ECE ECE145A (undergrad) and ECE218A (graduate)
Final Exam. Monday December 6, 2021, noon - 3 p.m.

Open book. You have 3 hrs.
Use all reasonable approximations (5% accuracy is fine.) ,
AFTER STATING and justifying THEM.
<u>Think before doing complex calculations.</u> Sometimes there is an easier way.

Problem	Points Received	Points Possible
1A		5
1B		5
1C		5
1D		5
1D		5
1F		5
1G		10 (218A only)
2		10
3		10
4A		10
4B		10 (218A only)
5A		5
5B		5
total		(145A), 114 (218A)

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_s S_{11})(1-\Gamma_L S_{22}) - S_{21}S_{12}\Gamma_s\Gamma_L|^2} \qquad G_P = \frac{1}{1-\|\Gamma_{in}\|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

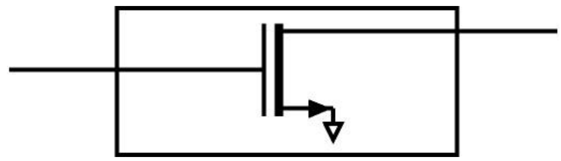
$$G_a = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2}\cdot|S_{21}|^2\cdot\frac{1}{1-\|\Gamma_{out}\|^2} \qquad G_{\max} = \frac{|S_{21}|}{|S_{12}|}\cdot\left[K-\sqrt{K^2-1}\right]\text{if } K>1$$

$$G_{MS} = \frac{|S_{21}|}{|S_{12}|}\cdot\text{if } K<1 \qquad K = \frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{21}S_{12}|} \quad \text{where } \Delta = \det[S]$$

Unconditionally stable if : (1) K>1 **and** (2) $\|\det[S]\|<1$

Problem 1, 30 points (145A), 40 points (218A)
Power Gain Definitions

part a, 5 points
At 10 GHz, the transistor has
S11=–0.5 , S21=+2, S12=0, S22=+0.5



The generator has 100 Ohms source impedance and 1 mW available power. The load is 25 Ohms.

If we place impedance-matching networks between the generator and the transistor, and between the transistor and the load, what RF powered will be deliver to the load ?

RF power delivered to the load = $64/9\ mW \doteq 71\ mW$

1 matched on in, out $\rightarrow$ MAG

1 $S_{12} = 0 \rightarrow$ unilateral

1 $$\rightarrow MAG = \frac{1}{1-\|S_{11}\|^2}\,\|S_{21}\|^2\,\frac{1}{1-\|S_{22}\|^2}$$

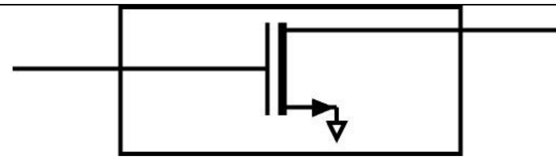
1 $$= \frac{1}{1-1/4}\quad 4\cdot\quad\frac{1}{1-1/4}$$

$$= \frac{4}{3}\cdot 4\cdot\frac{4}{3} = 2^6/9$$

1 $$P_L = MAG\cdot P_{avg} = \frac{1\,mW\cdot 2^6}{9}$$

$$= \frac{64}{9}\,mW \doteq 71.0\,mW$$

part b, 5 points

At 10 GHz, the transistor has S11=–0.5, S21=+2, S12=0, S22=+0.5 The generator has 50 Ohms source impedance and 1 mW available power. The load is 50 Ohms.	

If we directly connect the generator and load to the transistor, what RF power will be delivered to the load ?

RF power delivered to the load = 4mW

1 $Z_{gen} = Z_L = Z_0$; direct connection

2 $\rightarrow$ gain $= \|S_{21}\|^2 = 4$

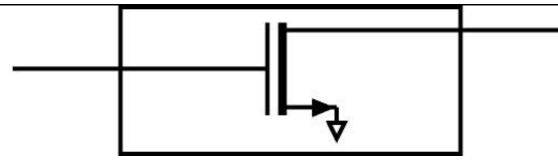
1 $P_L = 4 \cdot P_{avg}$

1 $= 4\,mW$

part c, 5 points

At 10 GHz, the transistor has
S11=–0.5, S21=+2, S12=0, S22=+0.5

The generator has 100 Ohms source impedance and 1 mW available power. The load is 25 Ohms.



If we directly connect the generator and load to the transistor, what RF power will be delivered to the load ?

