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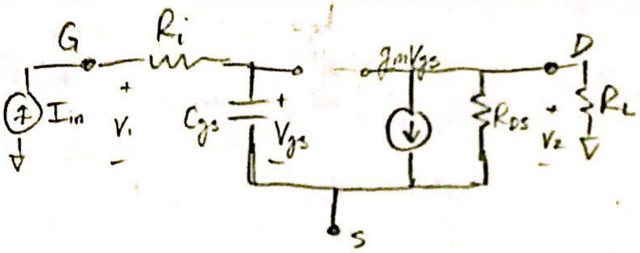
Problem 1



$g_m = 1.5\ mS/\mu m \cdot W_g$
$R_i = 1.5/g_m$
$C_{gd} = 0\ fF$
$C_{gs} = 0.75\ fF/\mu m \cdot W_g$
$G_{ds} = 0.1\ mS/\mu m \cdot W_g$

Part a)

$$G_{max} = \frac{P_{AVA}}{P_{in}} = \frac{P_{out}}{P_{in}}$$

$$I_{out} = -\frac{g_m V_{gs}}{2} = \frac{I_{in}}{j\omega C_{gs}}\frac{g_m}{2} \longrightarrow P_{out} = -\frac{I_{in}^2\, g_m^2}{(\omega^2 C_{gs}^2)} \cdot \frac{R_{DS}}{4}$$

$$P_{in} = I_{in}^2 R_i$$

$$\Rightarrow \boxed{G_{max}(\omega) = \frac{g_m^2}{\omega^2 C_{gs}^2} \cdot \frac{R_{DS}}{4R_i}} = \boxed{\frac{10 \times 10^{24}}{\omega^2}}$$

$$H_{21}(\omega) = \frac{g_m}{j\omega C_{gs}} = \boxed{\frac{2 \times 10^{12}}{\omega}}$$

Part b)

$$G_{max}\left(\frac{f_{max}}{3}\right) = \frac{10 \times 10^{24}}{(2\pi f_{max}/3)^2} = \frac{2.28 \times 10^{24}}{f_{max}^2}$$

$f = f_{max}$ @ $G_{max} = 1$

$$G_{max} = 1 = \frac{10 \times 10^{24}}{(2\pi f_{max})^2} \Rightarrow f_{max} = \frac{\sqrt{10 \times 10^{24}}}{2\pi} = 503.3\ GHz$$

$$\Rightarrow \boxed{G_{max}\left(\frac{f_{max}}{3}\right) = 9}$$

$$\boxed{Z_{gen}} = Z_{in}^* = R_i - \frac{1}{j\omega C_{gs}} = \boxed{10 + j\,12.65\ \Omega}$$

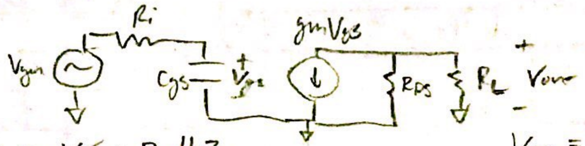
$$\boxed{Z_L = Z_{out}^* = R_{DS} = 100\ \Omega}$$

Part c)

$$\boxed{Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}} = \frac{I_1}{I_1(R_i + 1/j\omega C_{gs})} = \frac{j\omega C_{gs}}{j\omega C_{gs} R_i + 1}$$

$$\boxed{Y_{21}} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = +\frac{g_m V_{gs}}{V_1}, \quad V_{gs} = V_1\left(\frac{1/j\omega C_{gs}}{R_i + 1/j\omega C_{gs}}\right)$$

$$= \boxed{\frac{+g_m}{1 + j\omega C_{gs} R_i}}$$



$$V_{gs} = \frac{V_{gen}}{1 + j\omega C_{gs}(R_i + Z_0)}$$

$$\boxed{S_{21}} = \left.\frac{2V_{out}}{V_{gen}}\right|_{Z_L = Z_0} = \frac{-g_m \cancel{V_{gs}} \cdot R_L \| Z_0}{\cancel{V_{gs}}(1 + j\omega C_{gs}(R_i + Z_0))}$$

$$= \boxed{\frac{-2g_m (R_L \| Z_0)}{(1 + j\omega C_{gs}(R_i + Z_0))}}$$

Part D) Insertion Power Gain:

