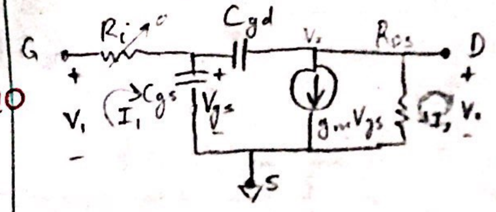
ECE 218A: HW2

Problem 1:

10/10



$R_i = 0\,\Omega$
$g_m = 200\,mS$
$R_{ds} = 50\,\Omega$
$C_{gs} = 100\,fF$
$C_{gd} = 0\,fF$

$$Y_{11}(\omega) = \frac{I_1}{V_1}\Big|_{V_2=0} = j\omega C_{gs} + j\omega \cancel{C_{gd}} = j\omega C_{gs}\ [\Omega^{-1}] = \boxed{j\omega\, 100\,fF}$$

$$Y_{12}(\omega) = \frac{I_1}{V_2}\Big|_{V_1=0} = \frac{-j\omega C_{gd} V_2}{V_2} = -j\omega C_{gd}^{\,\to 0} = \boxed{0\,[\Omega^{-1}]}$$

$$Y_{21}(\omega) = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{g_m V_{gs} - \cancel{j\omega C_{gd} V_1}}{\cancel{V_1}} = g_m\ [\Omega^{-1}] = \boxed{200\,mS}$$

$$Y_{22}(\omega) = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{\cancel{j\omega C_{gd} V_2} + \frac{1}{R_{DS}} V_2}{\cancel{V_2}} = \frac{1}{R_{DS}} = \boxed{\frac{1}{50}[\Omega^{-1}]}$$

Problem 2:

MOSFET Small Signal:

$R_i = 10\,\Omega,\ g_m = 200\,mS,\ R_{ds} = 50\,\Omega,\ C_{gs} = 100\,fF,\ C_{gd} = 5\,fF$

→ voltage divider

$$Y_{11}(\omega) = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{j\omega C_{gs} \cdot V_{gs} + j\omega C_{gd} V_{gs}}{V_1} = \frac{\cancel{j\omega C_{gs}} \cdot \frac{1}{\cancel{j\omega C_{gs}}}}{R_i + \frac{1}{j\omega C_{gs}}} + \frac{j\omega C_{gd}\,\frac{1}{j\omega C_{gs}}}{R_i + \frac{1}{j\omega C_{gs}}}$$

$$= \frac{j\omega C_{gs}\left(1 + \frac{C_{gd}}{C_{gs}}\right)}{j\omega C_{gs} \cdot R_i + 1} \Rightarrow \frac{(j\omega)^2 (C_{gs}^2 R_i + C_{gd} R_i) - j\omega (C_{gs} + C_{gd})}{-(\omega C_{gs} R_i)^2 - 1} = \boxed{\frac{(j\omega)^2 (50\times10^{-27}) + j\omega(105\times10^{-15})}{-\omega^2 (10^{-24}) - 1}}$$

$$Y_{12}(\omega) = \frac{I_1}{V_2}\Big|_{V_1=0} = \frac{-j\omega C_{gd} V_2}{V_2} = -j\omega C_{gd} = \boxed{-j\omega(5\times10^{-15})[\Omega^{-1}]}$$

$$Y_{21}(\omega) = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{g_m V_{gs} - j\omega C_{gd} V_{gs}}{V_1} = \frac{g_m \frac{1}{j\omega C_{gs}}}{R_i + \frac{1}{j\omega C_{gs}}} + \frac{j\omega C_{gd}\,\frac{1}{j\omega C_{gs}}}{R_i + \frac{1}{j\omega C_{gs}}}$$

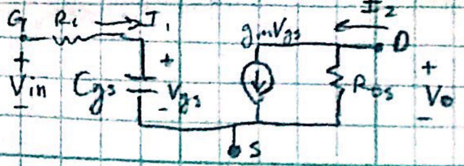
$$= \frac{\frac{g_m}{j\omega C_{gs}} + \frac{C_{gd}}{C_{gs}}}{R_i + \frac{1}{j\omega C_{gs}}} = \frac{g_m + j\omega C_{gd}}{j\omega C_{gs} R_i + 1} \Rightarrow \frac{(j\omega)^2 C_{gs} C_{gd} R_i + j\omega(C_{gs}\, g_m - C_{gd}) - g_m}{-\omega^2 (C_{gs} R_i)^2 - 1}$$

$$\Rightarrow \boxed{\frac{(j\omega)^2 (5\times10^{-27}) + j\omega(10\times10^{-15}) - 0.2}{-\omega^2(10^{-24}) - 1}}$$

$$Y_{22}(\omega) = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{j\omega C_{gd} V_2 + \frac{1}{R_{ds}} V_2}{V_2} = j\omega C_{gd} + \frac{1}{R_{ds}}$$

$$= \boxed{j\omega(5\times10^{-15}) + 0.02}$$

Problem 3:



