Problem Set 1 - Solution

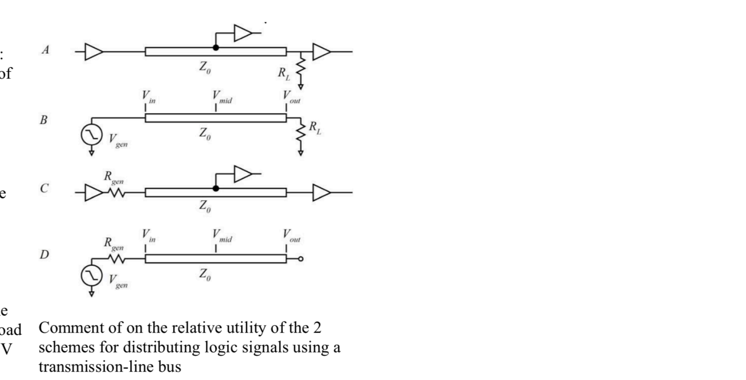
Monday, September 27, 2021 7:49 PM

Problem 1: A transmission line has 50 Ohms characteristic impedance and is load impedance of (a) 25 Ohms (b) 50 Ohms (c) 75 Ohms. Compute in each case the voltage reflection coefficient.

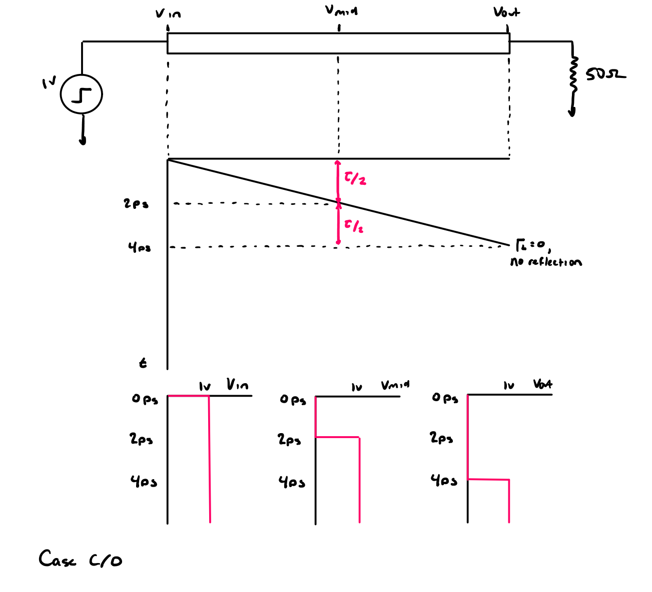
Problem 2: lattice diagrams: Draw lattice diagrams for the 2 cases (a/b and c/d), and draw clean plots of Vin, Vmid, and Vout.

Case a/b: logic gate with low (zero ohm) output impedance drives a 50 Ohm line of 1 mm length and $2.5 \cdot 10^8$ m/s propagation velocity. The equivalent circuit is at left. The load impedance is 50 Ohms and Vgen is a 1 V step-function.

Case c/d: logic gate with 50 ohm output impedance drives the same line. The load impedance is infinity and Vgen is a 1 V step function.



Case A/B



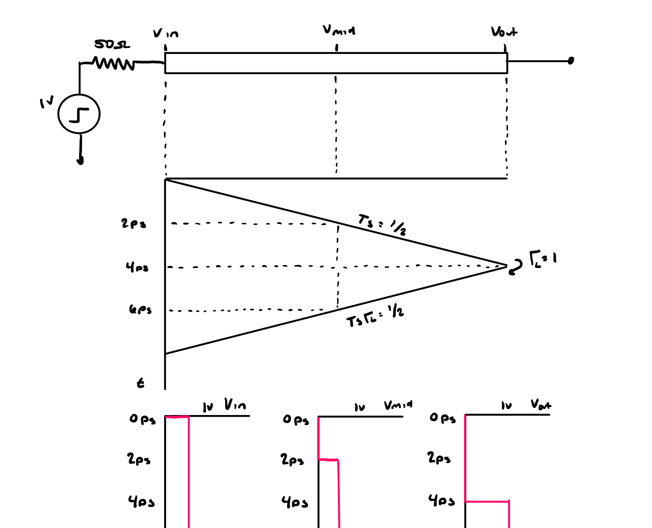
Case C/D

Now, Z_S is matched to the line and Z_L is ∞

$\Gamma_S = 0$, $\Gamma_L = \frac{\infty - 50\Omega}{\infty + 50\Omega} = 1$

$T_S = \frac{Z_0}{Z_0 + Z_S} = \frac{50}{100} = 1/2$

$\tau = 4ps$ (same as case A/B)



Case A/B has more utility since $T_S = 1$ and the load is matched to the line. In this case, the whole signal propagates through the line in 4ps. Using case C/D, $T_S = 1/2$, so the signal is propagated in 0.5V increments.

