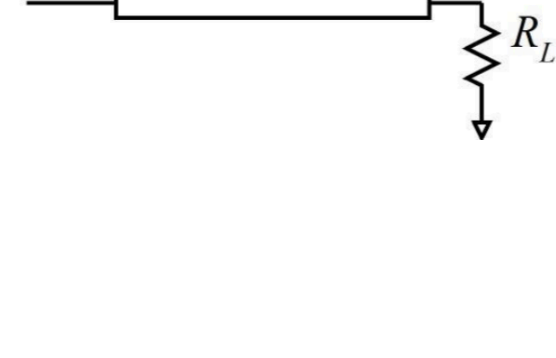


Problem Set 1 - Solution

Monday, September 27, 2021 7:49 PM

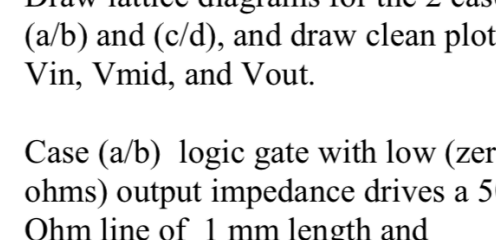
Problem 1: A transmission line has 50 Ohms characteristic impedance and a load impedance of (a) 25 Ohms (b) 50 Ohms (c) 75 Ohms. Compute in each case the voltage reflection coefficient.



$$Z_0 = 50 \Omega$$

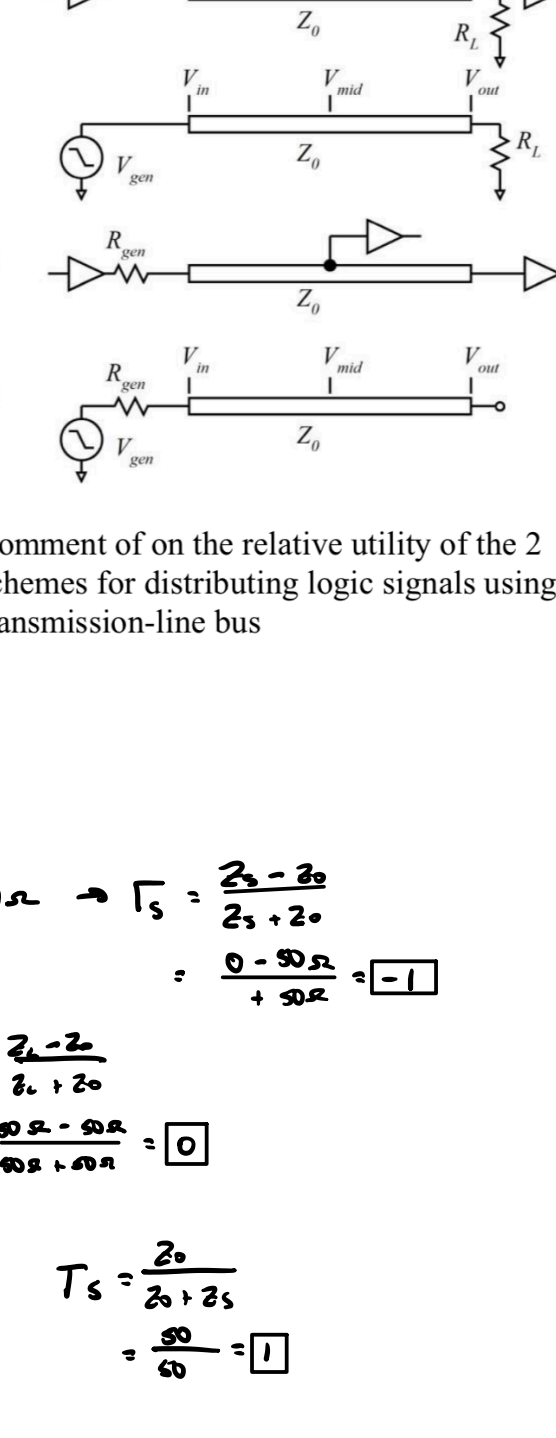
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \text{ or } \frac{Z_L - Z_0}{Z_L + Z_0}$$

- a. $Z_L = 25 \Omega$
 $\Gamma = \frac{25 - 50}{25 + 50} = -1/2$
- b. $Z_L = 50 \Omega$
 $\Gamma = \frac{50 - 50}{50 + 50} = 0$ (No reflections)
- c. $Z_L = 75 \Omega$
 $\Gamma = \frac{75 - 50}{75 + 50} = 1/6$



Case (a/b) logic gate with low (zero ohms) output impedance drives a 50 Ohm line of 1 mm length and 2.5×10^8 m/s propagation velocity. The equivalent circuit is in (b). The load impedance is 50 Ohms and Vgen is a 1V step-function.

Case (c/d) logic gate with 50 ohms output impedance drives the same line. The equivalent circuit is in (d). The load impedance is infinity and Vgen is a 1V step-function.