$R_i = 0\Omega$

$g_m = 200mS$

$R_{ds} = 50\Omega =$

$C_{gs} = 100fF$

$$S_{11}(\omega) = \left.\frac{(Z_{in}/Z_0)-1}{(Z_{in}/Z_0)+1}\right|_{\Gamma=0} = \frac{\frac{1}{j\omega C_{gs}Z_0}-1}{\frac{1}{j\omega C_{gs}Z_0}+1} = \frac{1-j\omega C_{gs}Z_0}{1+j\omega C_{gs}Z_0} = \frac{(j\omega)^2(C_{gs}Z_0)^2 - j\omega C_{gs}Z_0 + 1}{1+\omega^2 C_{gs}^2 Z_0^2}$$

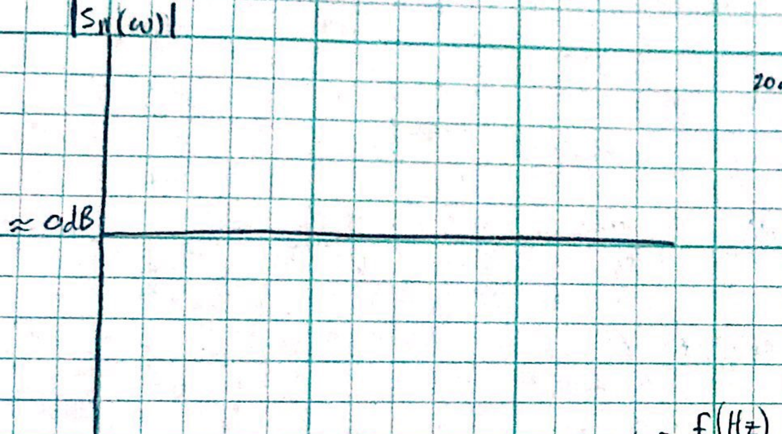
$$= \boxed{\frac{(j\omega)^2(25\times10^{-24}) - j\omega(5\times10^{-12}) + 1}{1+\omega^2(25\times10^{-24})}}$$

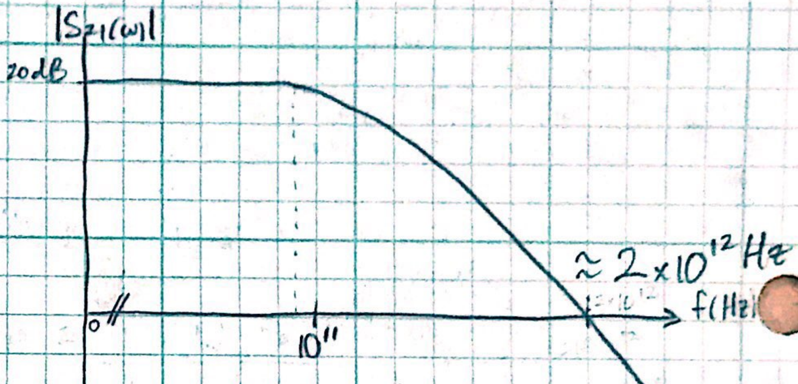
$$S_{22}(\omega) = \left.\frac{(Z_{out}/Z_0)-1}{(Z_{out}/Z_0)+1}\right|_{\Gamma_{in}=0} = \boxed{0}$$

$$S_{21}(\omega) = \left.\frac{2V_o}{V_{gen}}\right|_{Z_L=Z_0} = \frac{-2g_m V_{gs} Z_0}{\cancel{V_{gs}}(1+j\omega C_{gs}Z_0)} = \frac{-2g_m Z_0 \| R_{ds}}{1+j\omega C_{gs}Z_0} = \frac{-10}{1+j\omega(5\times10^{-12})} = \boxed{\frac{2\times10^{12}}{j\omega + 200\times10^{9}}}$$