$$\|S_{21}\|^2 = \left(\frac{2\,g_m(R_i \| Z_o)}{(1 + j\omega C_{gs}(R_i + Z_o))}\right)^2 = \left(\frac{5}{\sqrt{1^2 + (2.83)^2}}\right)^2 = \frac{25}{1 + 2.83^2} = 11.125$$

$$\boxed{G_I = 10\log_{10}(\|S_{21}\|^2) = 10.46\ dB}$$

Operating Power Gain: $Z_L = Z_o$

$$P_{in} = I_{in}^2 R_i = (V_{gs} \cdot j\omega C_{gs})^2 R_i$$

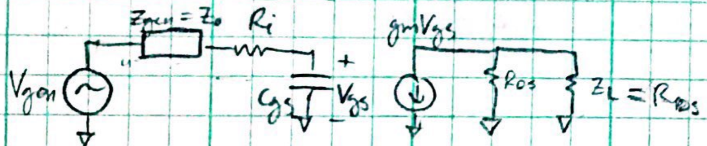
Current Divider

$$P_L = I_{out}^2 Z_L = \left(g_m V_{gs}\left[\frac{R_{DS}}{R_{DS} + Z_o}\right]\right)^2 \cdot Z_o$$

$$\boxed{G_P = \frac{P_L}{P_{in}}} = \frac{\left(g_m \cancel{V_{gs}}\left[\frac{R_{DS}}{R_{DS} + Z_o}\right]\right)^2 \cdot Z_o}{(\cancel{V_{gs}} \cdot j\omega C_{gs})^2 R_i}$$

$$= \frac{\left((150\,mS)\left[\frac{100\Omega}{100\Omega + 50\Omega}\right]\right)^2 \cdot 50\Omega}{(100\,GHz \cdot 2\pi \cdot 75\,fF)^2 \cdot 10\Omega} = \boxed{22.52} \Rightarrow \boxed{13.52\ dB}$$

Available Power Gain



$$V_{gs} = V_{gen}\frac{1/j\omega C_{gs}}{1/j\omega C_{gs} + R_i + Z_o} = V_{gen}\frac{1}{1 + j\omega C_{gs}(R_i + Z_o)}$$

$$P_{out} = (I_{out})^2 R_{DS} = \left(\frac{g_m V_{gs}}{2}\right)^2 R_{DS}$$

$$V_{gs} = \frac{I_{in}}{j\omega C_{gs}} \ ; \ I_{in} = \frac{V_{gen}}{R_i + Z_o + 1/j\omega C_{gs}} \Rightarrow V_{gs} = \frac{V_{gen}}{j\omega C_{gs}(R_i + Z_o) + 1}$$

$$\Rightarrow P_{out} = \frac{g_m^2}{4}\left(\frac{V_{gen}}{j\omega C_{gs}(R_i + Z_o) + 1}\right)^2 R_{DS}$$

$$P_{in} = \left(\frac{V_{gen}}{2}\right)^2 \frac{1}{Z_o} =$$

$$\boxed{G_A}\ \frac{P_{out}}{P_{in}} = \frac{\frac{g_m^2 R_{DS}}{4}\left(\frac{1}{1 + j\omega C_{gs}(R_i + Z_o)}\right)^2}{\frac{1}{4}\frac{1}{Z_o}} = 12.508 \Rightarrow \boxed{10.972\ dB}$$

$$\boxed{G_{Max}} = \frac{g_m^2}{\omega^2 C_{gs}^2} \frac{R_{DS}}{4R_i} \quad \text{(from Part a)}$$
$$= 25.33$$
$$= \boxed{14.04\ dB}$$