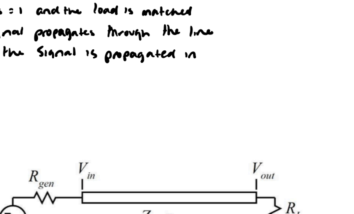
Problem 3: lumped-distributed relationships

In all cases: Z_o = 50 Ohms, $\tau = l/v$ = 50 ps, and V_{gen} is a 1 V step-function. For cases (a,b,c), Compute and plot $V_{in}(t)$ using lattice diagram methods.

Case a: R_L = 200 Ohms, R_{gen} = 200 Ohms.

Case b: R_L = 12.5 Ohms, R_{gen} = 12.5 Ohms.

Case c: R_L = 200 Ohms, R_{gen} = 12.5 Ohms.

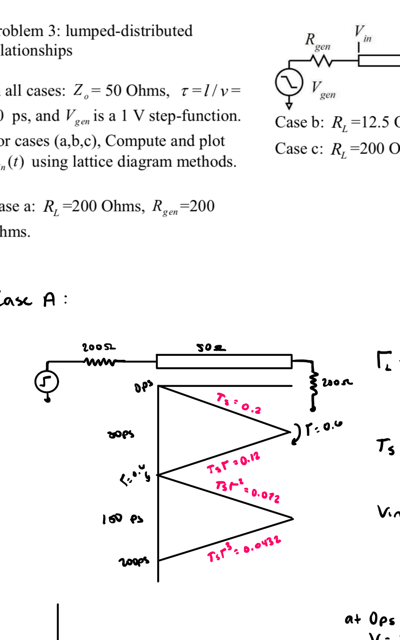


Case A:

$\Gamma_L = \Gamma_S = \frac{200/50 - 1}{200/50 + 1} = 3/5 = 0.6$

$T_S = \frac{Z_0}{Z_0 + R_S} = \frac{50}{250} = 0.2$

$V_{in} T_S \Gamma_L + V_{in} T_S \Gamma_L \Gamma_S$



Charges like a capacitor

at 0ps:
$V = V_{in} T_S u(t) = 200mV \cdot u(t)$

at 100ps:
$V_{added} = V_{in} T_S \Gamma_L \cdot u(t) + V_{in} T_S \Gamma_L \Gamma_S \cdot u(t)$
$= V_{in} T_S \Gamma_L (1 + \Gamma_S) \cdot u(t)$
$V_{added} = 192mV \cdot u(t)$
$V_{in}(100ps) = V(0ps) + V_{added}$
$V_{in}(100ps) = 392mV$

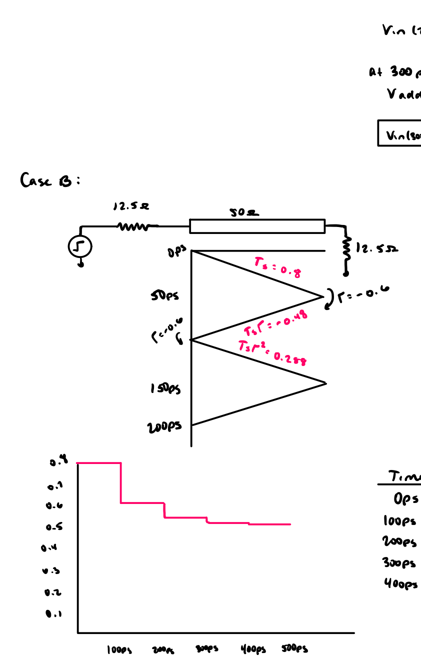
at 200ps:
$V_{added} = V_{in} T_S \Gamma_L^2 \Gamma_S (1 + \Gamma_S) \cdot u(t)$
$= 0.06912mV \cdot u(t)$
$V_{in}(200ps) = 431.13mV$

at 300ps:
$V_{added} = V_{in} T_S \Gamma_L^3 \Gamma_S^2 (1 + \Gamma_S) \cdot u(t)$
$= 0.02488mV \cdot u(t)$
$V_{in}(300ps) = 456mV$