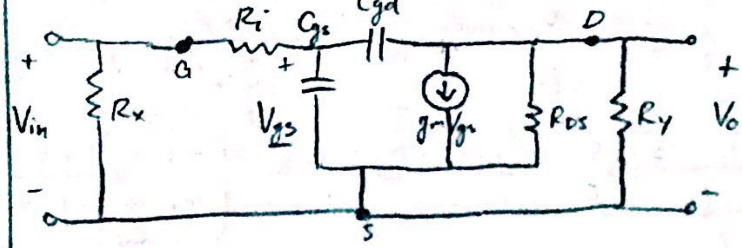
$V_{gen} = V_{gs}$

$$S_{12}(\omega) = \left.\frac{2V_{in}}{V_{gen}}\right|_{Z_{gen}=Z_0} = \boxed{0}$$



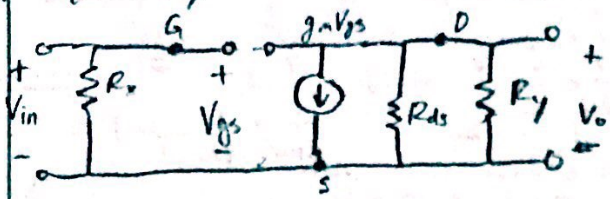


Problem 4



$R_i = 0\Omega$
$g_m = 200\,mS$
$R_{ds} = 50\Omega$
$C_{gs} = OFF$
$C_{gd} = OFF$

a.) Equivalently:



$\boxed{R_x = 50\Omega} = Z_{in}$

$Z_{out} = \dfrac{R_y \cdot R_{ds}}{R_y + R_{ds}} = 50 \Rightarrow \boxed{R_y = \infty}$

b.) $Z_{gen} = Z_L = Z_0 = 50\Omega$

$$S_{11}(w) = \frac{(Z_{in}/Z_0) - 1}{(Z_{in}/Z_0) + 1} = \boxed{0}$$

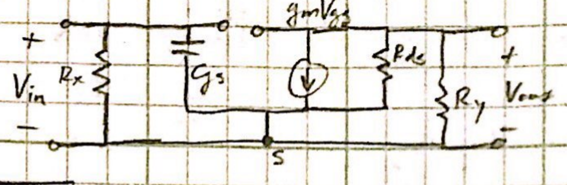
$$S_{22}(w) = \frac{(Z_{out}/Z_0) - 1}{(Z_{out}/Z_0) + 1} = \boxed{0}$$

$$S_{21}(w) = \frac{2V_o}{V_{gen}}\Bigg|_{Z_L = Z_0} = \frac{-2gm\, V_{gs} \cdot R_{ds}/\!/Z_0}{V_{gs}\left(\underbrace{\frac{R_x + Z_0}{R_x}}_{\text{V-divider}}\right)} \quad \frac{-2gm\, R_x\, R_{ds}/\!/Z_0}{R_x + Z_0} = \boxed{-5}$$

$$S_{12}(w) = \frac{2V_{in}}{V_{gen}}\Bigg|_{Z_{in} = Z_0} = \boxed{0}$$

c.)
- S_{11} is the input impedance given that the load impedance $Z_L = Z_0$.
- S_{22} is the output impedance given that the gen impedance $Z_{in} = Z_0$
- S_{21} is twice the voltage gain.

Problem 5



$$Z_{in} = R_x \parallel C_{gs} = \frac{R_x / j\omega C_{gs}}{R_x + \frac{1}{j\omega C_{gs}}} = \frac{R_x}{j\omega C_{gs} \cdot R_x + 1} \nearrow \frac{50}{j\omega(5\times10^{-12})+1}$$

a.) $$\boxed{S_{11}(j\omega)} = \frac{(Z_{in}/Z_o)-1}{(Z_{in}/Z_o)+1} = \frac{\frac{R_x}{(j\omega C_{gs}R_x+1)Z_o} - 1}{\frac{R_x}{(j\omega C_{gs}R_x+1)Z_o} + 1} = \frac{R_x - (j\omega C_{gs}R_x+1)Z_o}{R_x + (j\omega C_{gs}R_x+1)Z_o} = \boxed{\frac{-j\omega(250\times10^{-12})}{100 + j\omega(250\times10^{-12})}}$$

$$\boxed{S_{21}(j\omega)} = \frac{2V_o}{V_{gen}}\Bigg|_{Z_L = Z_o} = \frac{-2g_m \overset{\nearrow V_{in} = V_{gs}}{V_{gs}} R_{ds}}{\frac{(Z_o + Z_{in})V_{in}}{Z_{in}}} = \frac{-2g_m R_{ds} \cdot Z_{in}}{Z_o + Z_{in}} = \frac{\frac{-2g_m R_{ds} R_x}{j\omega C_{gs}R_x+1}}{Z_o + \frac{R_x}{j\omega C_{gs}R_x+1}} = \frac{-2g_m R_{ds} R_x}{(j\omega C_{gs}R_x+1)Z_o + R_x}$$