RF power delivered to the load = 2.85mW

1 [no matched on either gen or load → gain = G_T]

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_s S_{11})(1-\Gamma_L S_{22}) - S_{21}S_{12}\Gamma_s\Gamma_L|^2}$$

1/2 [$\Gamma_s = \dfrac{100/50 - 1}{100/50 + 1} = \dfrac{1}{3}$]

1/2 [$\Gamma_L = \dfrac{25/50 - 1}{25/50 + 1} = \dfrac{1/2 - 1}{1/2 + 1} = \dfrac{-1}{3}$]

1 [$S_{12} = 0 \rightarrow$

$$G_T = \frac{1 - ||\Gamma_s||^2}{|1 - \Gamma_s S_{11}|^2} ||S_{21}||^2 \frac{1 - ||\Gamma_L||^2}{||1 - \Gamma_L S_{22}||^2}$$
]

1 [$$= \frac{1 - 1/4}{\left(1 + \frac{1}{3}\frac{1}{2}\right)^2} \; 4 \; \frac{1 - 1/9}{\left(1 + \frac{1}{3}\frac{1}{2}\right)^2}$$

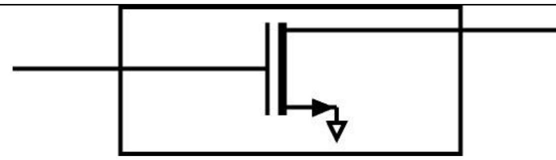
$$= \frac{3/4}{(7/6)^2} \; 4 \; \frac{8/9}{(7/6)^2} = 2.825$$
]

1 [$P_L = 2.825 \cdot 1mW = 2.825mW$.]

part d, 5 points

At 10 GHz, the transistor has
S11=–0.5, S21=+2, S12=0, S22=+0.5

The generator has 100 Ohms source impedance and 1 mW available power. The load is 25 Ohms.



If we impedance-match the generator to the transistor input, but directly connect the load to the transistor output, what RF power will be delivered to the load ?

RF power delivered to the load = 3.48mW

$$G_P = \frac{1}{1-\|\Gamma_{in}\|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

1 Impedance match on output, not input

→ gain = G_P

1/4 $\Gamma_s = \frac{100/50-1}{100/50+1} = \frac{1}{3}$

1/4 $\Gamma_L = \frac{25/50-1}{25/50+1} = \frac{1/2-1}{1/2+1} = -1/3$

1 because $S_{12}=0$, $S_{11} = \Gamma_{in}$

1 → $G_P = \frac{1}{1-\|S_{11}\|^2}\,\|S_{21}\|^2\,\frac{1-\|\Gamma_L\|^2}{|1-\Gamma_L S_{22}|^2}$

1/2
$$= \frac{1}{1-1/4}\cdot 4\,\frac{1-1/9}{(1+\frac{1}{3}\frac{1}{2})^2}$$

$$= \frac{4}{3}\cdot 4\cdot\frac{8/9}{(7/6)^2}$$

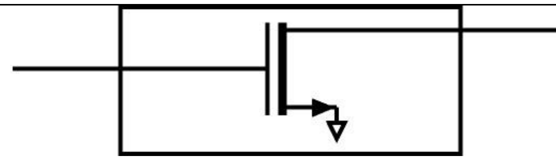
$$= 3.48$$

1 $P_L = 3.48\cdot 1\,\text{mW} = 3.48\,\text{mW}$

part e, 5 points

At 10 GHz, the transistor has
S11=–0.5, S21=+2, S12=0, S22=+0.5

The generator has 100 Ohms source impedance and 1 mW available power. The load is 25 Ohms.



If we directly connect the generator to the transistor input, but impedance-match the load to the transistor output, what RF power will be delivered to the load ?

RF power delivered to the load = 2.938 m

1 [Load matched, input not → $G = G_a$

1 [$\Gamma_{out} = S_{22}$ because $S_{12} = 0$

1/4 [$\Gamma_s = \dfrac{100/50 - 1}{100/50 + 1} = \dfrac{1}{3}$

1/4 [$\Gamma_L = \dfrac{25/50 - 1}{25/50 + 1} = \dfrac{1/2 - 1}{1/2 + 1} = -1/3$

1 [
$$G_a = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-||\Gamma_{out}||^2}$$

$$= \frac{1 - 1/4}{\left(1 + \frac{1}{3}\frac{1}{2}\right)^2} \; 4 \; \frac{1}{1 - 1/4}$$

1/2
$$= \frac{3/4}{(7/6)^2} \; 4 \cdot \frac{4}{3} = \frac{36}{49} \cdot 4$$

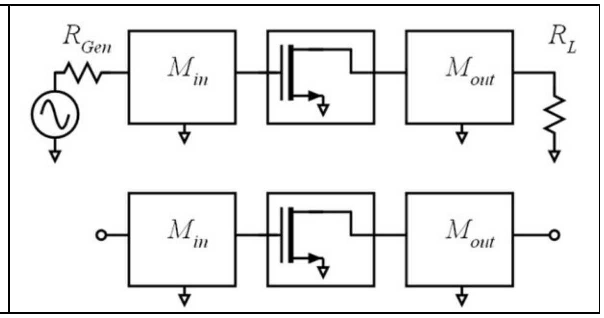
$G_a = 2.938$

1 [$P_L = G_a \cdot P_{avg} = 2.938$ mW.

part f, 5 points

At 10 GHz, the transistor has
S11=−0.5, S21=+2, S12=0, S22=+0.5

The generator has 25 Ohms source impedance and 1 mW available power. The load is 100 Ohms.



