omega$

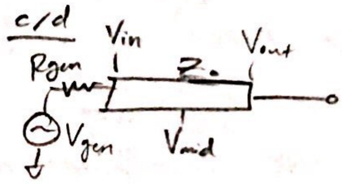
$l = 10\,cm$

$V_p = 2 \times 10^8\, m/s$

a/b



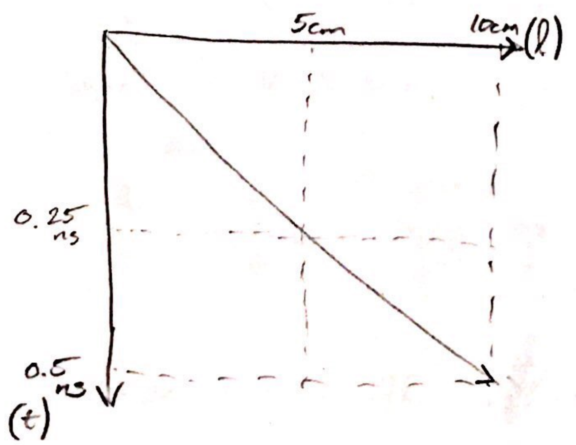
c/d

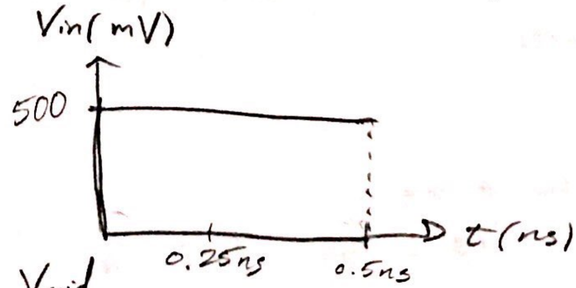


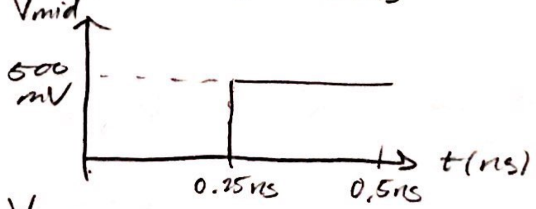
$R_{gen} = 50\Omega$

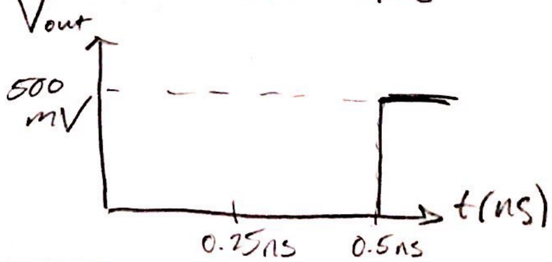
Case a/b) $R_L = 50\Omega$, $V_{gen} = 500\,mV\,u(t)$

$$\tau = \frac{l}{V_p} = \frac{10\,cm}{2 \times 10^8\, m/s} = \boxed{0.5\,ns} \qquad R_L = 50\Omega \Rightarrow \Gamma_L = 0$$