Part e) Waveforms Attached

Problem 2

$S_{11} = 0.25 \qquad S_{12} = 0 \qquad Z_0 = 50\Omega$

$S_{22} = 0.7071 \qquad S_{21} = 10$

a.) $Z_{gen} = 50\Omega \qquad Z_L = 50\Omega$

$P_{AVG} = 1mW$

$$G_T = \frac{\text{PWR to } Z_L}{P_{AVG}} = \|S_{21}\|^2 \Rightarrow \boxed{P_L} = \|S_{21}\|^2 P_{AVG} = (10)^2(1mW) = \boxed{0.1W}$$

b.) $Z_{gen} = 75\Omega \qquad Z_L = 25\Omega$

$P_{AVG} = 1mW$

$$G_T = \frac{P_L}{P_{AVG}} = \frac{(1-\|\Gamma_L\|^2)(1-\|\Gamma_S\|^2)\,\|S_{21}\|^2}{\|(1-S_{11}\Gamma_S)(1-\Gamma_L S_{22}) - \Gamma_S\Gamma_L S_{11}S_{22}\|^2}$$

$$\Gamma_S = \frac{75-50}{75+50} = 0.2 \qquad \Gamma_L = \frac{25-50}{25+50} = -0.333$$

$$= \frac{(1-(-0.33)^2)(1-(0.2)^2)(10)^2}{\left[(1-0.25\times0.2)(1-(-0.33)(0.7071)) - (0.2)(-0.33)(0.25)(0.7071)\right]^2}$$

$$\Rightarrow \boxed{P_L} = 60.95 \cdot 1mW = \boxed{60.95mW}$$

c) $Z_{gen} = Z_{in}^* \qquad Z_L = 30\Omega$

$P_{AVG} = 1mW$

$$\Gamma_L = \frac{30-50}{30+50} = \frac{-1}{4}$$

$$G_P = \frac{1}{1-\|\Gamma_{in}\|^2}\,\|S_{21}\|^2 \frac{1-\|\Gamma_L\|^2}{\|1-S_{22}\Gamma_L\|^2} = \left.\frac{P_L}{P_{AVG}}\right|_{\Gamma_S = \Gamma_{in}^*}$$

$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}\cancel{S_{12}}^{\,0}}{1-S_{22}\Gamma_L} = 0.25$$

$$= 72.21$$

$$\Rightarrow \boxed{P_L = 72.21\ mW}$$

d) **For transistor (Matched Both ends):**

$$G_{MAX} = \frac{1}{1-\|S_{11}\|^2}\|S_{21}\|^2\frac{1}{1-\|S_{22}\|^2} = 213.33$$

Gmax for transistor equals Gmax for amplifier:

$$G_{MAX} = \frac{1}{1-\|S_{11}^{amp}\|^2}\|S_{21}^{amp}\|^2\frac{1}{1-\|S_{22}^{amp}\|^2}$$

$S_{11}^{amp} = S_{22}^{amp} = 0$ because input and output are matched.

$$G_{MAX} = \|S_{21}^{amp}\|^2 = 213.33 \Rightarrow S_{21}^{amp} = 14.61$$

For Amplifier:

$$\Gamma_S = \frac{25-50}{25+50} = \frac{-1}{3}$$

$$\Gamma_L = \frac{100-50}{100+50} = \frac{1}{3}$$

$S_{12}^{amp} = 0$ for unilateral amp

$$G_T = \boxed{\frac{P_L}{P_{AVG}}} = \frac{(1-\|\Gamma_S\|^2)(1-\|\Gamma_L\|^2)\|S_{21}\|^2}{\|(1-S_{11}\Gamma_S)(1-\Gamma_L S_{22})-\Gamma_S\Gamma_L S_{11}S_{12}\|^2}$$

$$= \boxed{169.4 \text{ mW}}$$

Problem 3 $S_{11} = 0.1$ $S_{12} = 1$
$S_{22} = 0.125$ $S_{21} = 20$

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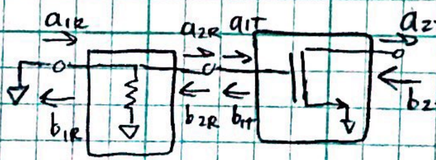
a)

$$S_{11} = \frac{Z_{in}/Z_0 - 1}{Z_{in}/Z_0 + 1};\ Z_{in} = 50\|Z_0 = 25\Omega, \Rightarrow \boxed{S_{11} = -1/3 = S_{22} \text{ by symmetry}}$$