$$V_{in} = \frac{V_{gen} \cdot Z_{in}}{Z_o + Z_{in}}$$

$$\boxed{= \frac{-500}{j\omega(250\times10^{-12}) + 100}}$$

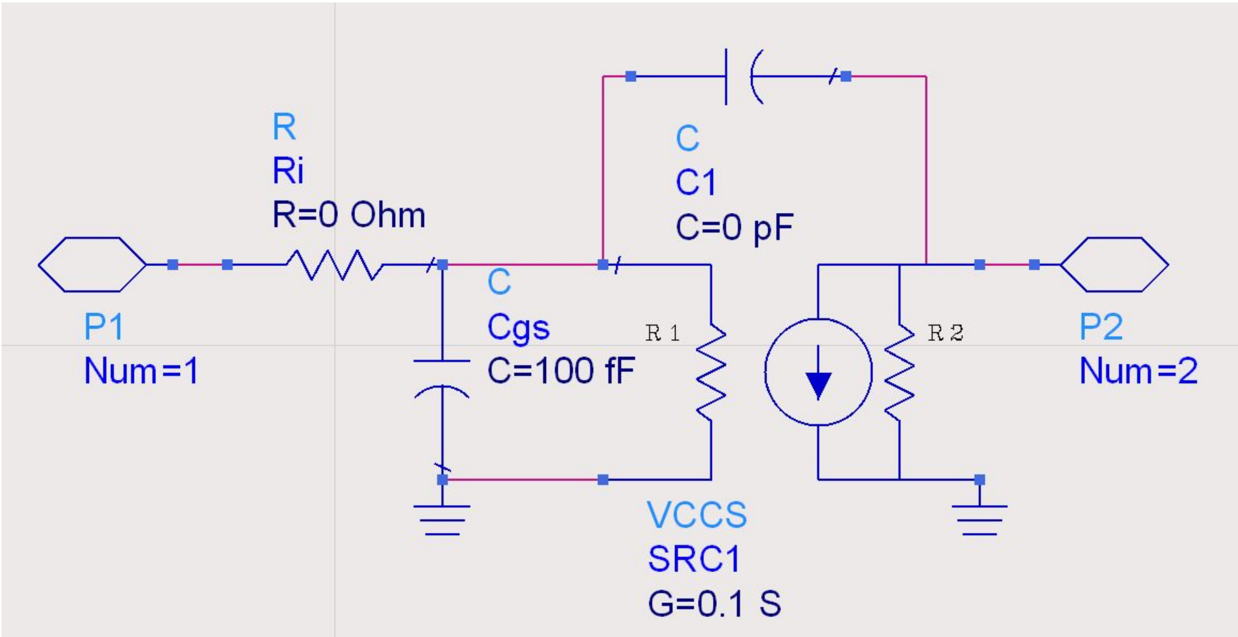
$$|S_{21}(\omega)| = \frac{1}{\sqrt{2}} = \left|\frac{-5}{1 + j\omega_{3dB}(2.5\times10^{-12})}\right|$$