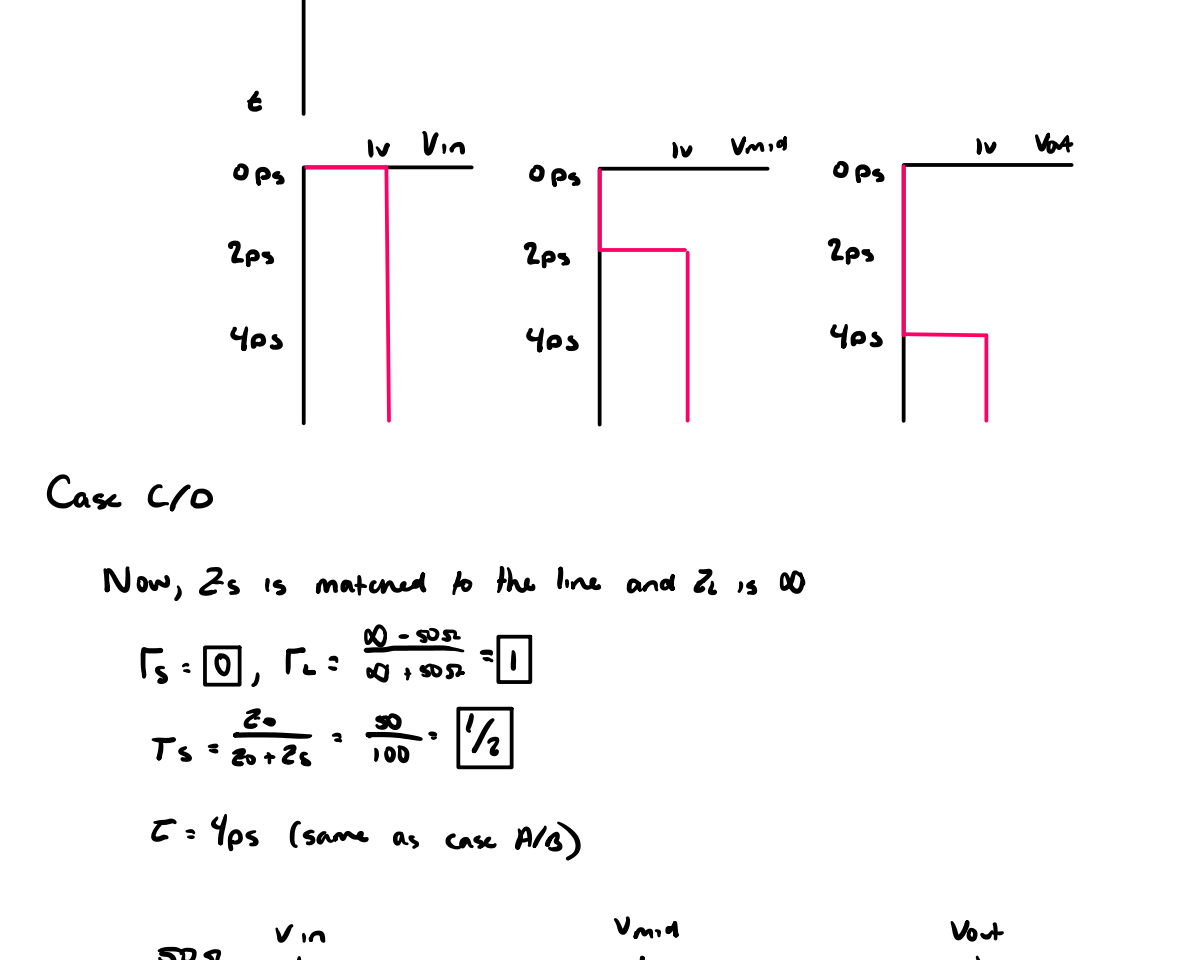


Case A/B

$V = 2.5 \times 10^8$ m/s

Logic gate is the source, $Z_s = 0 \Omega \rightarrow \Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0} = \frac{0 - 50}{0 + 50} = -1$

Load is 50 Ohm, $Z_L = 50 \Omega \rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - 50}{50 + 50} = 0$



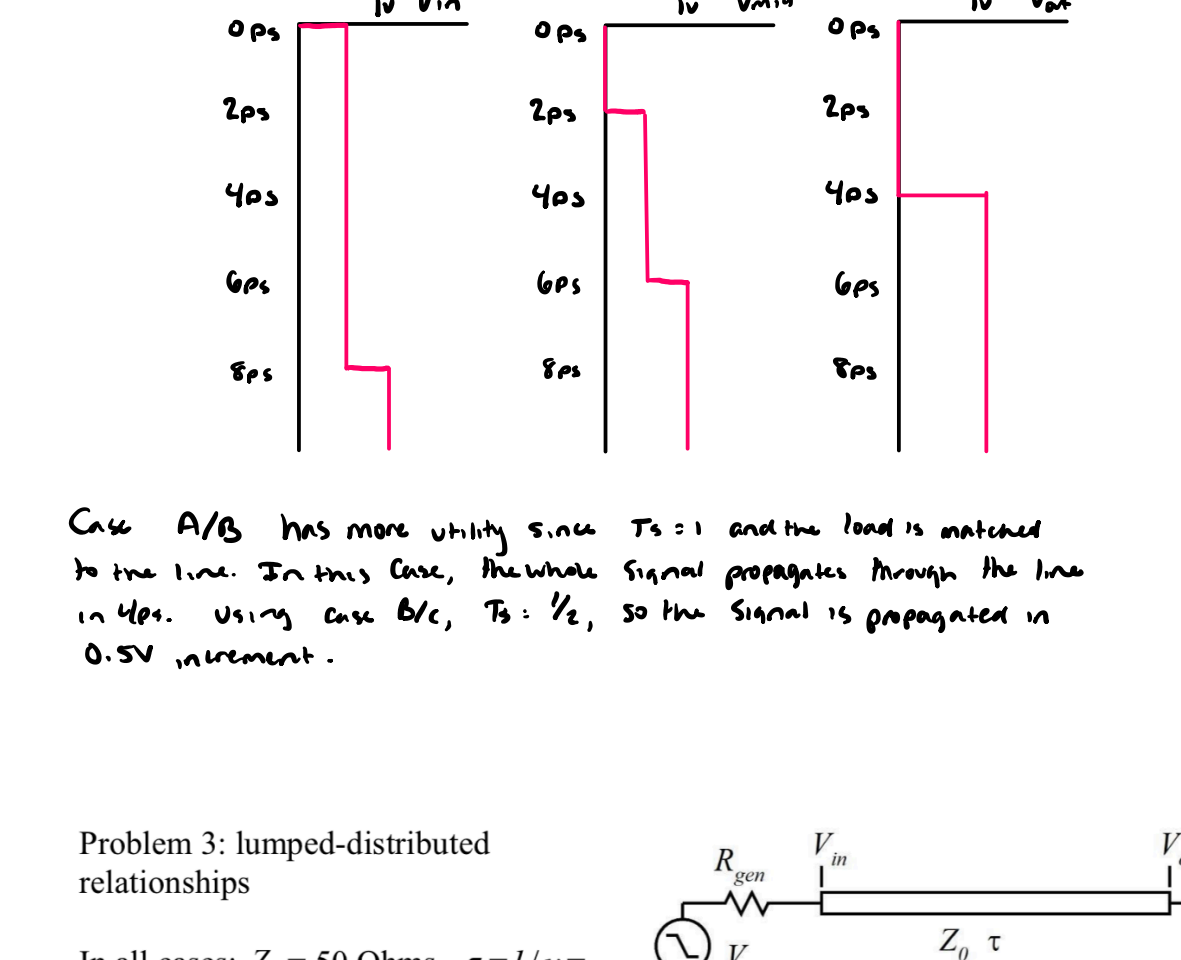
Case C/D

Now, Z_s is matched to the line and Z_L is ∞

$\Gamma_s = 0$, $\Gamma_L = \frac{\infty - 50}{\infty + 50} = 1$

$T_s = \frac{Z_0}{Z_0 + Z_s} = \frac{50}{50 + 0} = 1$

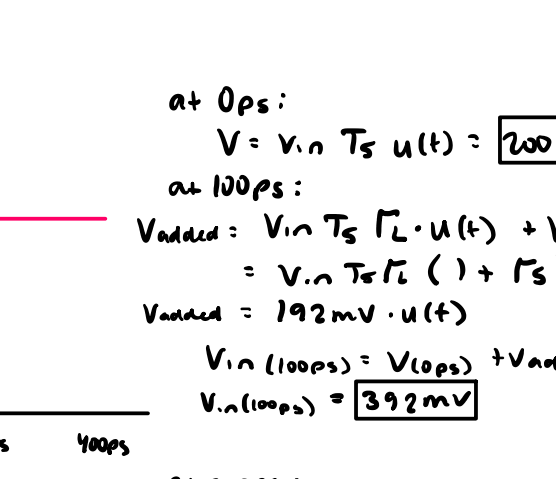
$C = 4$ ps (same as case A/B)



Case A/B has more utility since $T_s = 1$ and the load is matched to the line. In this case, the whole signal propagates through the line in 4ps. Using case B/C, $T_s = 1$, so the signal is propagating in 0.5V increments.

