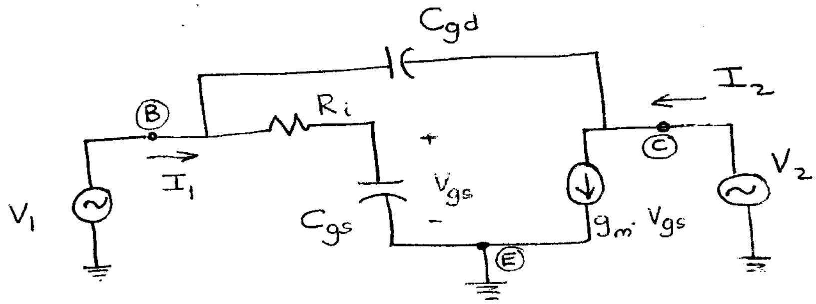
ECE 145C / 218C

HW Solution : 1

1 a)



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$\frac{1}{1+x} = 1 - x +$

$$I_1 = \frac{V_1 - V_2}{1/j\omega C_{gd}} + \frac{V_1 - V_{gs}}{R_i}$$

But $\quad \dfrac{V_1 - V_{gs}}{R_i} = \dfrac{V_{gs}}{1/j\omega C_{gd}} \quad \Rightarrow \quad V_{gs} = \dfrac{V_1}{1 + j\omega C_{gs} R_i}$

$$\therefore \; I_1 = (V_1 - V_2)\, j\omega C_{gd} + \frac{V_1}{R_i} - \frac{V_1}{R_i(1 + j\omega C_{gs} R_i)}$$

$$= V_1 \; j\omega C_{gd} + \frac{\left[(1 + j\omega C_{gs} R_i) - 1\right]}{R_i (1 + j\omega C_{gs} R_i)} - V_2\, j\omega C_{gd}$$

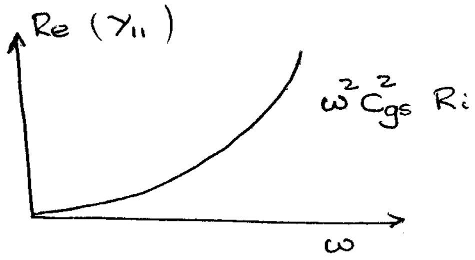
$$= V_1 \left[j\omega C_{gd} + \frac{j\omega C_{gs} R_i \cdot (1 - j\omega C_{gs} R_i)}{R_i (1 + \omega^2 C_{gs}^2 R_i^2)} \right] - V_2\, j\omega C_{gd}$$

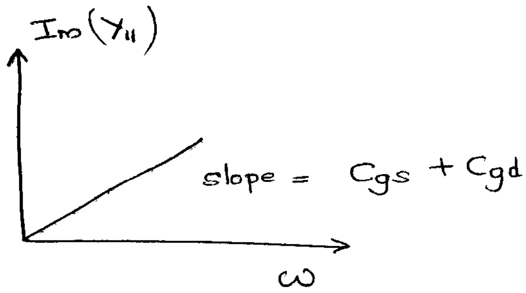
$$= V_1 \left[\omega^2 C_{gs}^2 R_i + j\omega (C_{gs} + C_{gd}) \right] - V_2\, j\omega C_{gd}.$$

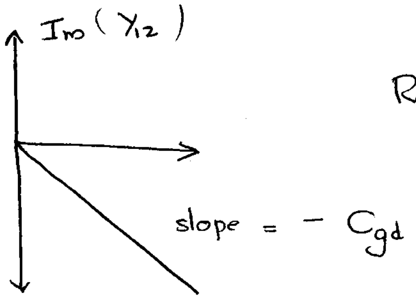
Assuming $1 \gg \omega^2 C_{gs}^2 R_i^2$.