We first impedance-match the generator to the transistor input and then impedance-match the load to the transistor output (upper diagram). We then disconnect the generator and the load (lower diagram), leaving us with the transistor and its input and output networks, which we can an "amplifier".

Please find the following:

1/4 [$\Gamma_s = \frac{25/50-1}{25/50+1} = \frac{1/2-1}{1/2+1} = -1/3$

S11 of the "amplifier"= ________________

1/4 [$\Gamma_L = \frac{100/50-1}{100/50+1} = \frac{+1}{3}$

S22 of the "amplifier"= ________________

~~||S21|| of the "amplifier"=~~ ________________

2.5 [After matchin
Z_{in} must be 25Ω, Z_{out} must be 100Ω

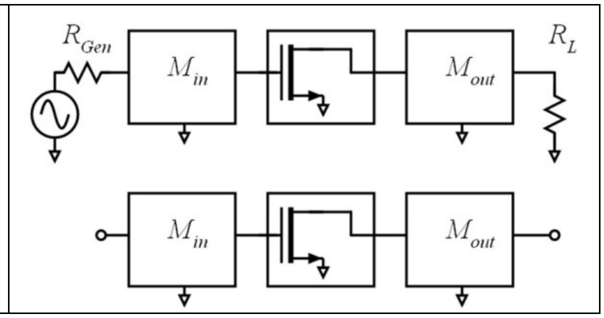
1 [so $S_{11} = \frac{25/50-1}{25/50+1} = -1/3$

1 [$S_{22} = \frac{100/50-1}{100/50+1} = +1/3$

part g, 10 points (218A only)

At 10 GHz, the transistor has
S11=–0.5, S21=+2, S12=0, S22=+0.5

The generator has 25 Ohms source impedance and 1 mW available power. The load is 100 Ohms.



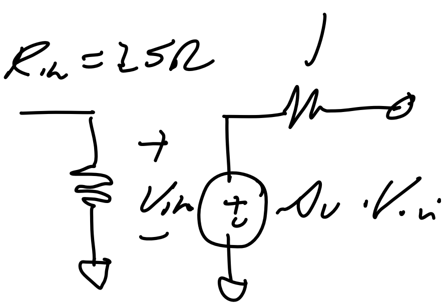
We first impedance-match the generator to the transistor input and then impedance-match the load to the transistor output (upper diagram). We then disconnect the generator and the load (lower diagram), leaving us with the transistor and its input and output networks, which we can an "amplifier".

Please find the following:

||S21|| of the "amplifier"= $16/27 = 0.59$

This will required some hard thinking

Amplifier model:



2

$$P_{avA} = (A_v V_{in})^2 / 4R_{out}$$
$$P_{in} = V_{in}^2 / R_{in}$$
$$MAG = \frac{P_{avA}}{P_{inA}} = \frac{A_v^2 R_{out}}{4/R_{in}}$$

2

But $MAG = \frac{1}{1-|S_{11}^T|^2} \|S_{21}^T\|^2 \frac{1}{1-\|S_{22}^T\|^2}$ given $S_{12}^T = 0$

$$= \frac{1}{1-1/4} \cdot 4 \cdot \frac{1}{1-1/4} = \frac{4}{3} \cdot 4 \cdot \frac{4}{3}$$
$$= 64/9$$

2

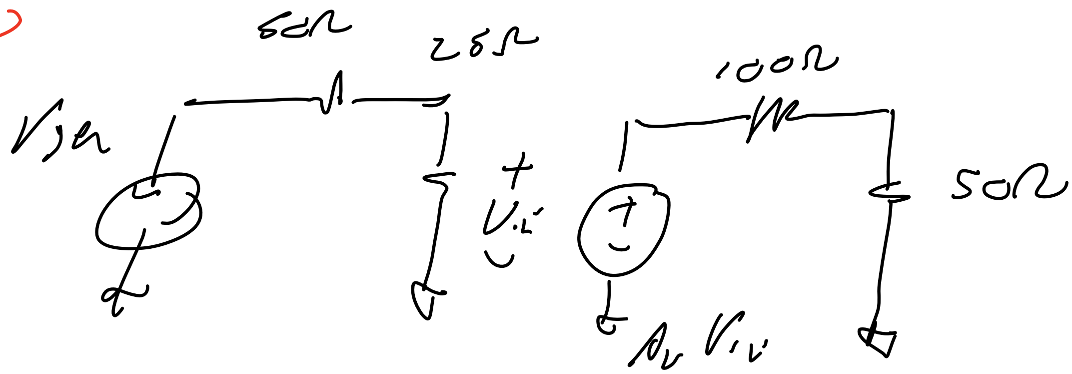
2

$$\rightarrow \frac{64}{9} = \|A_v\|^2 \frac{R_{out}}{4R_{in}} = \|A_v\|^2 \frac{100\Omega}{4 \cdot 25\Omega}$$

$$|A_v| = \sqrt{\frac{64}{9}} = \frac{8}{3}$$

now compute $\|S_{21}\|$:

2



$$|S_{21}| = \left| 2 \frac{V_o}{V_{gen}} \right|_{Z_L = Z_g = Z_0}$$

$$= 2 \cdot \frac{25\Omega}{75\Omega} \cdot \frac{8}{3} \cdot \frac{50\Omega}{150\Omega}$$

$$= 2 \cdot \frac{1}{3} \cdot \frac{8}{3} \cdot \frac{1}{3}$$

$$= \frac{16}{27}$$

$$= 0.59$$

Can also work with s parameters

27

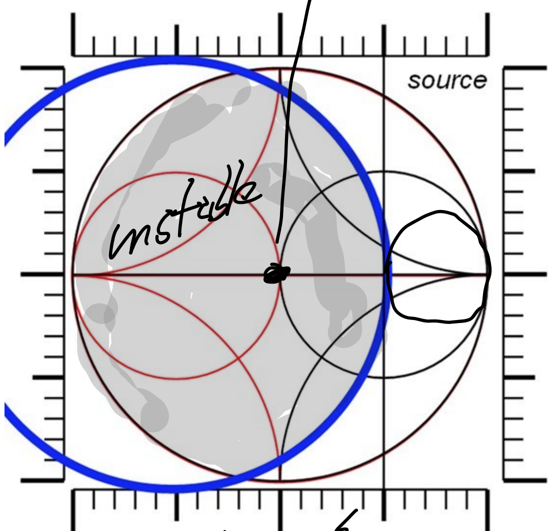
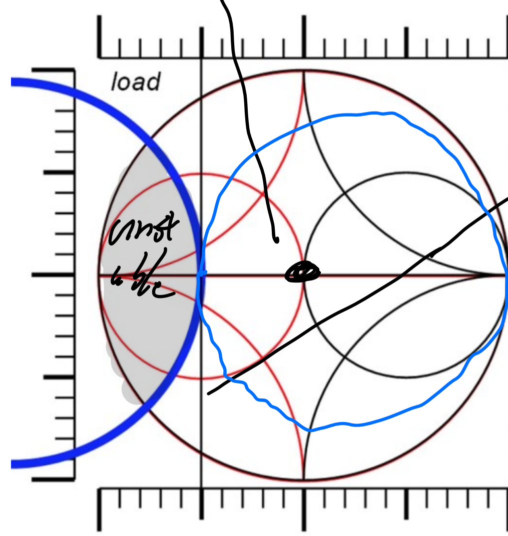
at this point, $\Gamma_s = 0$, $\Gamma_{out} = S_{22}$, $|S_{22}| >$ so UNstable!

here $\Gamma_L = 0$, $\Gamma_{in} = S_{11}$, $|S_{11}| < 1 \rightarrow$ stable

2

Problem 2, 10 points

Stabilization

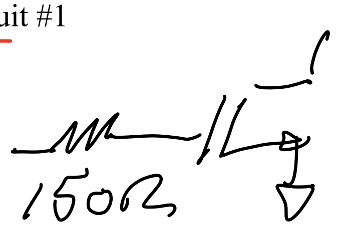
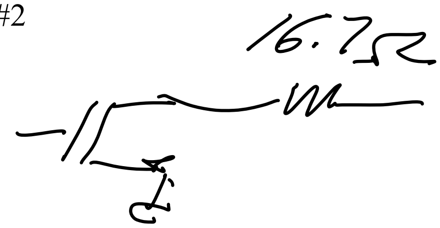
Source stability circle	load stability circle
	

$\Gamma = -1/2 \rightarrow R = 50\Omega \frac{1-1/2}{1+1/2} = 16.7$

1

$\Gamma = 1/2$; $R = Z_0 \frac{1+\Gamma}{1-\Gamma} = 50\Omega \frac{1+1/2}{1-1/2} = 150\Omega$ 1

A MOSFET in common-source mode has $\|S_{11}\| < 1$ and $\|S_{22}\| > 1$. Source and load stability circles are as shown. The Smith charts use 50 Ohms impedance normalization. Draw **2*** circuit diagrams, giving resistor values, of methods of stabilizing the transistor. ***Please draw your answers in the 2 boxes below***

circuit #1	circuit #2
2 	2 

$$\Gamma_{out} = S_{22} + \frac{S_{21}S_{12}\Gamma_s}{1 - S_{11}\Gamma_s} = S_{22} \text{ , if } \Gamma_s = 0$$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = S_{11} \text{ , if } \Gamma_L = 0$$

$$G_a = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-\|\Gamma_{out}\|^2}$$