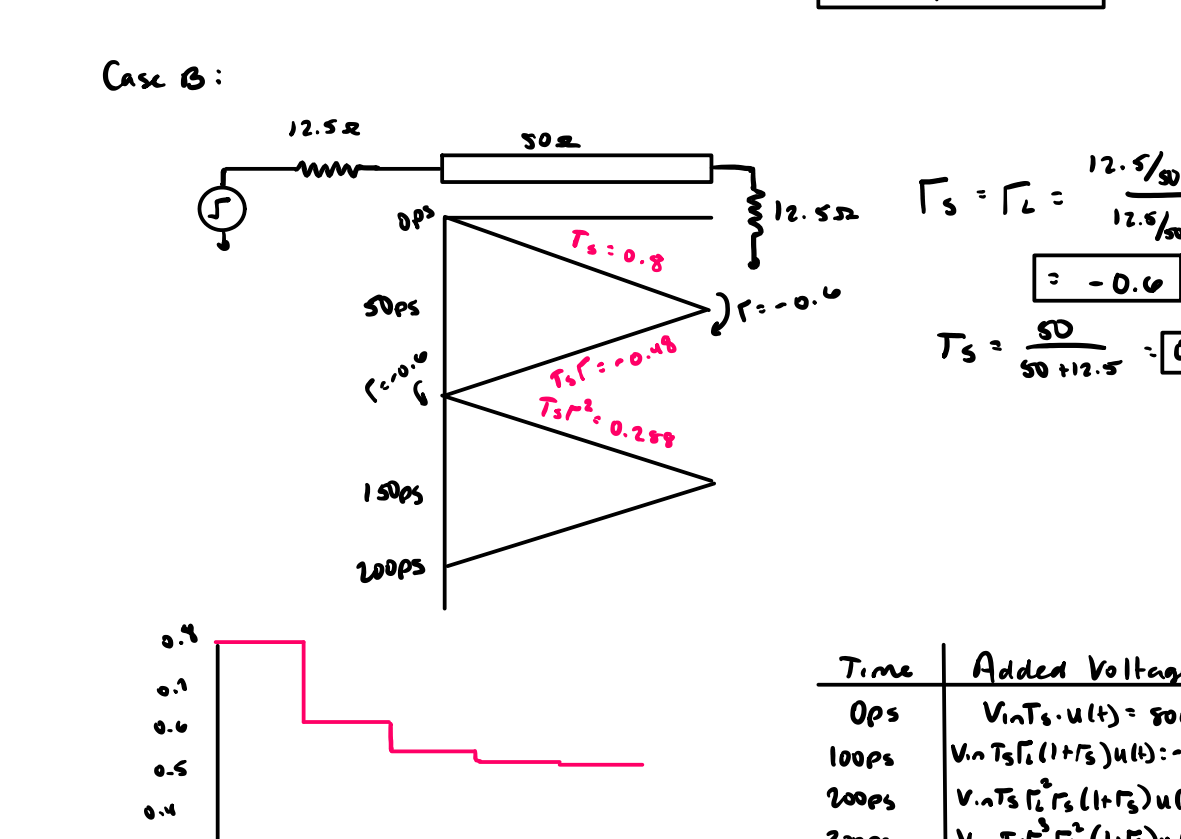
Problem 3: lumped-distributed relationships

In all cases: $Z_0 = 50$ Ohms, $\tau = l/v = 50$ ps, and V_{gen} is a 1V step-function. For cases (a,b,c), compute and plot $V_{in}(t)$ using lattice diagram methods.



Case a: $R_L = 200$ Ohms, $R_{gen} = 200$ Ohms.

Case A:



Changes like a capacitor

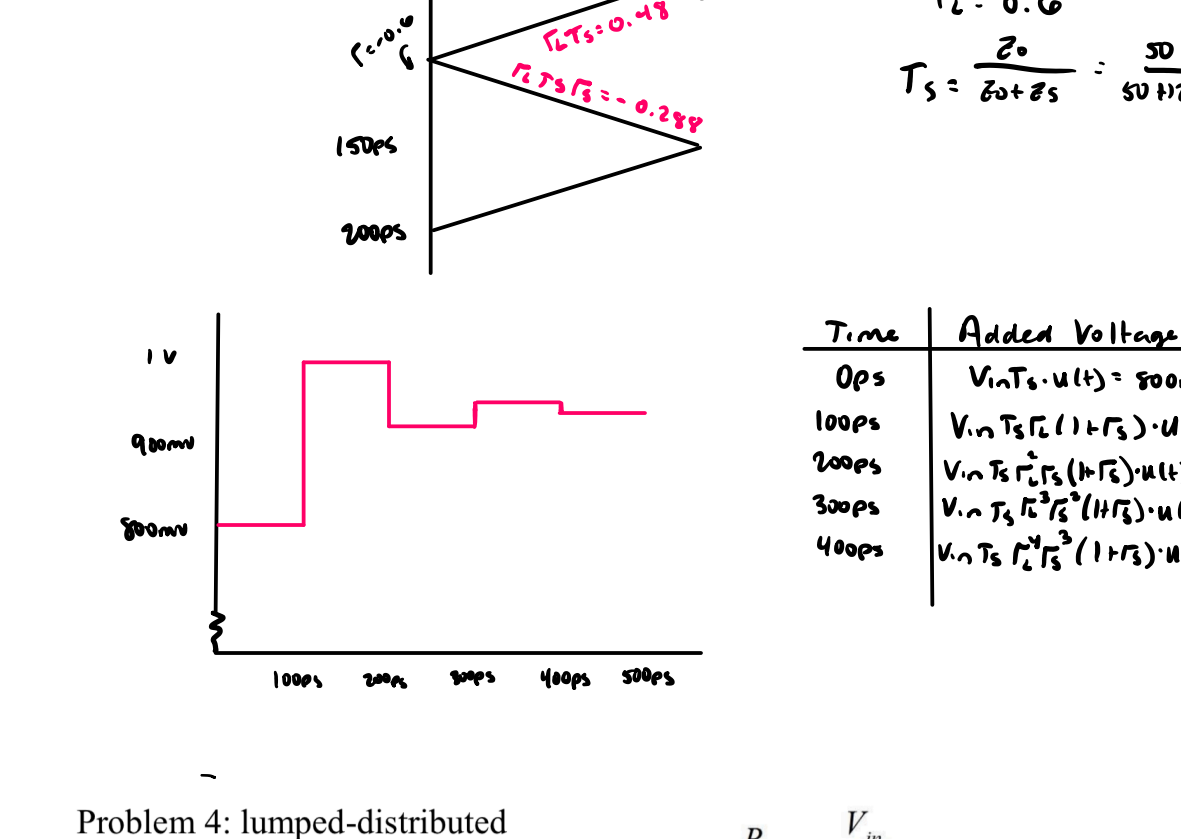
At 0ps: $V = V_{in} T_s u(t) = 200 \text{ mV} \cdot u(t)$