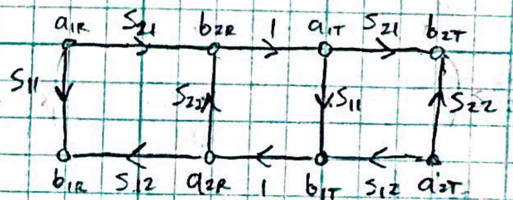
$$S_{21} = 2\frac{V_{out}}{V_{gen}}\Big|_{Z_L=Z_0};\ V_{out} = \frac{V_{gen}\cdot(Z_0\|50)}{Z_0+(Z_0\|50)} = \frac{V_{gen}}{3}$$

$$= \boxed{2/3} = S_{12} \text{ by symmetry}$$

b)

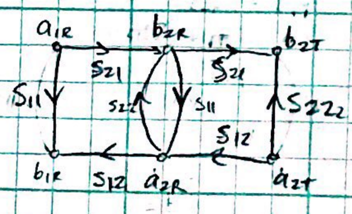


$\Rightarrow$



$b_{2R} = a_{1T}$
$a_{2R} = b_{1T}$

$\Rightarrow$



$S_{11} = \dfrac{b_{1R}}{a_{1R}}$ $\quad S_{12} = \dfrac{b_{1R}}{a_{2T}}$ $\quad$ loops: $S_{11T} S_{22R}$

$S_{21} = \dfrac{b_{2T}}{a_{1R}}$ $\quad S_{22} = \dfrac{b_{1T}}{a_{2T}}$

$$S_{11} = \frac{b_{1R}}{a_{1R}} = \frac{S_{11R}(1 - S_{11T} S_{22R}) + S_{21R} S_{11T} S_{12R}}{1 - S_{11T} S_{22R}} = \boxed{-0.29}$$

$$S_{12} = \frac{b_{1R}}{a_{2T}} = \frac{S_{12R} S_{12T}}{1 - S_{11T} S_{22R}} = \frac{(2/3)(1)}{1-(0.1)(-1/3)} = \boxed{0.645}$$

$$S_{21} = \frac{b_{2T}}{a_{1R}} = \frac{S_{21T} S_{21R}}{1 - S_{11T} S_{22R}} = \frac{(20)(2/3)}{1-(0.1)(-1/3)} = \boxed{12.91}$$

$$S_{22} = \frac{b_{2T}}{a_{2T}} = \frac{S_{22T}(1 - S_{11T} S_{22R}) + S_{12T} S_{22R} S_{21T}}{1 - S_{11T} \cdot S_{22R}} = \boxed{-6.33}$$

$$S_{iso} = \begin{bmatrix} b_{1i}/a_{1i} & b_{1i}/a_{2i} \\ b_{2i}/a_{1i} & b_{2i}/a_{2i} \end{bmatrix}$$

Problem 4

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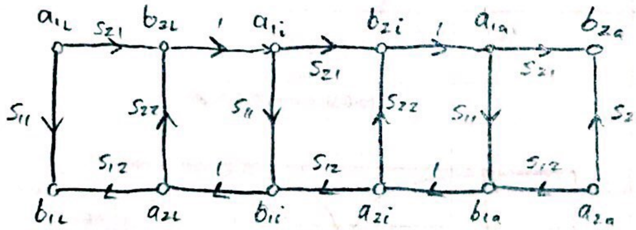
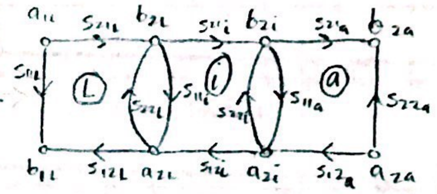
$$S_{laser} = \begin{bmatrix} 0.1 & 10 \\ 10 & 0.1 \end{bmatrix} \quad S_{isolator} = \begin{bmatrix} 0 & 0.25 \\ 0.5 & 0 \end{bmatrix} \quad S_{powermeter} = \begin{bmatrix} R & T \\ T & R \end{bmatrix} = \begin{bmatrix} b_{1a}/a_{1a} & b_{1a}/a_{2a} \\ b_{2a}/a_{1a} & b_{2a}/a_{2a} \end{bmatrix} = \begin{bmatrix} S_{11a} & S_{12a} \\ S_{21a} & S_{22a} \end{bmatrix}$$