Depends on Γ_S

2

$$G_P = \frac{1}{1-\|\Gamma_{in}\|^2} \cdot |S_{21}|^2 \cdot \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

Depends on Γ_L

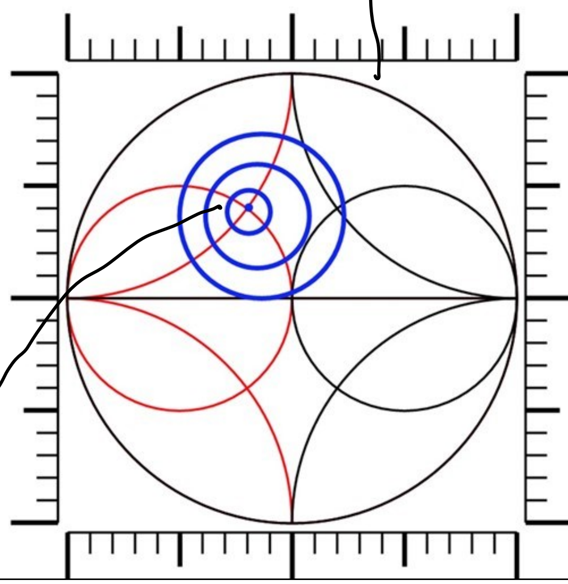
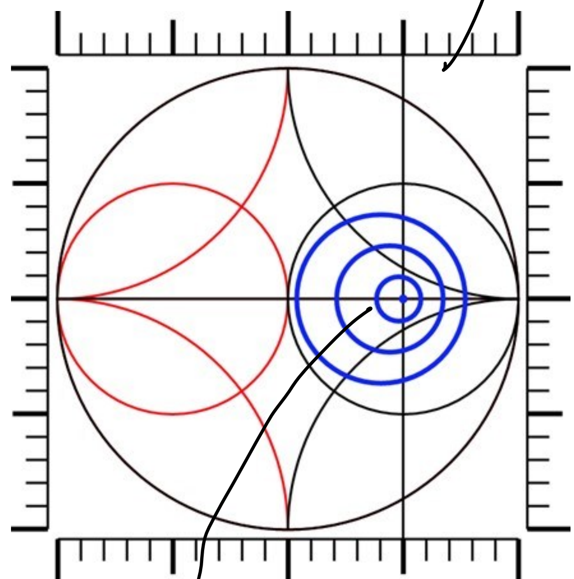
2

Γ_S plane

Γ_L plane

Problem 3, 10 points

Gain circles

Available gain circles	Operating gain circles
	

$y_S = 1 - j1$ ← 1

$\Gamma_L = 0.5 + j0$ ← 1

A FET in common-source mode has operating and available gain circles as shown (50 Ohm impedance normalization). Find the optimum generator and load impedances (in complex Ohms).

optimum source impedance= $25\Omega + j25\Omega$

optimum load impedance= $150\Omega + j0\Omega$

1

$y_{s\,opt} = 1 - j1 \rightarrow Y_{s,opt} = \frac{1}{50\Omega}(1-j1)$

$\Rightarrow Z_{s,opt} = \frac{50\Omega}{1-j1} = \frac{50\Omega(1+j1)}{(1-j1)(1+j1)} = 25\Omega + j25\Omega$

1

$\Gamma_{L,opt} = 1/2$

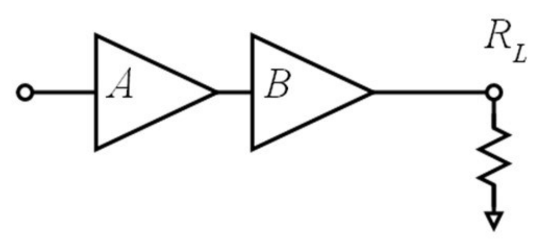
1

$\rightarrow Z_{L\,opt} = 50\Omega \frac{1+1/2}{1-1/2} = 150\Omega$

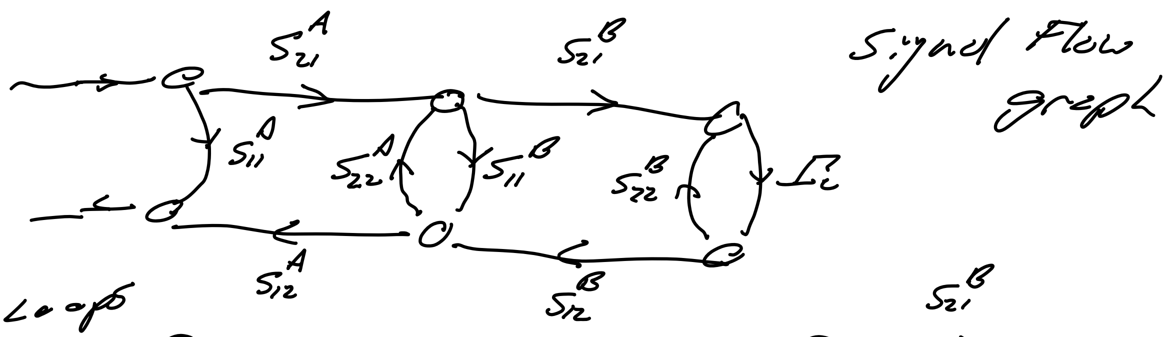
1

Problem 4, 10 points (145A), 20 points (218A)
2-port parameters and signal flow graphs