At 100ps: $V_{added} = V_{in} T_s \Gamma_L u(t) + V_{in} T_s \Gamma_s \Gamma_L u(t)$

At 200ps: $V_{added} = V_{in} T_s \Gamma_L^2 (1 + \Gamma_s) u(t)$

At 300ps: $V_{added} = V_{in} T_s \Gamma_L^3 \Gamma_s (1 + \Gamma_s) u(t)$

Case B:



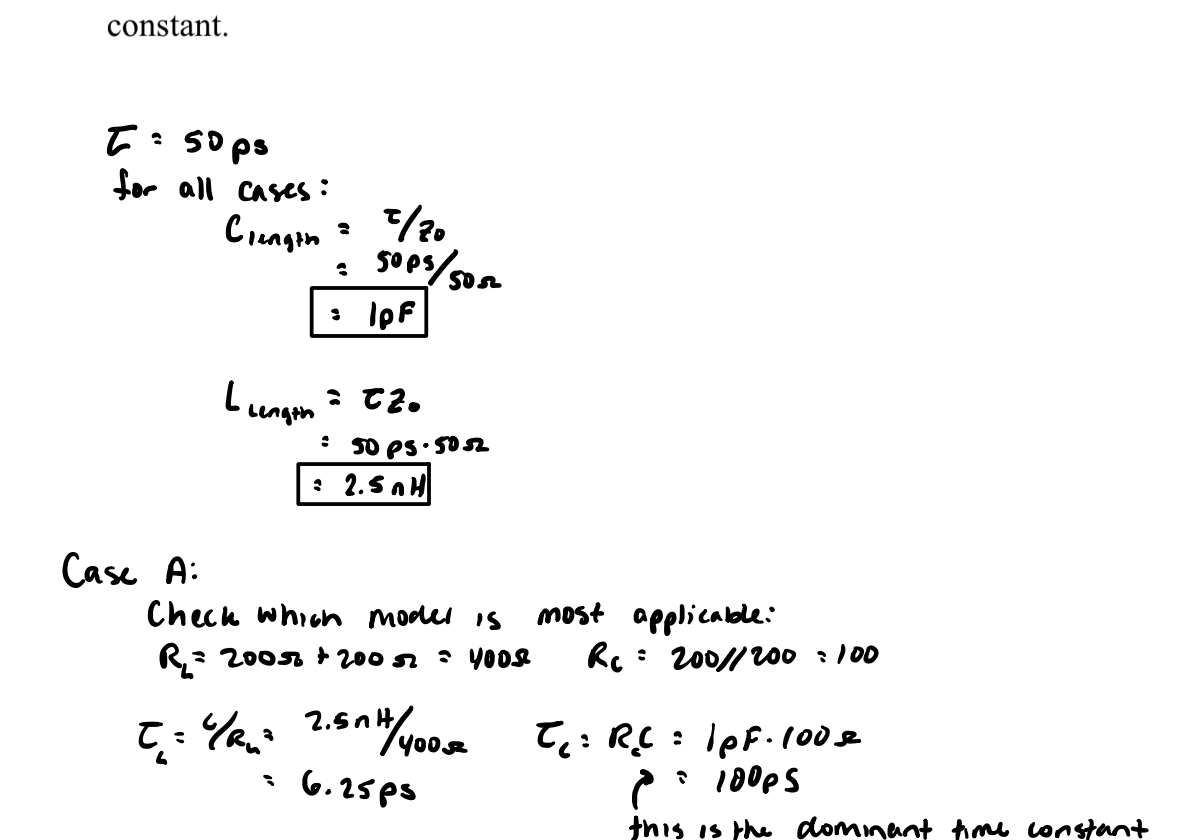
From parts A, B:

$\Gamma_L = 0.6$

$\Gamma_s = 0.6$

$T_s = \frac{Z_0}{Z_0 + Z_s} = \frac{50}{50 + 200} = 0.2$

Case C:



From parts A, B:

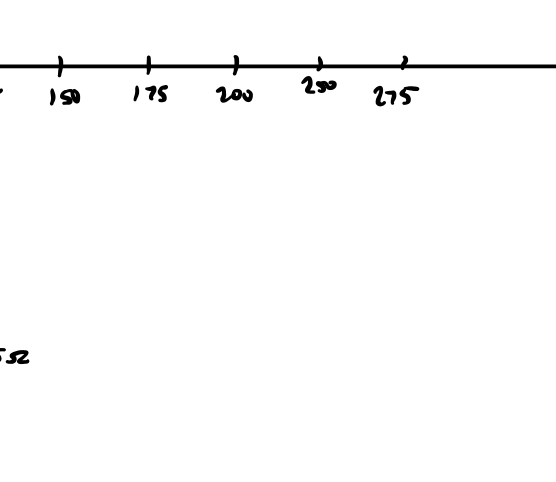
$\Gamma_L = 0.6$

$\Gamma_s = 0.6$

$T_s = \frac{Z_0}{Z_0 + Z_s} = \frac{50}{50 + 200} = 0.2$

Problem 4: lumped-distributed relationships

In all cases: $Z_0 = 50$ Ohms, $\tau = l/v = 50$ ps, and V_{gen} is a 1V step-function. For cases (a,b,c), replace the transmission-line with a T or Pi equivalent circuit, compute and plot $V_{in}(t)$ using basic circuit analysis.



In (a) and (b) first compute L/R and RC time constants, and ignore either L or C if the associated time constant is more than 10:1 less than the dominant time constant.