Case B:

$\Gamma_S = \Gamma_L = \frac{12.5/50 - 1}{12.5/50 + 1} = -0.6$

$T_S = \frac{50}{50 + 12.5} = 0.8$

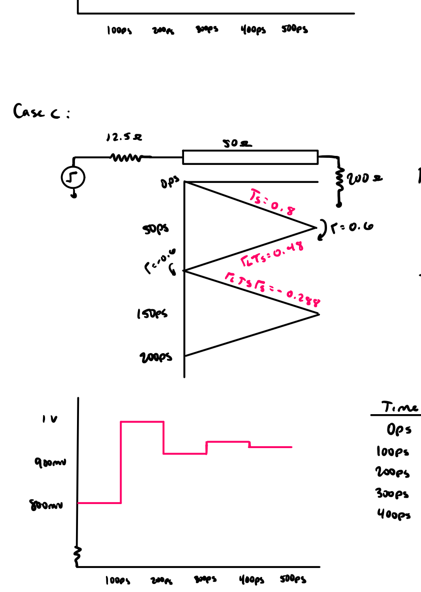


Time	Added Voltage	Vin
0ps	$V_{in} T_S \cdot u(t) = 800mV \cdot u(t)$	800mV
100ps	$V_{in} T_S \Gamma_L (1+\Gamma_S) u(t) = -192mV \cdot u(t)$	608mV
200ps	$V_{in} T_S \Gamma_L^2 \Gamma_S (1+\Gamma_S) u(t) = -0.06912 u(t)$	538.5mV
300ps	$V_{in} T_S \Gamma_L^3 \Gamma_S^2 (1+\Gamma_S) u(t) = -0.02488 u(t)$	514mV
400ps	$V_{in} T_S \Gamma_L^4 \Gamma_S^3 (1+\Gamma_S) u(t) = -0.00895 u(t)$	505mV

Case C:

From parts A, B:

$\Gamma_S = -0.6$
$\Gamma_L = 0.6$
$T_S = \frac{Z_0}{Z_0 + Z_S} = \frac{50}{50 + 12.5} = 0.8$



Time	Added Voltage	Vin
0ps	$V_{in} T_S \cdot u(t) = 800mV \cdot u(t)$	800mV
100ps	$V_{in} T_S \Gamma_L (1+\Gamma_S) \cdot u(t) = 192mV \cdot u(t)$	992mV
200ps	$V_{in} T_S \Gamma_L^2 \Gamma_S (1+\Gamma_S) \cdot u(t) = -69.12mV \cdot u(t)$	922.88mV
300ps	$V_{in} T_S \Gamma_L^3 \Gamma_S^2 (1+\Gamma_S) \cdot u(t) = 24.88mV \cdot u(t)$	947.76mV
400ps	$V_{in} T_S \Gamma_L^4 \Gamma_S^3 (1+\Gamma_S) \cdot u(t) = -8.95mV \, u(t)$	938.81mV

Problem 4: lumped-distributed relationships

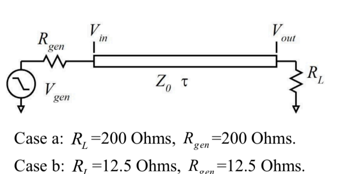
In all cases: Z_o = 50 Ohms, $\tau = l/v$ = 50 ps, and V_{gen} is a 1 V step-function. For cases (a,b,c), replace the transmission-line with a T or Pi equivalent circuit, compute and plot $V_{in}(t)$ using basic circuit analysis.

In (a) and (b) first compute L/R and RC time constants, and ignore either L or C if the associated time constant is more than 10:1 less than the dominant time constant.

Case a: R_L = 200 Ohms, R_{gen} = 200 Ohms.

Case b: R_L = 12.5 Ohms, R_{gen} = 12.5 Ohms.

Case c: R_L = 200 Ohms, R_{gen} = 12.5 Ohms.