$\Rightarrow$

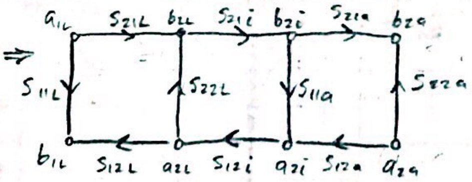


$b_{2i} = a_{1a}$

$a_{2i} = b_{1a}$

$S_{11a} = \dfrac{b_{1a}}{a_{1a}}$

$\Rightarrow$



$$S_{11} = \frac{b_{1L}}{a_{1L}} = \frac{S_{11L}(1 - S_{21i} S_{11a} S_{12i} S_{22L}) + S_{21L} S_{21i} S_{11a} S_{12i} S_{12L}}{(1 - S_{21i} S_{11a} S_{12i} S_{22L})} = S_{11L} + \frac{S_{21L} R S_{21i} S_{12i} S_{12L}}{1 - R S_{21i} S_{12i} S_{12L}}$$

$$= \boxed{0.1 + \frac{12.5R}{1 - 0.0125R}}$$

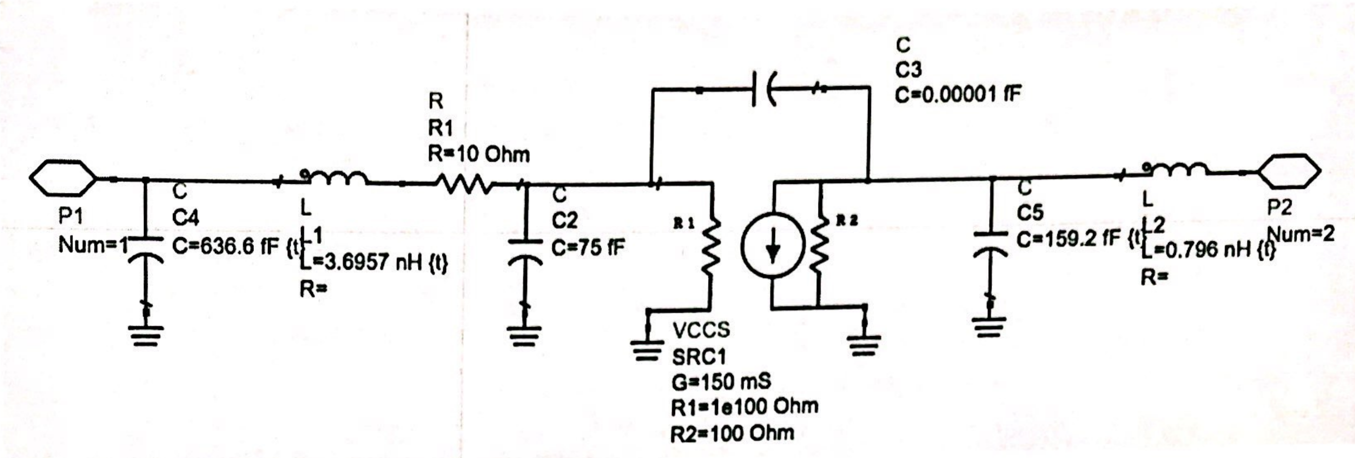
$$S_{12} = \frac{b_{1L}}{a_{2a}} = \frac{S_{12a} S_{12i} S_{12L}}{1 - S_{21i} S_{11a} S_{12i} S_{22L}} = \boxed{\frac{2.5T}{1 - 0.0125R}}$$

$$S_{21} = \frac{b_{2a}}{a_{1L}} = \frac{S_{21L} S_{21i} S_{21a}}{1 - S_{21i} S_{11a} S_{12i} S_{22L}} = \boxed{\frac{5T}{1 - 0.0125R}}$$

$$S_{22} = \frac{b_{2a}}{a_{2a}} = \frac{S_{22a}(1 - S_{21i} S_{11a} S_{12i} S_{22L}) + S_{12a} S_{12i} S_{22L} S_{21i} S_{21a}}{1 - S_{21i} S_{11a} S_{12i} S_{22L}} = \boxed{\frac{0.125T^2}{1 - 0.0125R}}$$

Jason Rivera

Problem 1, Part E:



m1	m2
freq=10.00GHz	freq=10.00GHz
S(1,1)=5.389E-5 / -9.417	S(2,2)=2.086E-4 / 136.856
impedance = Z0 * (1.000 - j1.764E-5)	impedance = Z0 * (1.000 + j2.851E-4)