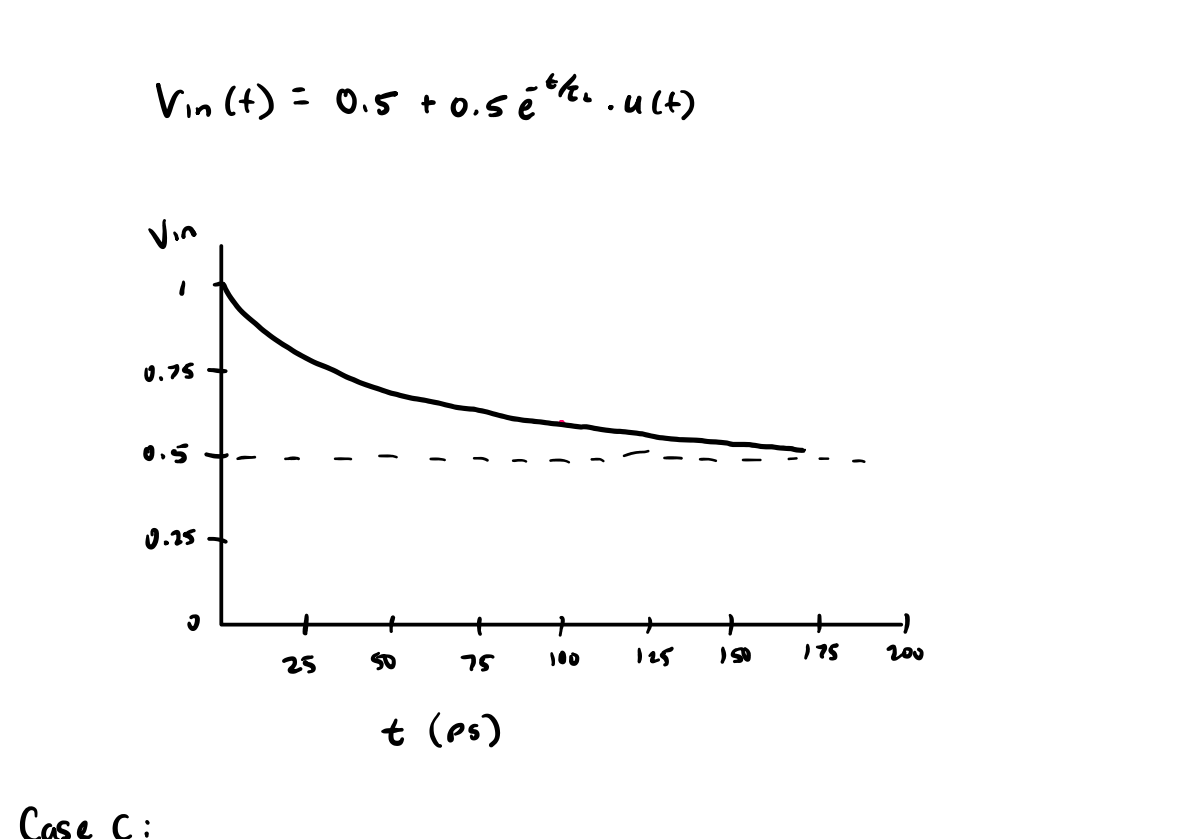
$\tau = 50$ ps

for all cases:

$C_{min} = \frac{\tau}{Z_0} = \frac{50}{50} = 1$ pF

$L_{min} = \tau Z_0 = 50 \times 50 = 2.5$ nH

Case A:



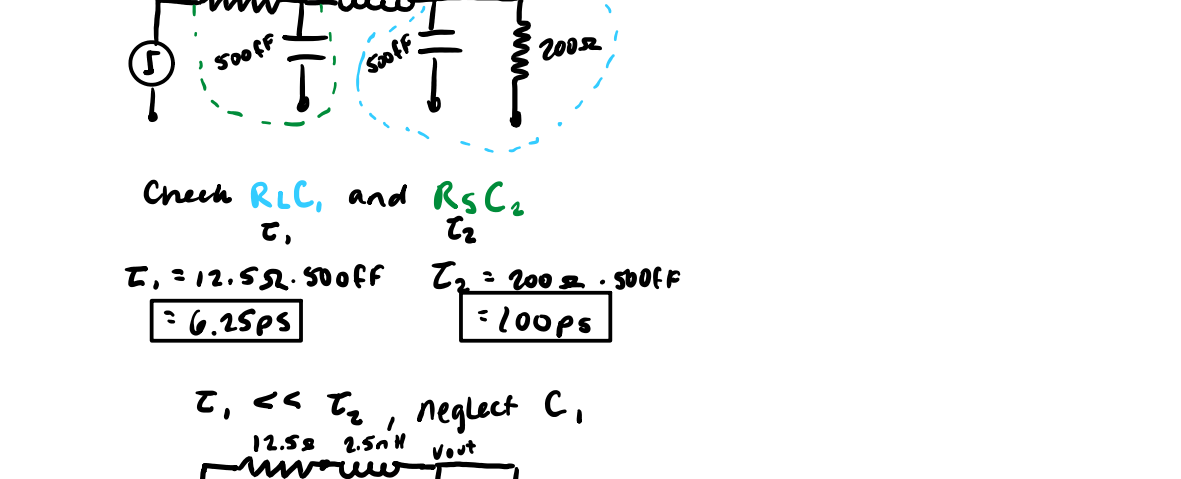
Check which model is most applicable:

$R_L = 200 \Omega + 200 \Omega = 400 \Omega$, $R_C = 200 \Omega / 200 = 100 \Omega$

$\tau_C = \frac{C}{R_C} = \frac{1 \text{ pF}}{100 \Omega} = 10 \text{ ps}$

$\tau_L = \frac{L}{R_L} = \frac{2.5 \text{ nH}}{400 \Omega} = 6.25 \text{ ps}$

Use T-model!