Part a, 10 points

Amplifiers A and B have S-parameters S_{ij}^{A} and S_{ij}^{B}. The output is connected to a load with reflection coefficient Γ_L. As a function of these given parameters, compute Γ_{in}, the input reflection coefficient.	

2



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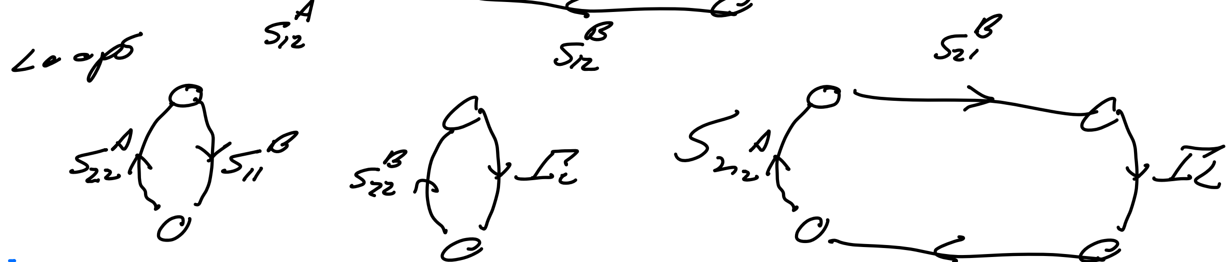
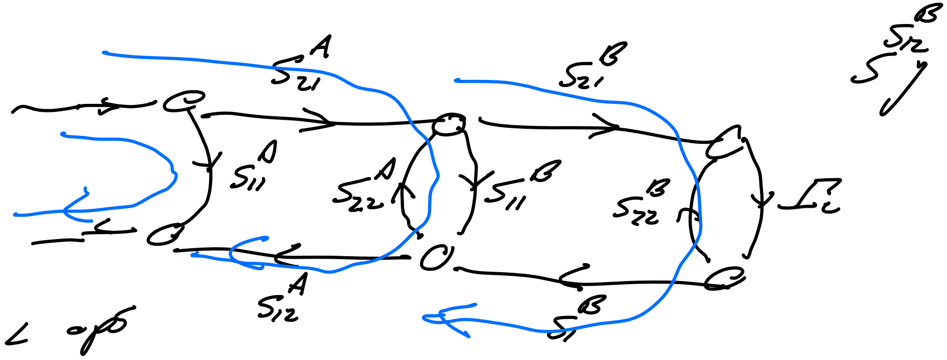


2



$$\Gamma_{in} = \frac{1}{1 - S_{11}^{B} S_{22}^{A} - S_{22}^{B}\Gamma_L + S_{11}^{B} S_{22}^{A} S_{22}^{B}\Gamma_L - S_{21}^{B} S_{12}^{B} S_{22}^{A}\Gamma_L}$$

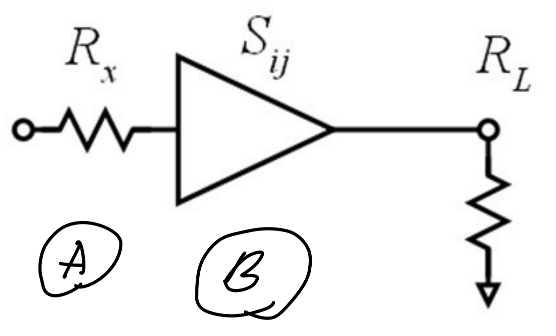
$$\times \Big[S_{11}^{A}\left(1 - S_{11}^{B} S_{22}^{A} - S_{22}^{B}\Gamma_L + S_{11}^{B} S_{22}^{A} S_{22}^{B}\Gamma_L - S_{21}^{B} S_{12}^{B} S_{22}^{A}\Gamma_L\right)$$
$$+ S_{21}^{A} S_{11}^{B} S_{12}^{A}\left(1 - \Gamma_L S_{22}^{B}\right) \qquad \text{or 4pt.}$$
$$+ S_{21}^{A} S_{21}^{B}\Gamma_L S_{12}^{A} S_{12}^{B} \Big]$$