$$Y_{11} = \omega^2 C_{gs}^2 R_i + j\omega (C_{gs} + C_{gd})$$

$$Y_{12} = -j\omega C_{gd}.$$







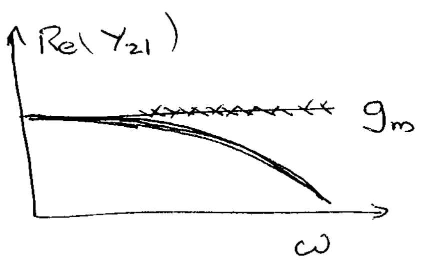
$$Re(Y_{12}) = 0 .$$

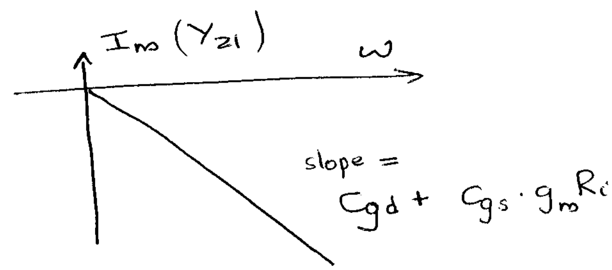
$$I_2 = g_m \cdot V_{gs} - j\omega C_{gd} (V_1 - V_2)$$

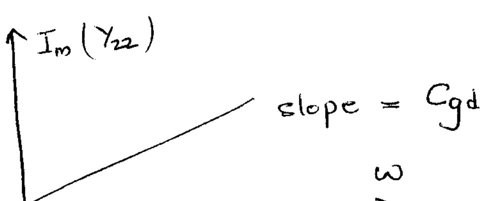
$$= g_m \cdot \frac{V_1}{1 + j\omega C_{gs} R_i} - j\omega C_{gd} (V_1 - V_2) \qquad \text{since } V_{gs} = \frac{V_1}{1 + j\omega C_{gs} R_i}$$

$$= V_1 \left(\frac{g_m (1 - j\omega C_{gs} R_i)}{1 + \omega^2 C_{gs}^2 R_i^2} - j\omega C_{gd} \right) + j\omega C_{gd} \cdot V_2$$

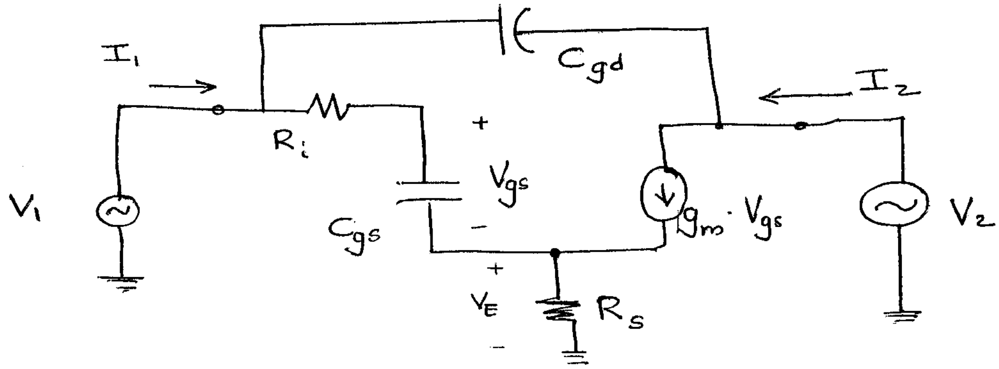
$$\approx V_1 \underbrace{\left[g_m(1 - \omega^2 C_{gs}^2 R_i^2) - j\,\omega (C_{gs} R_i g_m + C_{gd}) \right]}_{Y_{21}} + V_2 \cdot \underbrace{j\omega C_{gd}}_{Y_{22}}$$







b.