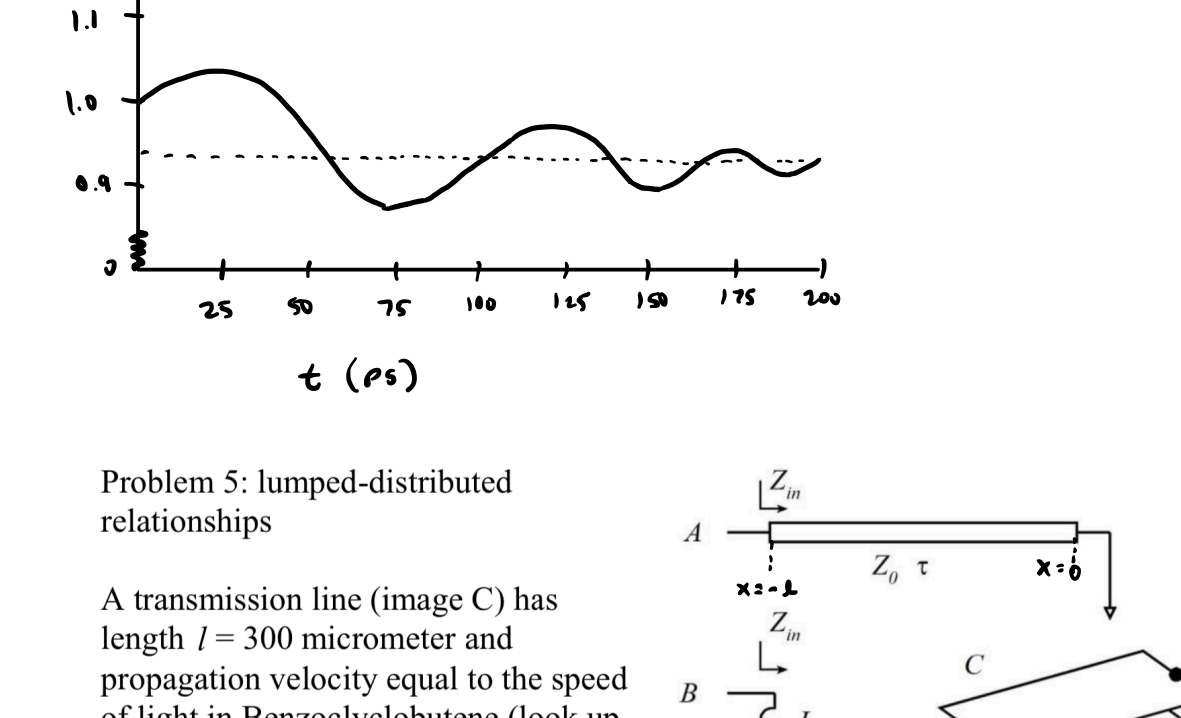


$\epsilon \rightarrow \infty$; C: OPEN

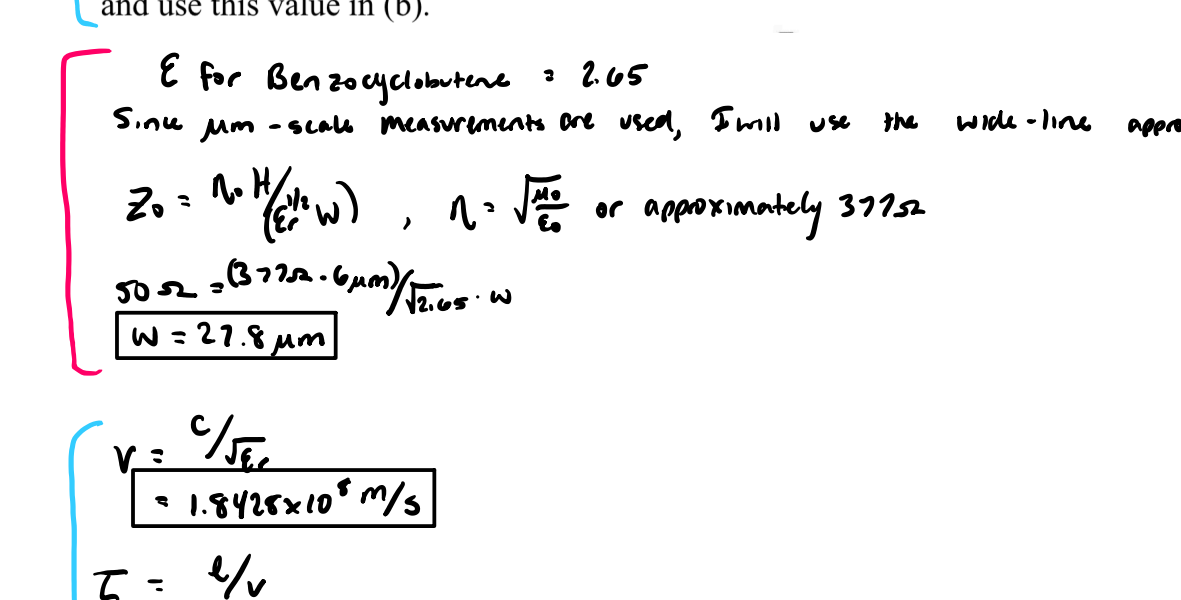
V_{in} as $\epsilon \rightarrow \infty = \frac{1V}{200 \Omega + 200 \Omega} = 0.25V$

$V_{in}(t) = 0.5V(1 - e^{-t/\tau}) \cdot u(t)$

At $t = \tau$, $V_{in} = 0.67 \cdot V_{in}(\infty) \approx 0.33$



Case B:

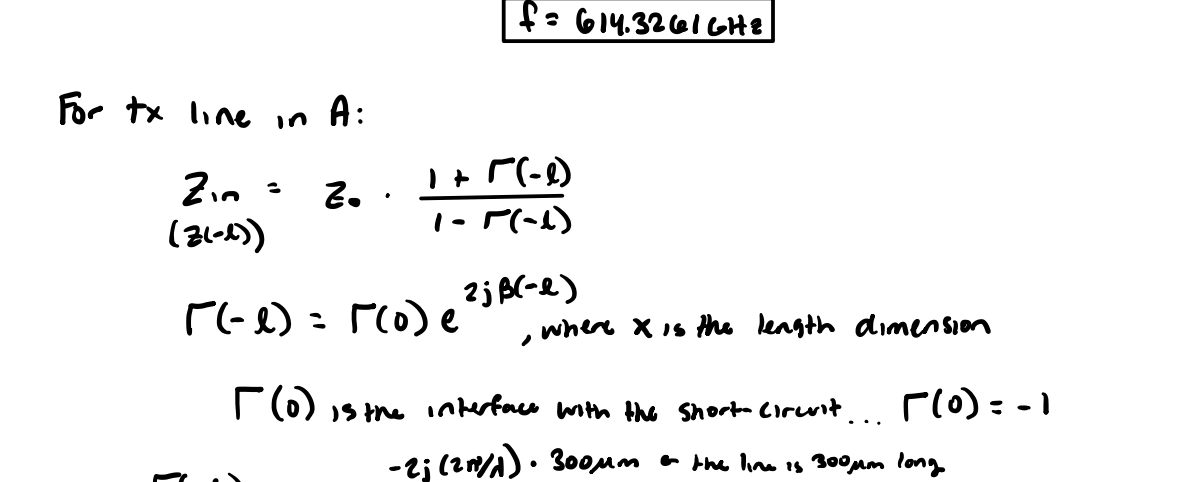


$R_L = 12.5 \Omega + 12.5 \Omega = 25 \Omega$, $R_C = 12.5 \Omega / 12.5 = 1 \Omega$