$$\Gamma_{in} = S_{11}^{A} + \frac{S_{21}^{A} S_{11}^{B} S_{12}^{A}\left(1 - \Gamma_L S_{22}^{B}\right) + S_{21}^{A} S_{21}^{B}\Gamma_L S_{12}^{A} S_{12}^{B}}{1 - S_{11}^{B} S_{22}^{A} - S_{22}^{B}\Gamma_L + S_{11}^{B} S_{22}^{A} S_{22}^{B}\Gamma_L - S_{21}^{B} S_{12}^{B} S_{22}^{A}\Gamma_L} \qquad \text{or 4pt}$$

$$\Gamma_{in} = S_{11}^{A} + \frac{S_{21}^{A} S_{11}^{B} S_{12}^{A}\left(1 - \Gamma_L S_{22}^{B}\right) + S_{21}^{A} S_{21}^{B}\Gamma_L S_{12}^{A} S_{12}^{B}}{\left(1 - S_{11}^{B} S_{22}^{A}\right)\left(1 - S_{22}^{B}\Gamma_L\right) - S_{21}^{B} S_{12}^{B} S_{22}^{A}\Gamma_L} \qquad \text{or } \underline{\text{4pt}}$$

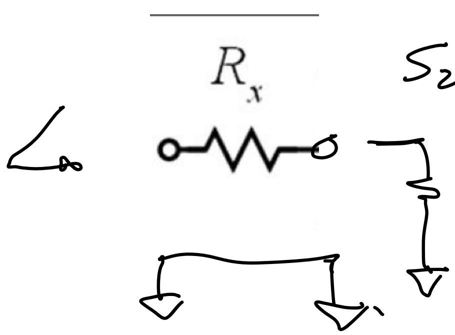
all 3 forms of the answer can be written by inspection from the graph

Part b, 10 points (218a only)

The amplifier has (50 Ohm normalization) S11=–0.5, S21=+2, S12=1/5, S22=+0.5. The load is 100 Ohms, and a resistor, Rx=50Ohms, is connected to the input. Using properties of S-parameters and signal flow graphs, find Γ_{in} of the resulting network	

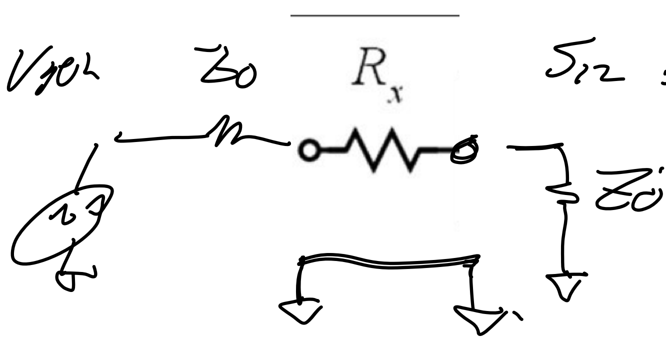
2

$Z_{in} = R_x + Z_0$



$$S_{22} = S_{11} = \frac{R_x + Z_0 - Z_0}{R_x + Z_0 + Z_0} = \frac{R_x}{R_x + 2Z_0} = \frac{50\Omega}{50\Omega + 100\Omega} = 1/3$$

2

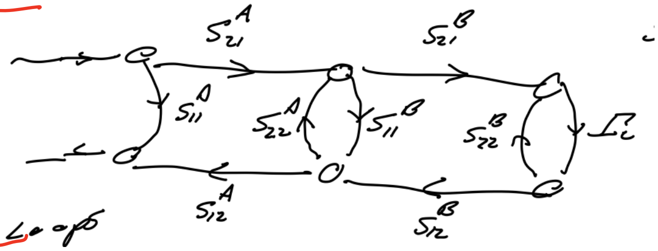


$$S_{12} = S_{21} = 2\frac{V_0}{V_g}\Big|_{Z_L = Z_S = Z_0} = \frac{2Z_0}{2Z_0 + R_x} = \frac{2(50\Omega)}{2(50\Omega) + 50\Omega} = \frac{2}{3}$$

1

$$\Gamma_L = \frac{100\Omega - 50\Omega}{100\Omega + 50\Omega} = 1/3$$

1



$$S_A = \begin{bmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix} \qquad \Gamma_L = 1/3$$

$$S_B = \begin{bmatrix} -1/2 & 1/5 \\ 2 & 1/2 \end{bmatrix}$$

3

$$\Gamma_{in} = S_{11}^A + \frac{S_{21}^A S_{11}^B S_{12}^A (1 - \Gamma_L S_{22}^B) + S_{21}^A S_{21}^B \Gamma_L S_{12}^A S_{12}^B}{(1 - S_{11}^B S_{22}^A)(1 - S_{22}^B \Gamma_L) - S_{21}^B S_{12}^B S_{22}^A \Gamma_L}$$