$\tau = 50ps$
for all cases:
$C_{length} = \tau / Z_0 = 50ps / 50\Omega = 1pF$

$L_{length} = \tau Z_0 = 50ps \cdot 50\Omega = 2.5nH$

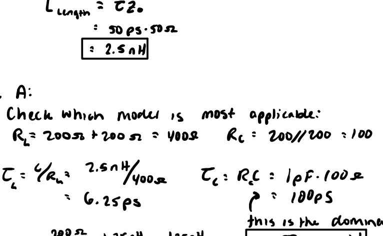
Case A:

Check which model is most applicable:
$R_L = 200\Omega + 200\Omega = 400\Omega$ $\quad R_C = 200 // 200 = 100$

$\tau_L = L/R_L = 2.5nH / 400\Omega = 6.25ps$ $\quad \tau_C = RC = 1pF \cdot 100\Omega = 100ps$

this is the dominant time constant

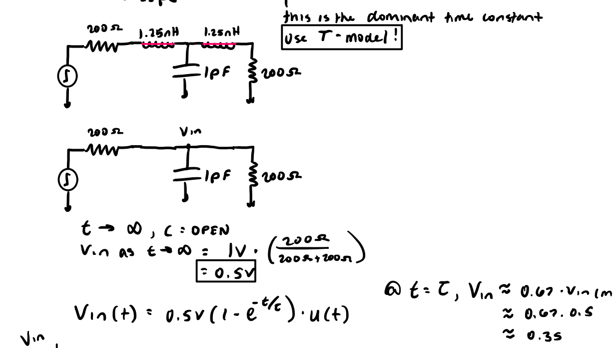
Use T-model!



$t \to \infty$, C = OPEN
V_{in} as $t \to \infty = 1V \cdot \left(\frac{200\Omega}{200\Omega + 200\Omega}\right) = 0.5V$

$V_{in}(t) = 0.5V(1 - e^{-t/\tau}) \cdot u(t)$

@ $t = \tau$, $V_{in} \approx 0.67 \cdot V_{in}(max)$
$\approx 0.67 \cdot 0.5$
≈ 0.35

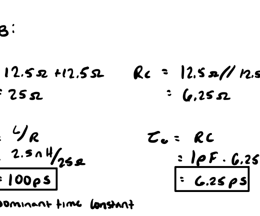


Case B:

$R_L = 12.5\Omega + 12.5\Omega = 25\Omega$ $\quad R_C = 12.5\Omega // 12.5\Omega = 6.25\Omega$

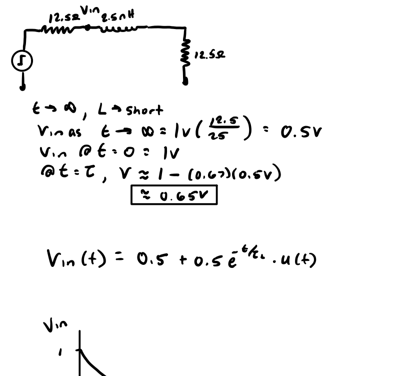
$\tau_L = L/R = 2.5nH / 25\Omega = 100ps$ (Dominant time constant) $\quad \tau_C = RC = 1pF \cdot 6.25\Omega = 6.25ps$

Use pi-model



$t \to \infty$, L = short
V_{in} as $t \to \infty = 1V \left(\frac{12.5}{25}\right) = 0.5V$
V_{in} @ $t = 0$: 1V
@ $t = \tau$, $V \approx 1 - (0.67)(0.5V)$
$\approx 0.65V$

$V_{in}(t) = 0.5 + 0.5 e^{-t/\tau_L} \cdot u(t)$

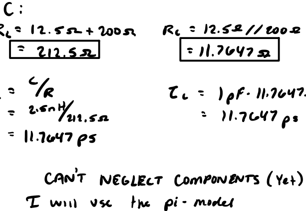


Case C:

$R_L = 12.5\Omega + 200\Omega = 212.5\Omega$ $\quad R_C = 12.5\Omega // 200\Omega = 11.7647\Omega$

$\tau_L = L/R = 2.5nH / 212.5\Omega = 11.7647ps$ $\quad \tau_C = 1pF \cdot 11.7647\Omega = 11.7647ps$

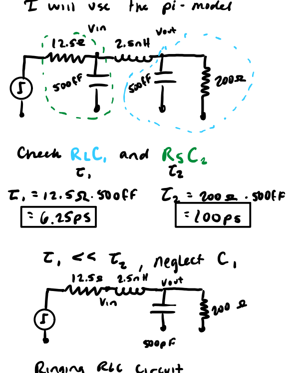
CAN'T NEGLECT COMPONENTS (Yet)
I will use the pi-model