$\tau_C = \frac{C}{R_C} = \frac{1 \text{ pF}}{1 \Omega} = 1 \text{ ps}$

$\tau_L = \frac{L}{R_L} = \frac{2.5 \text{ nH}}{25 \Omega} = 100 \text{ ps}$

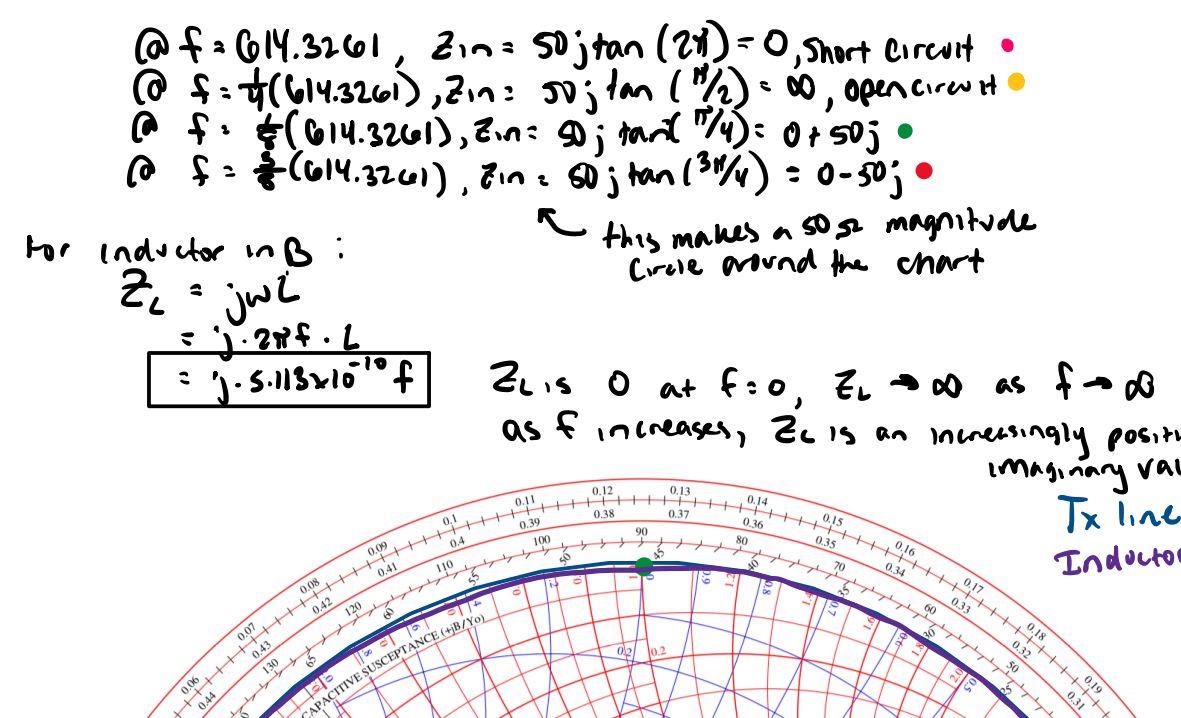
Use pi-model



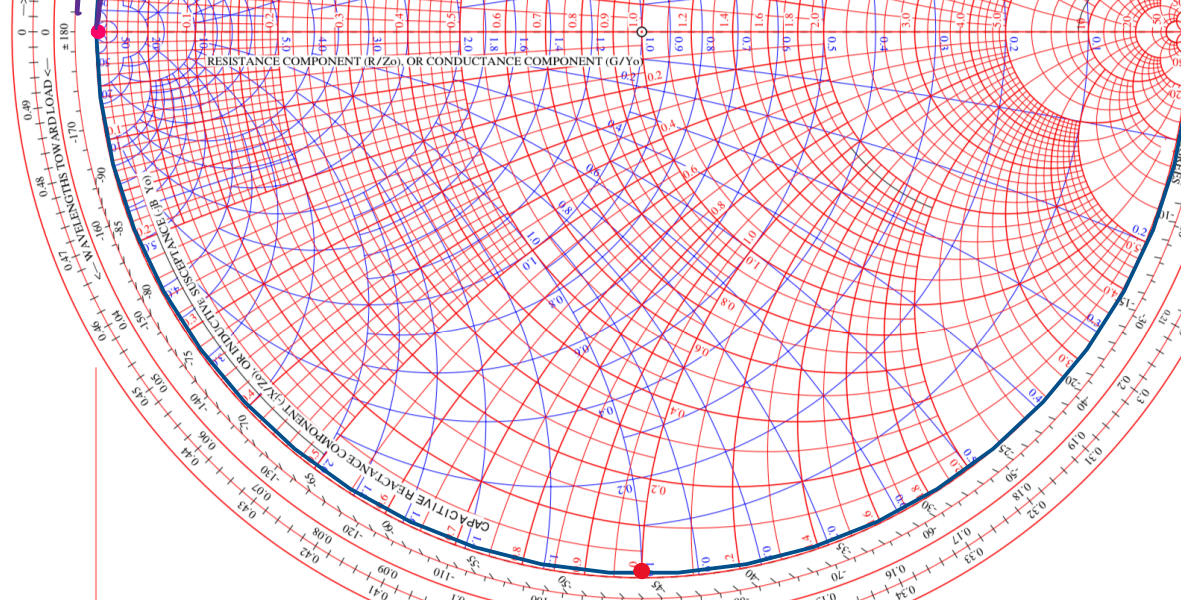
$\epsilon \rightarrow \infty$; L: short

V_{in} as $\epsilon \rightarrow \infty = V \frac{R_C}{R_C + Z_0} = 0.5V$

$V_{in}(t) = 0.5 + 0.5e^{-t/\tau} \cdot u(t)$



Case C:



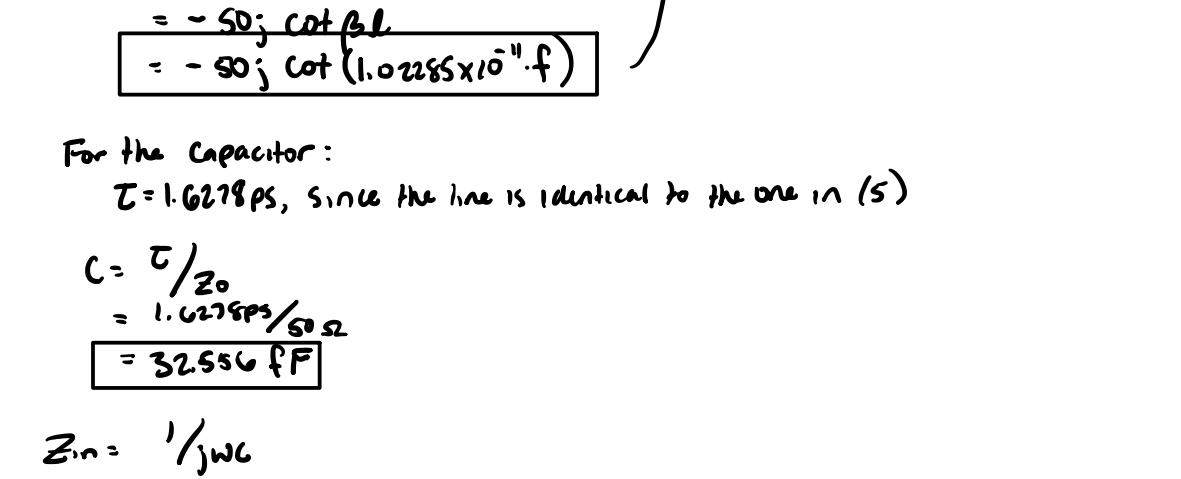
$R_L = 12.5 \Omega + 200 \Omega = 212.5 \Omega$, $R_C = 12.5 \Omega / 12.5 = 1 \Omega$

$\tau_C = \frac{C}{R_C} = \frac{1 \text{ pF}}{1 \Omega} = 1 \text{ ps}$

$\tau_L = \frac{L}{R_L} = \frac{2.5 \text{ nH}}{212.5 \Omega} = 11.76 \text{ ps}$

CAN'T NEGLECT COMPONENTS (Yes)