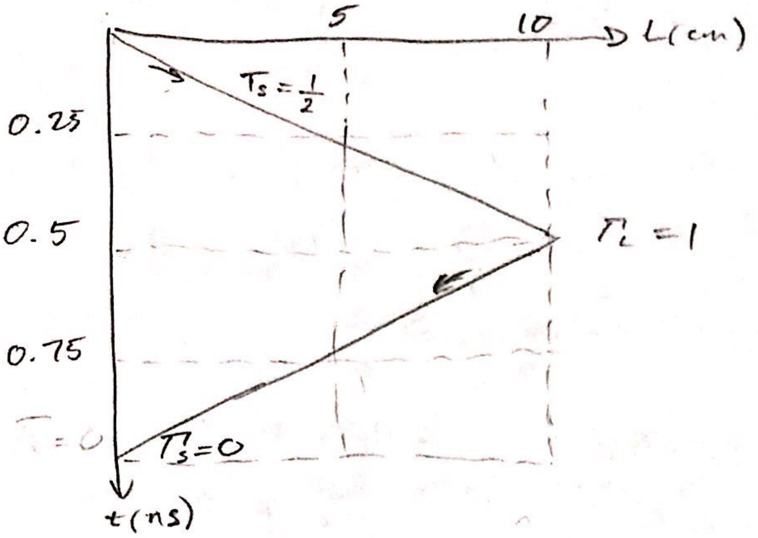


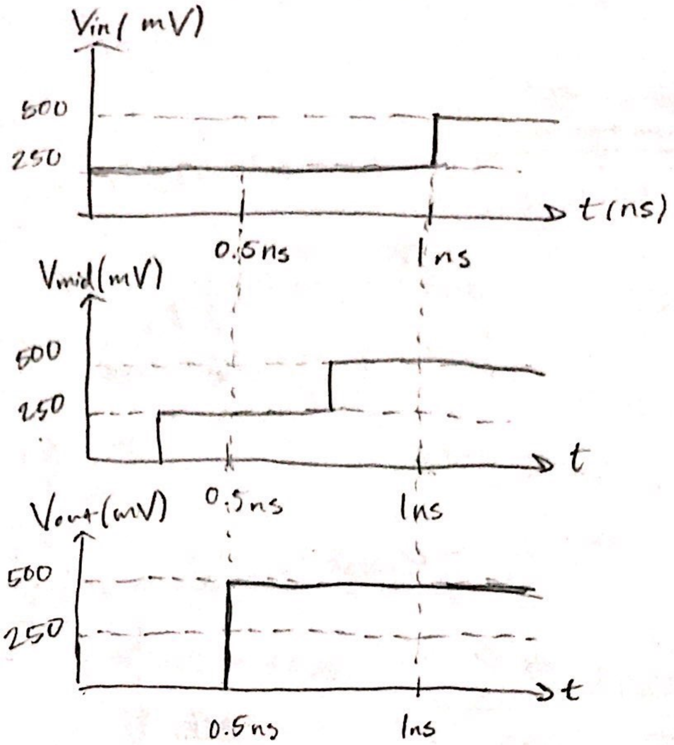




Case c/d :) $R_L = \infty$ $V_{gen} = 500mV\,u(t)$

$\Gamma_L = 1$ $T_s = \frac{Z_0}{Z_0 + R_{gen}} = \frac{1}{2}$ $\Gamma_s = 0$

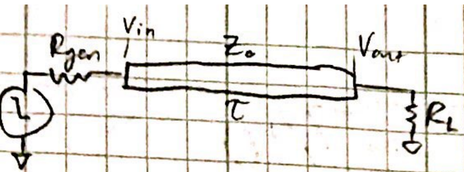




Comment: It is generally more desirable to have rising/falling edges that meet the full magnitude as in case a/b. Thus, it is more desirable to have low series source resistance, and have a load termination.

3/3

3.



$Z_o = 50\Omega$

$\tau = l/v = 300ps$

$V_{gen} = 1\ V\ u(t)$

Case a.) $R_L = 250\ \Omega$, $R_{gen} = 250\Omega$

$\Gamma_L = \dfrac{R_L/Z_o - 1}{R_L/Z_o + 1} = 0.67$

$\Gamma_s = \dfrac{R_{gen}/Z_o - 1}{R_{gen}/Z_o + 1} = 0.67$

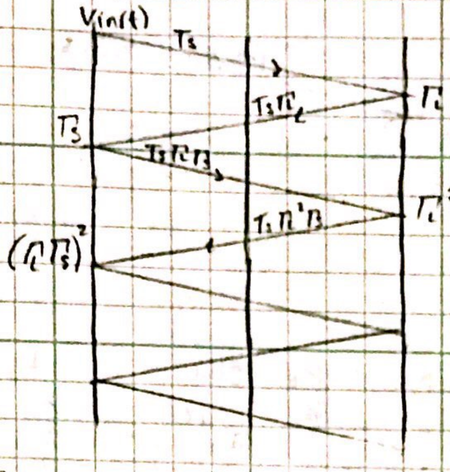
$T_s = \dfrac{Z_o}{Z_o + R_{gen}} = \dfrac{1}{6}$