$$I_1 = (V_1 - V_2)\, j\omega C_{gd} + V_{gs} j\omega C_{gs}$$

$$V_E = \left[V_{gs} \cdot j\omega C_{gs} + g_m \cdot V_{gs}\right] \cdot R_s$$

$$V_1 = V_{gs}\left(g_m R_s + j\omega C_{gs} R_s\right) + V_{gs} + V_{gs} \cdot j\omega C_{gs} \cdot R_i$$

$$= V_{gs}\left[(1 + g_m R_s) + j\omega C_{gs}(R_s + R_i)\right]$$

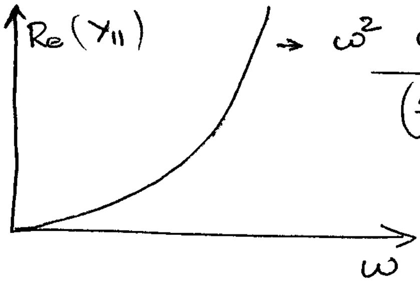
$$\therefore \quad I_1 = (V_1 - V_2)\, j\omega C_{gd} + \frac{V_1 \cdot j\omega C_{gs}}{(1 + g_m R_s) + j\omega C_{gs}(R_s + R_i)}$$

$$= (V_1 - V_2)\, j\omega C_{gd} + V_1 \cdot \frac{j\omega C_{gs}\left[(1 + g_m R_s) - j\omega C_{gs}(R_s + R_i)\right]}{(1 + g_m R_s)^2 + \omega^2 C_{gs}^2 (R_s + R_i)^2}$$

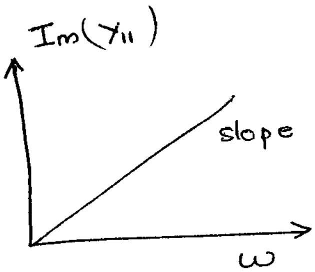
Assume $\omega^2 C_{gs}^2 (R_s + R_i)^2 \ll 1$

$$\Rightarrow \quad I_1 = V_1 \underbrace{\left[\frac{\omega^2 C_{gs}^2 (R_s + R_i)}{(1 + g_m R_s)^2} + j\left[\frac{\omega C_{gs}}{1 + g_m R_s} + \omega C_{gd}\right]\right]}_{\hookrightarrow\, Y_{11}} +$$

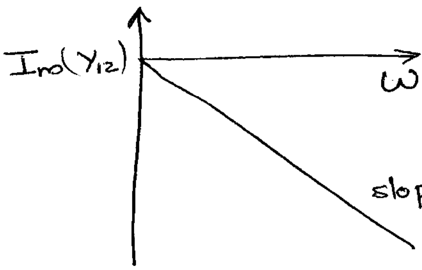
$$+ \underbrace{V_2\left(-j\omega C_{gd}\right)}_{\hookrightarrow\, Y_{12}}$$



$\rightarrow \dfrac{\omega^2 C_{gs}^2 (R_i + R_s)}{(1 + g_m \cdot R_s)^2}$



$\text{slope} = C_{gd} + \dfrac{C_{gs}}{1 + g_m R_s}$

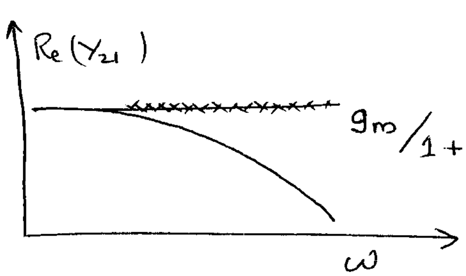


$\text{slope} = -C_{gd}$

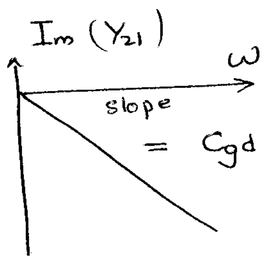
$$I_2 = (V_2 - V_1)\, j\omega C_{gd} + g_m V_{gs}$$

$$= (V_2 - V_1) \cdot j\omega C_{gd} + g_m V_1 \frac{\left[(1 + g_m R_s) - j\omega C_{gs}(R_i + R_s)\right]}{(1 + g_m R_s)^2 + \omega^2 C_{gs}^2 (R_i + R_s)^2}$$