Check $R_1 C_1$ and $R_2 C_2$

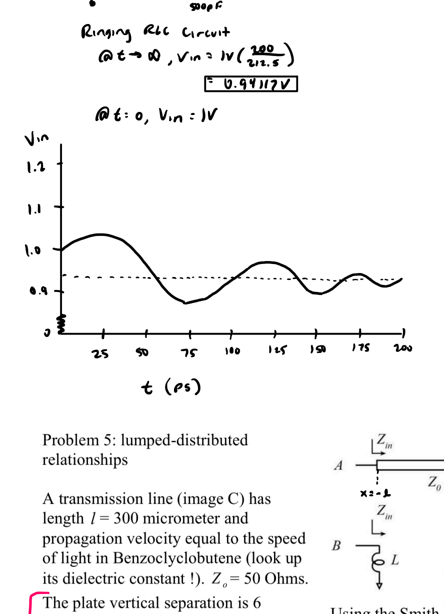
$\tau_1 = 12.5\Omega \cdot 500fF = 6.25ps$ $\quad \tau_2 = 200\Omega \cdot 500fF = 100ps$

$\tau_1 << \tau_2$, neglect C_1

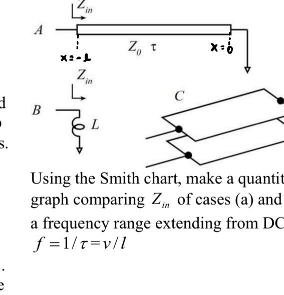


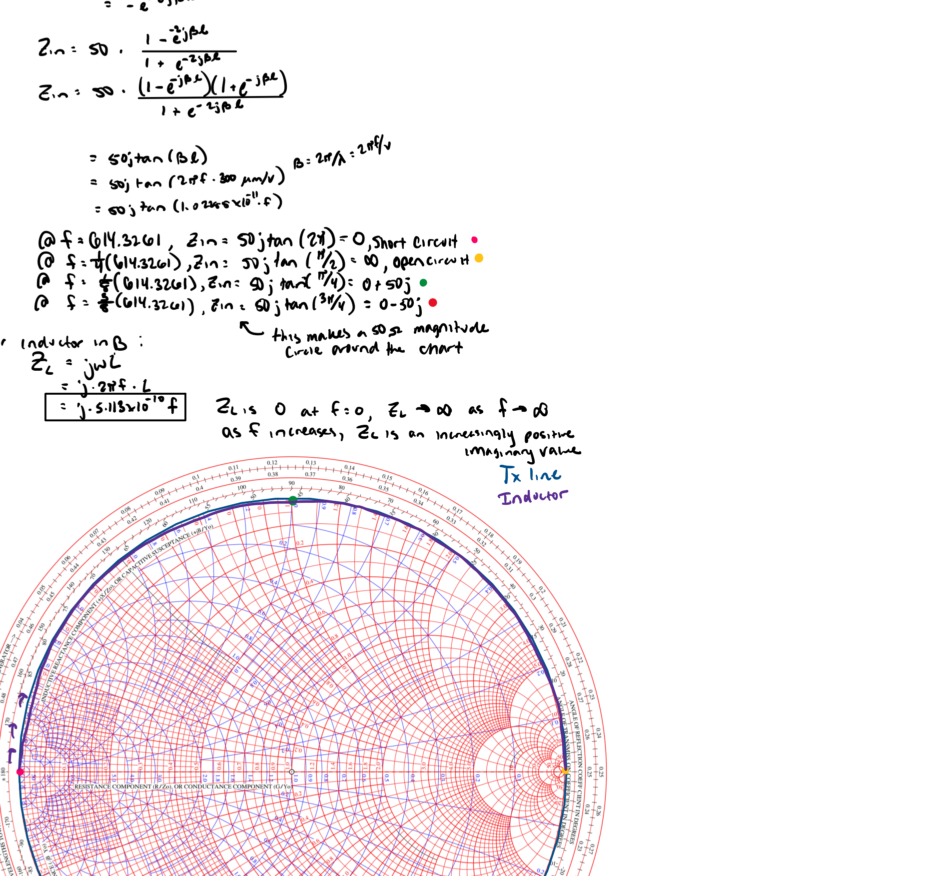
Ringing RLC Circuit
@ $t \to \infty$, $V_{in} = 1V\left(\frac{200}{212.5}\right) = 0.94117V$

@ $t = 0$, $V_{in} = 1V$



Problem 5: lumped-distributed relationships





Problem 5: lumped-distributed relationships