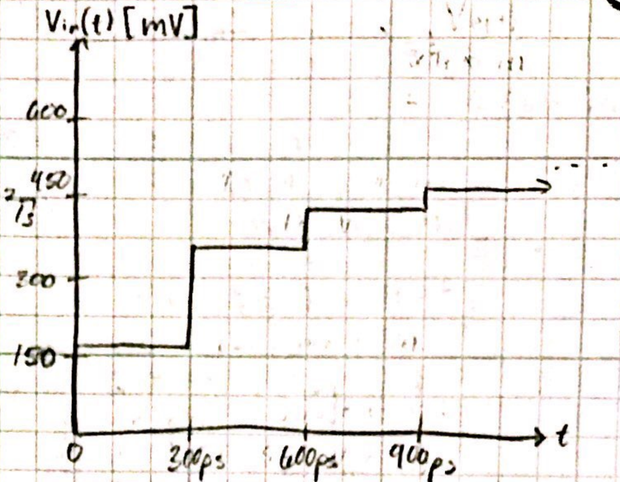
$T_s \cdot 1V u(t) = 0.167\ u(t)$ +

$T_s \cdot \Gamma_L(1+\Gamma_s) \cdot u(t) = 0.185\ V\ u(t)$ +

$T_s \Gamma_L^2 \Gamma_s (1+\Gamma_s) u(t) = 82.3\ mV\ u(t)$

$T_s \Gamma_L^3 \Gamma_s^2 (1+\Gamma_s) u(t) = 36.58\ mV$





Case b.) $R_L = 5\Omega$, $R_{gen} = 5\Omega$

$\Gamma_L = \dfrac{R_L/Z_o - 1}{R_L/Z_o + 1} = -818.2\ mV$

$\Gamma_s = -0.8182$

$T_s = 0.909$

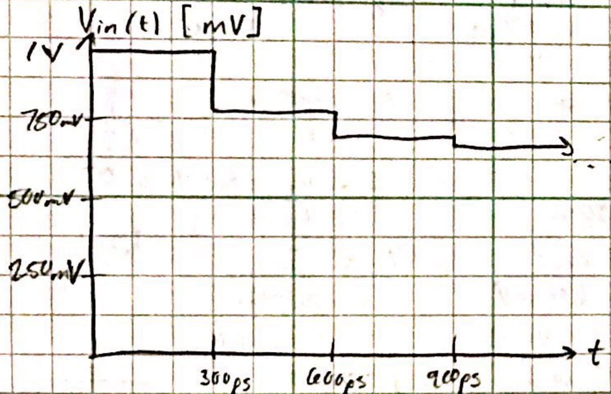
$T_s \cdot 1V u(t) = 909.09 mV$ +

$T_s \cdot \Gamma_L(1+\Gamma_s) u(t) = -135.23\ mV$ +

$T_s \Gamma_L^2 \Gamma_s (1+\Gamma_s) u(t) = -90.53\ mV$ +

$T_s \Gamma_L^3 \Gamma_s^2 (1+\Gamma_s\ u(t) = -60.604\ mV$ +

$\rightarrow -40.57 mV$



Case C.) $R_L = 250\Omega$, $R_{gen} = 5\Omega$

$\Gamma_L = 0.67$

$\Gamma_s = -0.8182$

$T_s = 0.909$

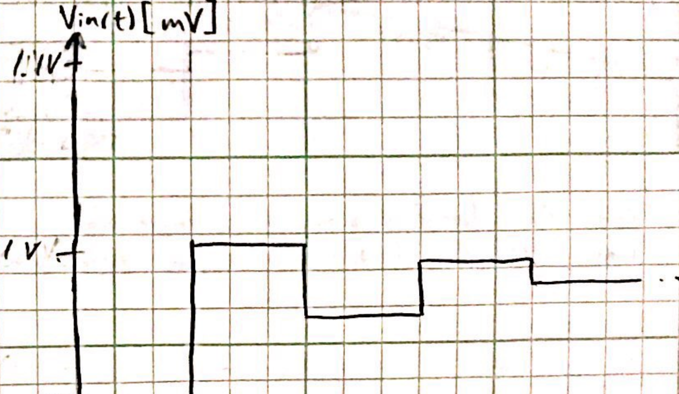