$$\Rightarrow \boxed{f_{3dB} = \frac{\omega_{3dB}}{2\pi} = 63.66\ \text{GHz}}$$

$$\boxed{S_{21}(0)} = \frac{-500}{100} = \boxed{-5}$$

Same as Problem 4

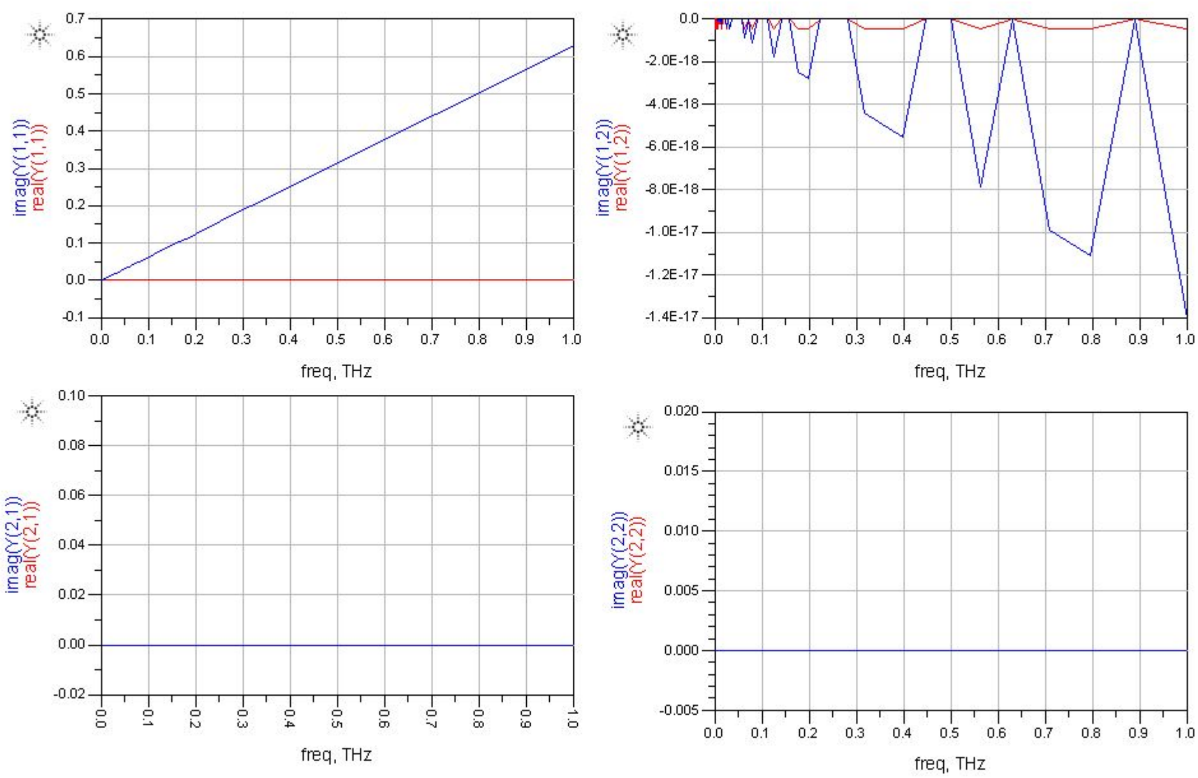
Problem 1:



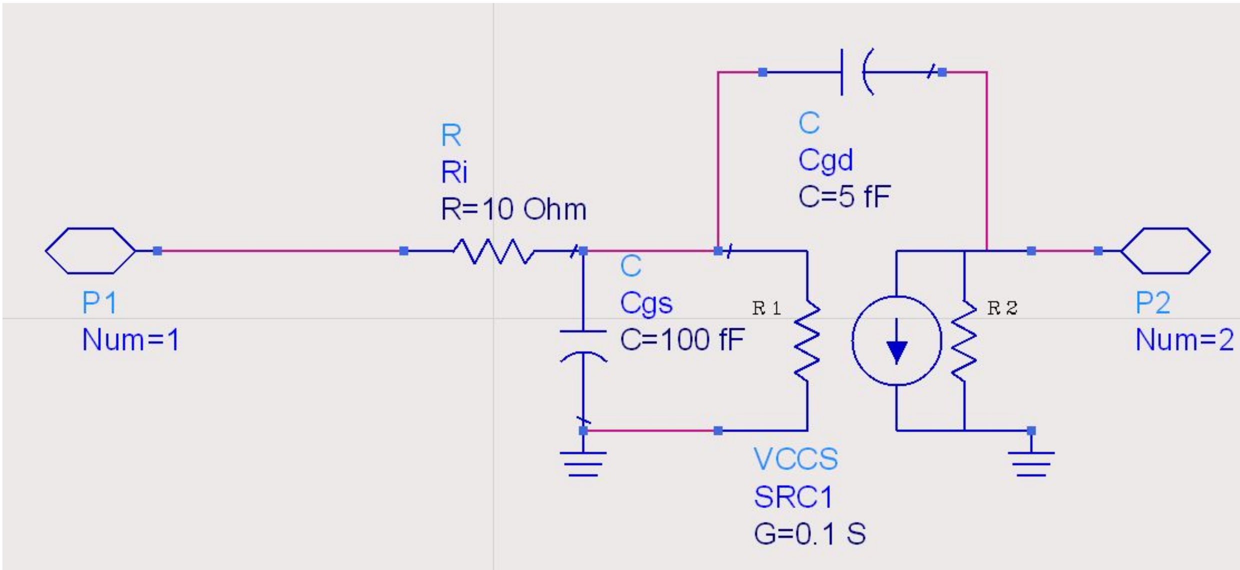
R1 = ∞

R2 = Rds = 50Ω

G = gm = 200mS, so real(Y(S21)) should equal 200mS



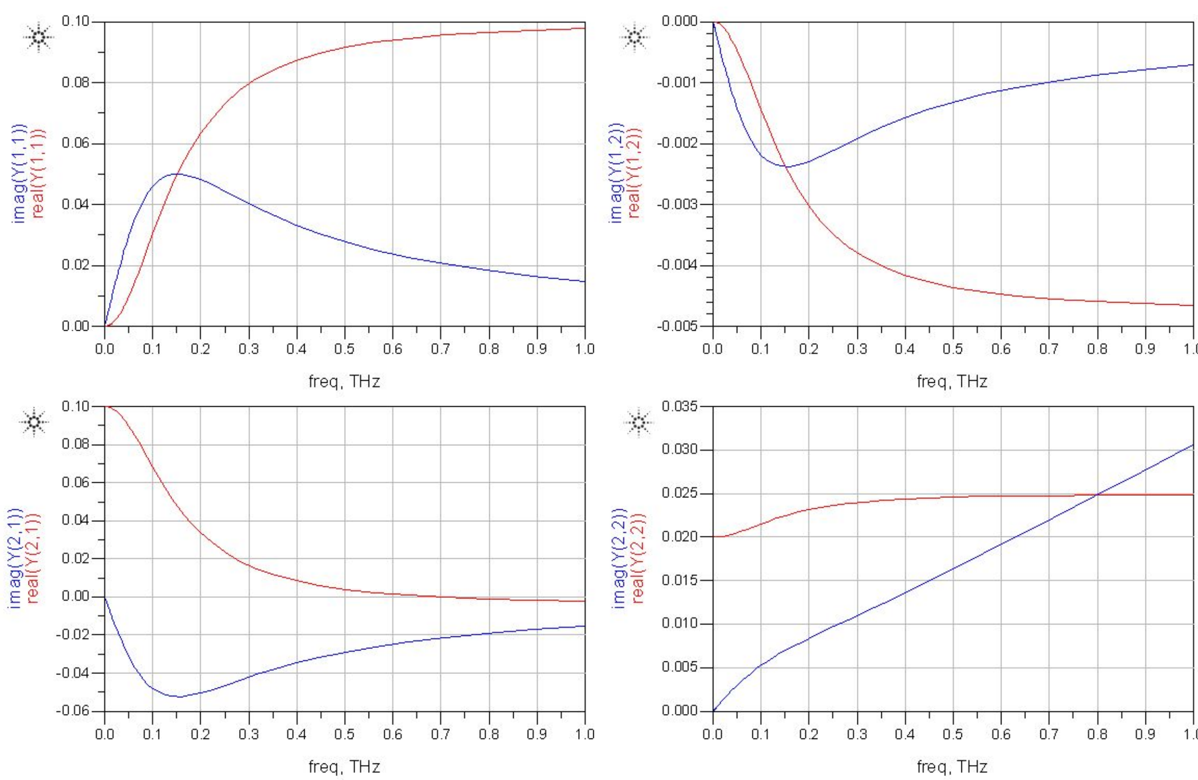
Problem 2:



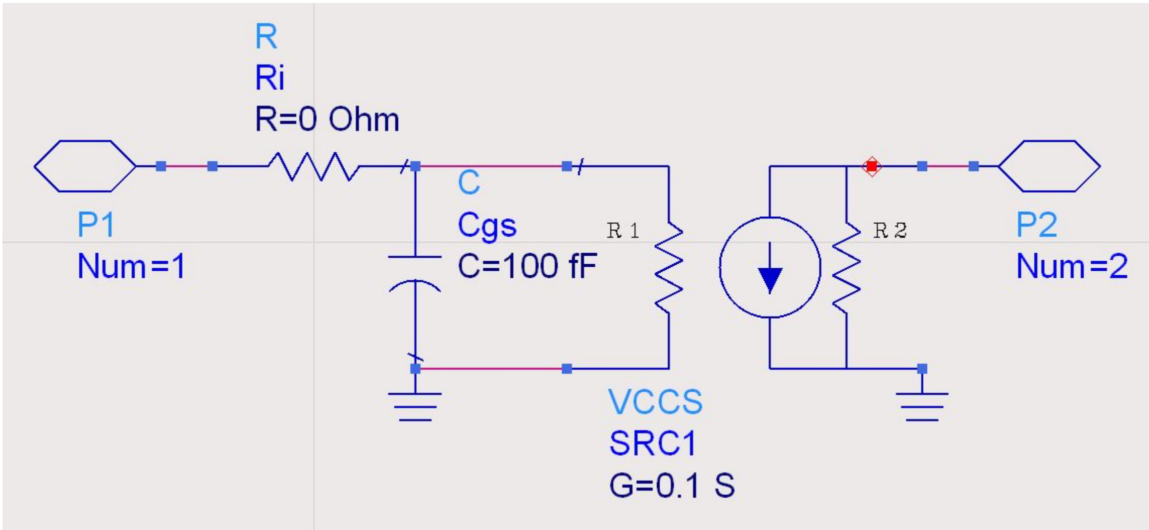
R1 = ∞

R2 = Rds = 50Ω

G = gm = 200mS, so real(Y(2,1)) at DC should equal 200mS, not 100mS - and should exhibit an increased downward slope due to a higher DC magnitude.