I will use the pi-model



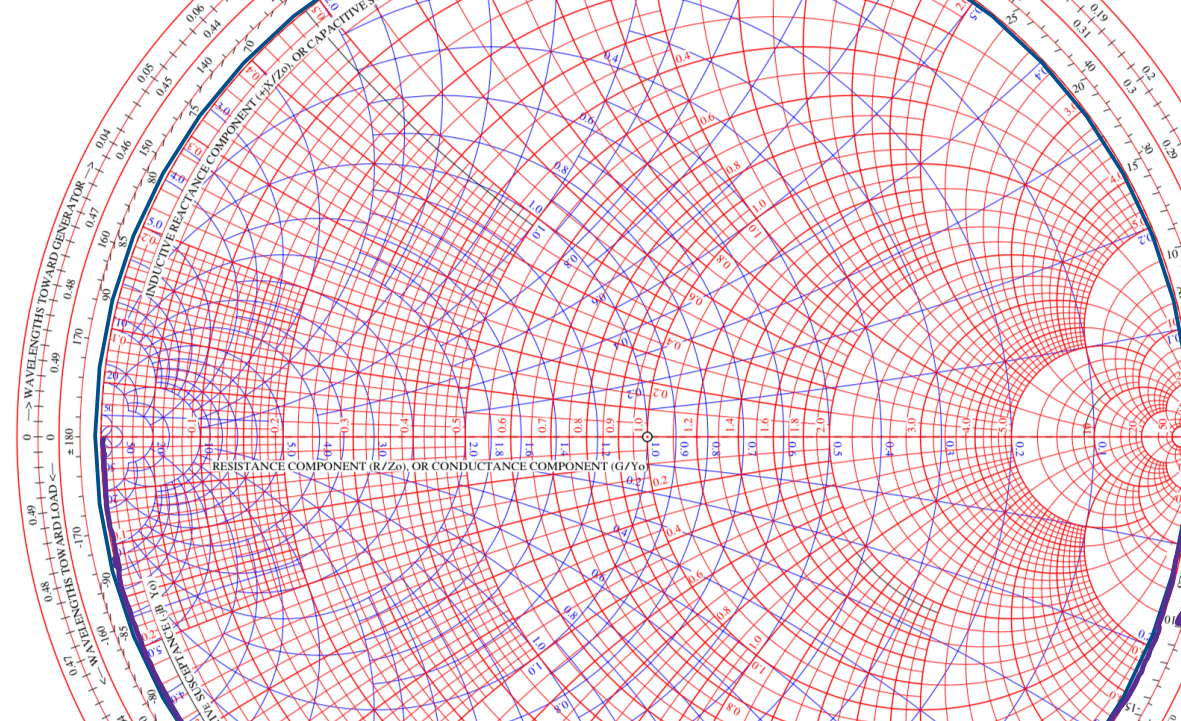
Check $R_L C$ and $R_C L$

$\tau_C = 1 \text{ ps}$, $\tau_L = 11.76 \text{ ps}$

$\tau_C \ll \tau_L$, neglect C_1

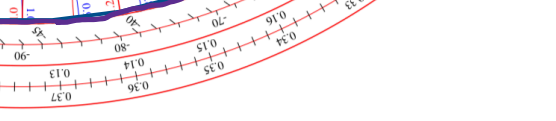
Ringing RC circuit

At $t = 0$, $V_{in} = V \frac{R_C}{R_C + Z_0} = 0.9477V$



Problem 5: lumped-distributed relationships

A transmission line (image C) has length $l = 300$ micrometer and propagation velocity equal to the speed of light in Benzocyclobutene (look up its dielectric constant!). $Z_0 = 50$ Ohms.



The plate vertical separation is 6 micrometer. Using approximate microstrip-line formulas, compute the Z_0 of the conductors.

This leads to the equivalent circuit (a). From this, compute the line inductance and use this value in (b).

ϵ for Benzocyclobutene = 2.65

Since μ_m -scale measurements are used, ϵ_{eff} will use the wide-line approx:

$Z_0 = \frac{60}{\sqrt{\epsilon_{eff}}} \ln \left(\frac{4h}{\pi a} \right)$, $\epsilon_{eff} \approx \frac{\epsilon + 1}{2}$ or approximately 372.2

$h = 3 \mu m$, $a = 2.7 \mu m$

$v = \frac{c}{\sqrt{\epsilon_{eff}}} = 1.8415 \times 10^8$ m/s

$\tau = \frac{l}{v} = 1.6278$ ps

$L = \tau Z_0 = 81.329$ pH

frequency range: DC to $\frac{1}{\tau} = 614.326$ GHz

For Tx line in A:

$Z_{in} = Z_0 \frac{1 + \Gamma(-l)}{1 - \Gamma(-l)}$

$\Gamma(-l) = \Gamma(0) e^{2j\beta l}$, where x is the length dimension

$\Gamma(0)$ is the interface with the short-circuit... $\Gamma(0) = -1$

$\Gamma(-l) = -\frac{1 - e^{-2j\beta l}}{1 + e^{-2j\beta l}}$

$Z_{in} = 50 \frac{1 - \frac{1 - e^{-2j\beta l}}{1 + e^{-2j\beta l}}}{1 - \frac{1 - e^{-2j\beta l}}{1 + e^{-2j\beta l}}}$

$Z_{in} = 50 \frac{1 - 1 + e^{-2j\beta l}}{1 + 1 - e^{-2j\beta l}} = 50 \frac{e^{-2j\beta l}}{2 - e^{-2j\beta l}}$

$Z_{in} = 50 \frac{1 - 2j\beta l}{1 - 2j\beta l} = 50 \frac{1 - 2j\beta l}{1 - 2j\beta l} = 50 \frac{1 - 2j\beta l}{1 - 2j\beta l}$

$Z_{in} = 50 \frac{1 - 2j\beta l}{1 - 2j\beta l} = 50 \frac{1 - 2j\beta l}{1 - 2j\beta l}$

for inductor in B:

$Z_{in} = j\omega L$

$Z_{in} = j\omega L = j\omega \cdot 81.329 \text{ pH}$

Z_L is 0 at $f=0$, $Z_L \rightarrow \infty$ as $f \rightarrow \infty$

as f increases, Z_L is an increasingly positive imaginary value

Tx line inductor

Problem 6: lumped-distributed relationships

The same transmission line is now open-circuited

This leads to the equivalent circuit (a). From this, compute the line capacitance and use this value in (b).

Using the Smith chart, make a quantitative graph comparing Z_0 of cases (a) and (b) over a frequency range extending from DC to $f = 1/\tau = v/l$

The line calculation is similar, except $\Gamma(x=0) = 1$ for an open circuit.

$\Gamma(-l) = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\infty - Z_0}{\infty + Z_0} = 1$

$Z_{in} = Z_0 \frac{1 + \Gamma(-l)}{1 - \Gamma(-l)} = \frac{1 + 1}{1 - 1} = \infty$

$Z_{in} = Z_0 \frac{1 + e^{-2j\beta l}}{1 - e^{-2j\beta l}} = Z_0 \frac{1 + \cos(2\beta l) - j\sin(2\beta l)}{1 - \cos(2\beta l) + j\sin(2\beta l)}$

$Z_{in} = Z_0 \frac{1 + \cos(2\beta l)}{1 - \cos(2\beta l)} = Z_0 \frac{2 \cos^2(\beta l)}{2 \sin^2(\beta l)} = Z_0 \cot^2(\beta l)$

For the Capacitor:

$Z_L = 1/j\omega C$, since the line is identical to the one in (5)

$C = \frac{\tau}{Z_0} = \frac{1.6278 \text{ ps}}{50 \Omega} = 32.556$ pF

$Z_{in} = 1/j\omega C = \frac{1}{j\omega \cdot 32.556 \text{ pF}}$

at $f = 0$, $Z_{in} = 0$ is open circuit

as $f \rightarrow \infty$, $Z_{in} \rightarrow \infty$ short circuit

as f increases, Z_{in} is a decreasing imaginary number