$T_s \cdot 1V u(t) = 0.909\ V$

$\Rightarrow 110.7\ mV$

$\Rightarrow -60.7\ mV$

$\Rightarrow 33.27\ mV$

$\Rightarrow -18.24\ mV$



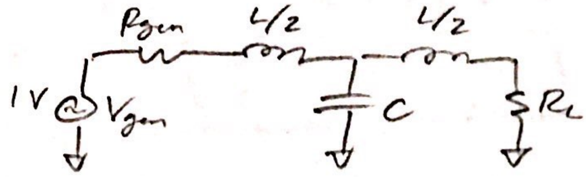
Problem 4

$Z_0 = 50\Omega$ $\qquad L = \tau Z_0 = 15nH$

$\tau = l/v = 300ps$ $\qquad C = \frac{\tau}{Z_0} = 6pF$

$V_{gen} = 1V\ u(t)$

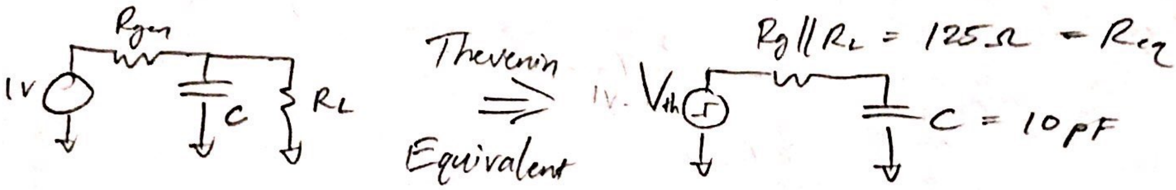
Case a) T-section $R_L = 250\Omega$, $R_{gen} = 250\Omega$



$\tau_{RC} = C \cdot (R_L \| R_{gen}) = 0.75ns$

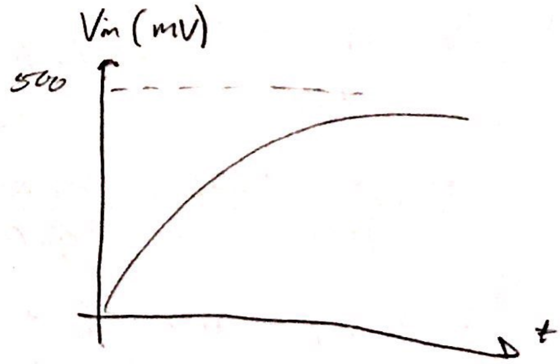
$\tau_{RL} = \frac{L}{(R_{gen} + R_L)} = 30ps$

$\tau_{RC} > \tau_{RL} \rightarrow$ ignore effects of inductor.

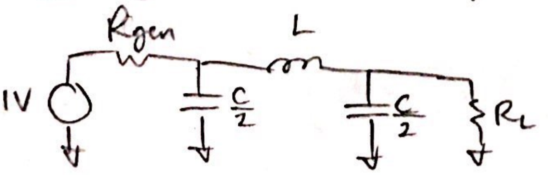


$V_{th} = 1V \cdot \frac{R_L}{R_L + R_{gen}} = 0.5V$

$$\Rightarrow \boxed{\begin{aligned} V_{in}(t) &= 0.5(1 - e^{-t/RC}) \\ &= 0.5(1 - e^{-t/1.25ns}) \end{aligned}}$$