$$= \frac{1}{3} + \frac{2/3 \left(\frac{-1}{2}\right) \frac{2}{3} \left(1 - \frac{1}{2}\frac{1}{3}\right) + \frac{2}{3}\, 2\, \frac{1}{3}\, \frac{2}{3}\, \frac{1}{5}}{\left(1 + \frac{1}{2}\frac{1}{3}\right)\left(1 - \frac{1}{2}\frac{1}{3}\right) - 2\frac{1}{5}\frac{1}{3}\frac{1}{3}}$$

$$= \frac{1}{3} + \frac{2/3 \left(\frac{-1}{\cancel{2}}\right) \frac{\cancel{2}}{3} \left(\frac{5}{6}\right) + \frac{2}{3}\, 2\, \frac{1}{3}\, \frac{2}{3}\, \frac{1}{5}}{\left(\frac{7}{6}\right)\left(\frac{5}{6}\right) - \frac{2}{5 \cdot 9}}$$

$$= \frac{1}{3}\; \frac{\frac{-2 \cdot 5}{9 \cdot 6} + \frac{8}{27 \times 5}}{\frac{35}{36} - \frac{2}{45}} = \frac{1}{3}\; \frac{\frac{-10}{54} + \frac{8}{27 \times 5}}{\frac{35}{36} - \frac{2}{45}}$$

$$= -0.0452$$

Problem 5, 10 points
Power amplifier design

part a, 5 points

You are working in some mm-wave CMOS technology. The maximum safe current is 1 mA per micrometer of gate width. For wide bandwidth (high fmax), the maximum gate width is 1.0 micrometers; set the gate width at this value, but use multiple gate fingers to further increase maximum gate current to some desired value.

The maximum safe drain-source voltage is 1.2V, and the minimum (knee) voltage is 0.2 Volts.

What is the maximum RF power per 1 micron gate finger ? (1/8). mW

If the minimum impedance we could tune to were 10 Ohms, how many parallel gate fingers would we use in the power transistor, so that the required load impedance were 10 Ohms? 100
(please round the answer to the nearest integer)

What output power would that cell produce ? 12.5 mW.

What would be the drain efficiency ? 1/2.8 = 35.7%.

1/2 $\Delta V = 1.2V - 0.2V = 1.0V$

1 1 μm finger → I_{max} = 1 mA → $P_{max} = \frac{1}{8} \cdot 1V \cdot 1mA$ $= 1/8$ mW

1/2 $R_L = 10\Omega \rightarrow \Delta I = 1V/10\Omega = 100mA$

1 # fingers $= \dfrac{100mA}{1mA/finger} = 100$

1 $P_{total} = 100 \cdot 1/8\,mW = \frac{100}{8}\,mW = 12.5\,mW$

1 $\eta_{Drain} = \dfrac{1}{2} \dfrac{V_{max} - V_{min}}{V_{max} + V_{min}} = \dfrac{1}{2} \dfrac{1V}{1.4V} = \dfrac{1}{2.8} = 35.7\%$

1/2 $\left[g_m = 2\,mS \cdot 100 = 200\,mS \right.$

1/2 $\left[R_i = \frac{1}{g_m} = 5\,\Omega \right.$

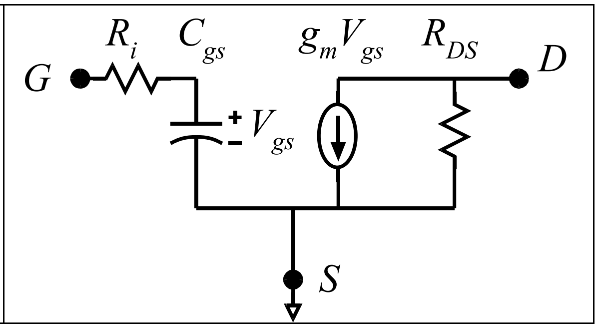
part c, 5 points

The transistor is now modelled by the the equivalent circuit to the right.

g_m=2 mS*(number gate fingers)

f_τ =100 GHz

R_{ds}=infinity, R_i=1/ g_m Ohms,



Given a 10 Ohm load, and the number of gate fingers you have found earlier, what input power at 10 GHz is necessary to produce this maximum output power ?

1/2 [Note simply:

– max pp o/p current = 100mA pp.

1 [– given $f_\tau = 100$ GHz & $f_s = 10$ GHz, $I_o / I_{in} = 10$.

1/2 → $I_{in} = 10$ mA peak-peak, 5mA peak

1/2 $= 5 \cdot \sqrt{2}$ mA RMS

1 $\boxed{P_{in} = \| I_{in,\,RMS} \|^2 \cdot R_{in}}$

1/2 $\left\{ \begin{aligned} &= \left(\frac{5\,mA}{\sqrt{2}}\right)^2 5\,\Omega \\ &= \frac{125}{2} \cdot 10^{-6}\,W = 62.5\,\mu W \end{aligned} \right.$

(unrealistically high gain)