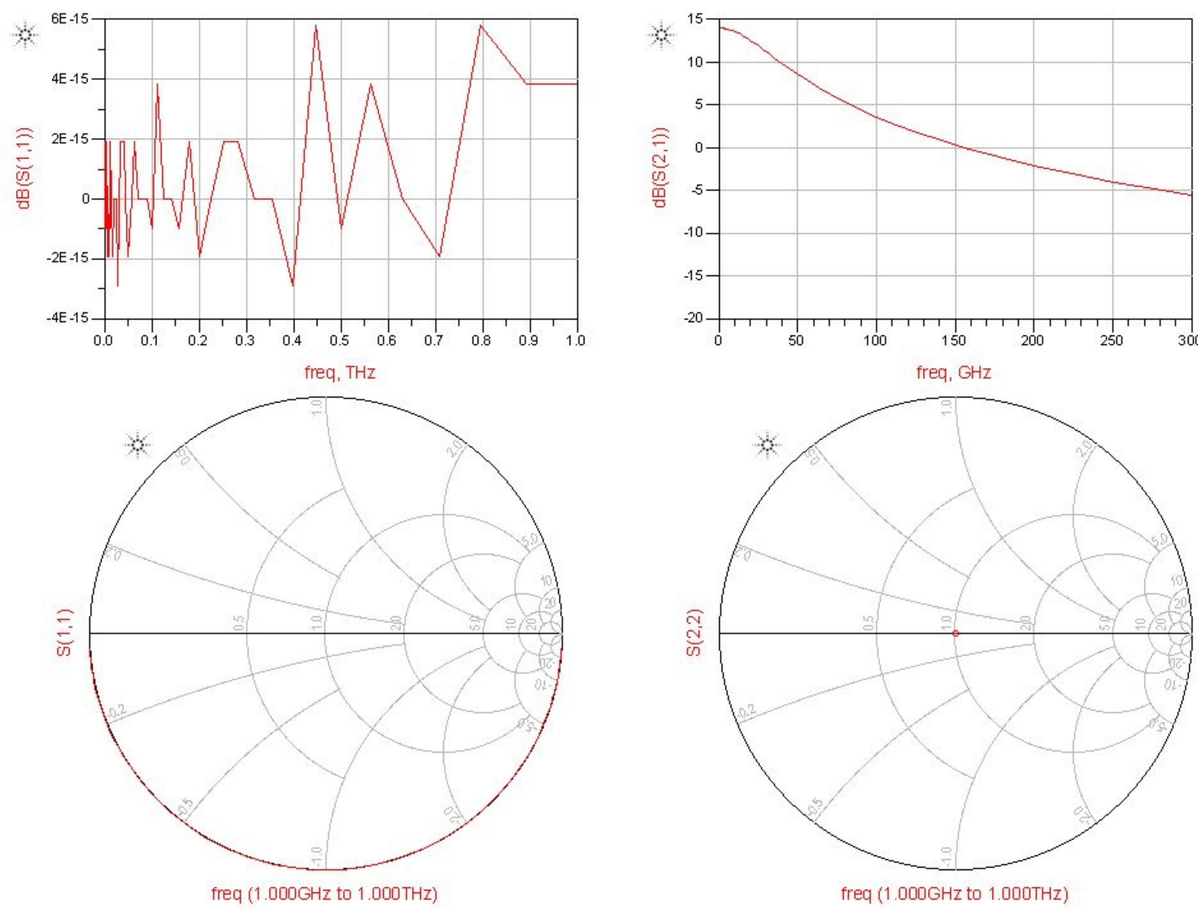
Problem 3:

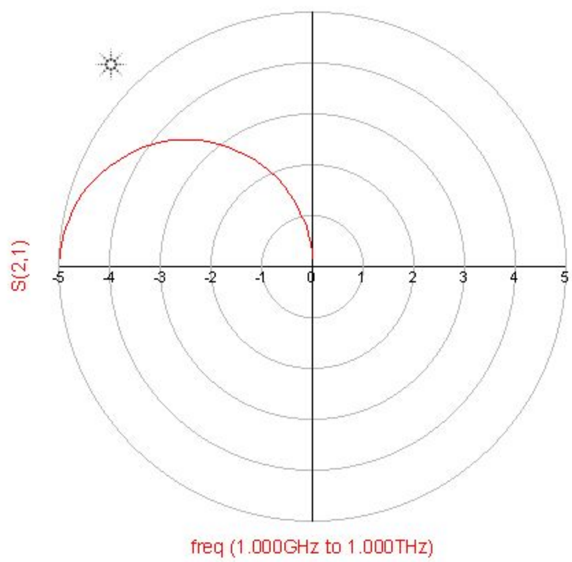


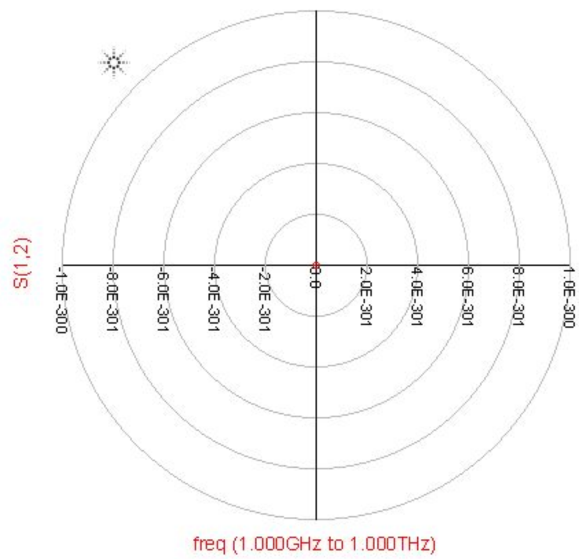
R1 = ∞

R2 = Rds = 50Ω

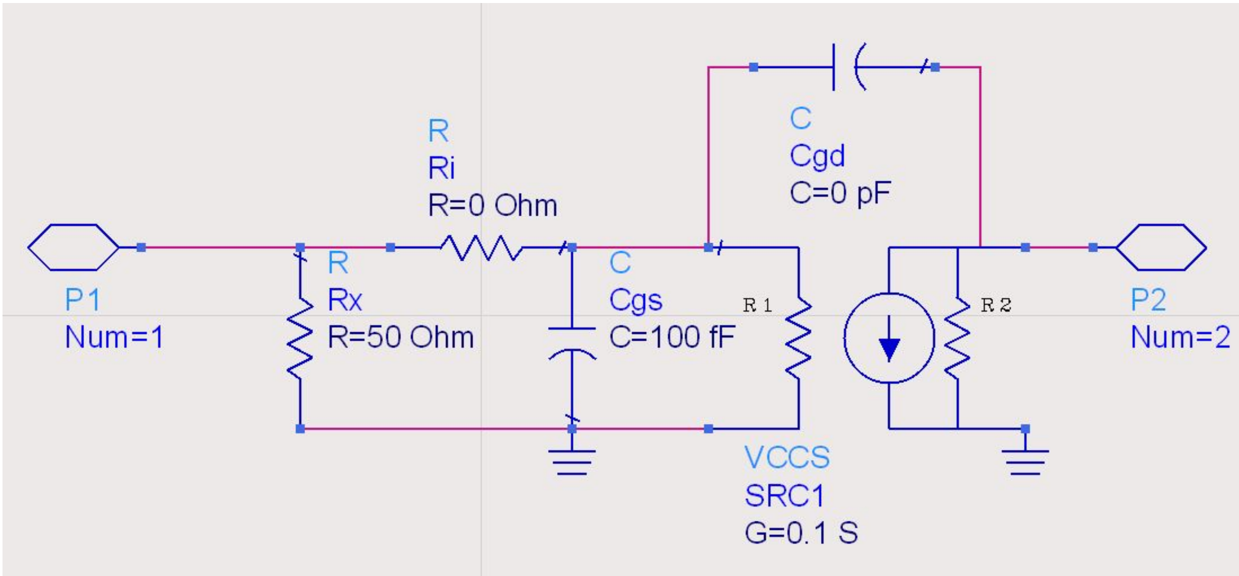
G = gm = 200mS, so S21 should be 20dB at DC, and exhibits a similar slope.




S(2,1)
-5
-4
-3
-2
-1
0
1
2
3
4
5
freq (1.000GHz to 1.000THz)


S(1,2)
-1.0E-300
-8.0E-301
-6.0E-301
-4.0E-301
-2.0E-301
0.0
2.0E-301
4.0E-301
6.0E-301
8.0E-301
1.0E-300
freq (1.000GHz to 1.000THz)

Problem 5:



R1 = ∞

R2 = Rds = 50Ω

G = gm = 200mS, so S21 should have a low frequency magnitude of 13.98dB, not 7.958dB shown below.