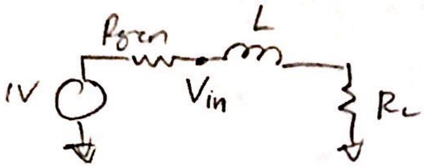


Case b) $R_L = 5\Omega$, $R_{gen} = 5\Omega$

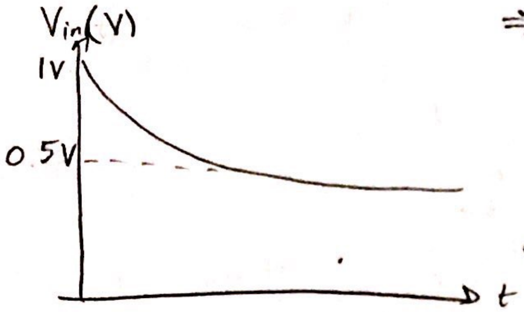


$\tau_{RC} = C(R_{gen} \| R_L) \qquad \tau_{RL} = \frac{L}{R_{gen} + R_L} \Rightarrow$ ignore capacitor

$= 15ps \qquad\qquad = 1.5ns$



$$I(t) = \frac{V_{gen}}{R_{gen}+R_L}\left(1 - e^{-t/\tau_{RL}}\right)$$
$$= 100\,mA\left(1 - e^{-t/1.5ns}\right)$$



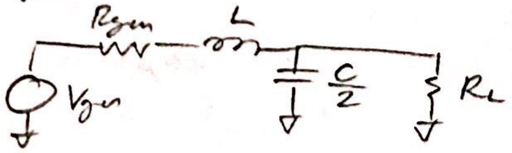
$$\Rightarrow V_{in}(t) = V_{gen} - I(t)R_{gen}$$
$$= 1V - (5\,\Omega)(100\,mA \cdot)$$
$$= 1 - 0.5\,V\left(1 - e^{-t/1.5ns}\right)$$
$$\boxed{V_{in}(t) = 0.5V\left(1 + e^{-t/1.5ns}\right)}$$

Case C: $R_L = 250\,\Omega$, $R_{gen} = 5\,\Omega$

$R_L >> R_{gen}$ → neglect time constant associated with R_{gen}.

⇒ neglect first $C/2$

⇒



$$\begin{cases} \dfrac{V_{in}(t) - V_{gen}}{R_{gen}} + \dfrac{V_{in}(t) - V_{out}(t)}{j\omega L} = 0 \\[2ex] \dfrac{V_{out}(t) - V_{in}(t)}{j\omega L} + \dfrac{V_{out}(t)}{2/j\omega C} + \dfrac{V_{out}(t)}{R_L} = 0 \end{cases}$$

$$\Rightarrow V_{in}(\omega) = \frac{j\omega L\left(\frac{2}{j\omega C} + R_L\right) + \left(\frac{2}{j\omega C} \cdot R_L\right)}{R_{gen}\left(\frac{2}{j\omega C} + R_L\right) + j\omega L\left(\frac{2}{j\omega C} + R_L\right) + \left(\frac{2}{j\omega C} \cdot R_L\right)}$$

$$= \frac{\left(\frac{2L}{C} + j\omega R_L \cdot L\right) + \left(\frac{2R_L}{j\omega C}\right)}{\left(\frac{2R_{gen}}{j\omega C} + R_L R_{gen}\right) + \left(\frac{2L}{C} + j\omega L R_L\right) + \left(\frac{2R_L}{j\omega C}\right)}$$

$$\Rightarrow V_{in}(s) = \frac{s^2 R_L \cdot L + \frac{2L}{C} \cdot s + \frac{2R_L}{C}}{\frac{2R_{gen}}{C} + s R_L R_{gen} + s\frac{2L}{C} + s^2 R_L \cdot L + \frac{2R_L}{C}}$$

$$= \frac{s^2 + s \cdot \frac{2}{R_L C} + \frac{2}{LC}}{s^2 + s\frac{R_{gen}}{L} + s\frac{2}{R_L C} + \frac{2}{LC} + \frac{2R_{gen}}{C}}$$

$$\boxed{a = \frac{2}{R_L C} \quad b = \frac{2}{LC} \quad c = \frac{R_{gen}}{L} + \frac{2}{R_L C}}$$

$$\frac{s^2 + sa + b}{s^2 + sc + b + \frac{2R_{gen}}{C}}$$

$\Rightarrow \ll \frac{2}{LC}$ ... negligible

Step function $u(t) \rightarrow \frac{1}{s}$ in freq. domain $\Rightarrow$

$$\Rightarrow \frac{s^2 + sa + b}{s^3 + s^2 c + bs}$$

$a = 1.33 \times 10^9$

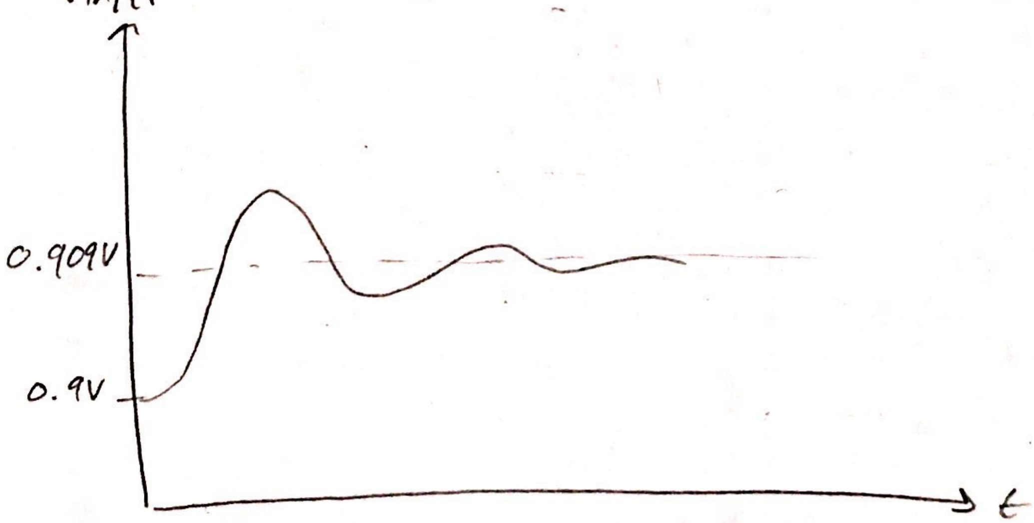
$b = 22.22 \times 10^{18}$

$c = 1.667 \times 10^9$

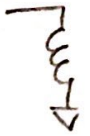
$\mathcal{L}^{-1}$ inverse fourier

$$\Rightarrow V_{in}(t) = 1 - \frac{(a-c)\left(e^{t\left(-\frac{1}{2}\sqrt{c^2-4b} - \frac{c}{2}\right)} - e^{t\left(\frac{1}{2}\sqrt{c^2-4b} - \frac{c}{2}\right)}\right)}{\sqrt{c^2-4b}}$$

$$= 1 - \frac{\left(+337 \times 10^6\left(e^{t(-833.5\times10^6 - 4.64\times10^9 j)} - e^{t(833.5\times10^6 + 4.64\times10^9 j)}\right)\right)}{9.279 \times 10^9 j}$$



Problem 5



L

$l = 300\mu m$

$Z_o = 50\Omega$

$$Z_o = \frac{\eta_o}{\sqrt{\varepsilon_r}} \frac{H}{W+H} \Rightarrow 50\,\Omega = \frac{377\,\Omega}{\sqrt{2.65}} \frac{10\mu m}{W + 10\mu m}$$

$$\Rightarrow \boxed{W = 36.32\,\mu m}$$

$$V_p = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{\sqrt{2.65}} = 1.84 \times 10^8\ m/s$$

$$\tau = \frac{l}{V_p} = 1.63 ps \quad \Rightarrow \boxed{L = Z_o \tau = 81.39\,pH}$$

$$f = 1/\tau = 613.5\ GHz$$

Smith Chart Attached $\Rightarrow$

Problem 6

$\tau = 1.63\ ps$

$$\boxed{C = \frac{\tau}{Z_o} = 32.6\ fF}$$