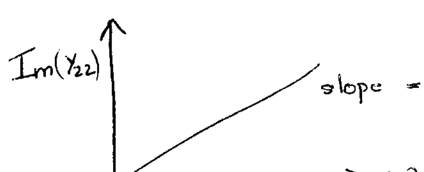
$$= V_1 \left[\underbrace{\frac{g_m \left[1 - \frac{\omega^2 C_{gs}^2 (R_i + R_s)^2}{(1 + g_m R_s)^2}\right]}{1 + g_m R_s}}_{Re(Y_{21})} - j\omega \underbrace{\left[\frac{C_{gs}(R_i + R_s) g_m}{(1 + g_m R_s)^2} + C_{gd}\right]}_{Im(Y_{21})} \right] + \underbrace{V_2 \cdot j\omega C_{gd}}_{Im(Y_{22})}$$



$g_m / 1 + g_m R_s$

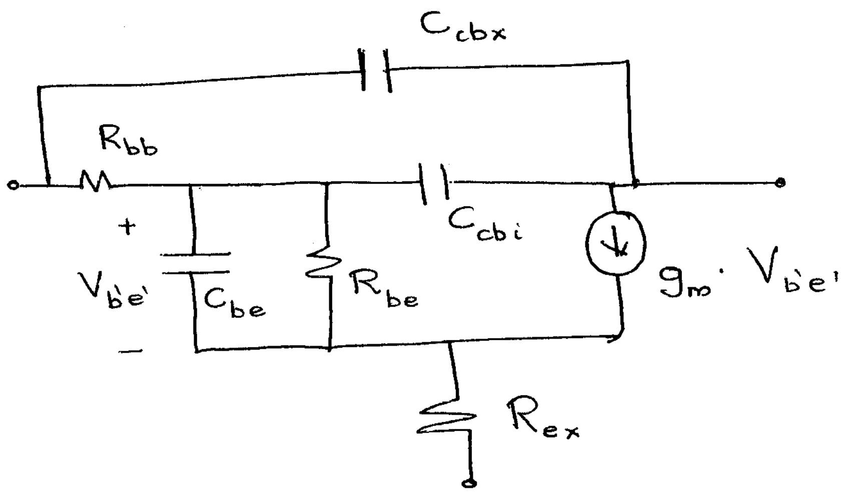


$\text{slope} = C_{gd} + \dfrac{C_{gs}(R_s + R_i)\, g_m}{(1 + g_m R_s)^2}$



$\text{slope} = C_{gd}$

2.



$I_c = 3\,mA/2$

$$g_m = \frac{I_c}{V_t} = \frac{3\,mA/2}{26\,mV} = 57\,mS$$

$\tau_f = 0.44\,ps, \quad C_{be,depl} = 17\,fF$

$$\therefore C_{be} = g_m \cdot \tau_f + C_{be,depl} = 57\times10^{-3} \times .44\times10^{-12} + 17\times10^{-15} = \underline{42\,fF}$$

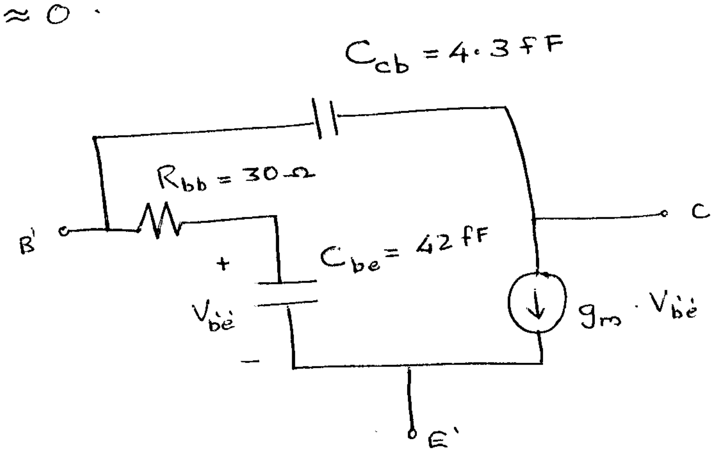
$R_{bb} = 30\,\Omega.$

$$R_{be} = \frac{\beta+1}{g_m} \approx \infty \quad \text{as } \beta\to\infty \;;$$

$C_{cbi} \approx 0. \rightarrow C_{cb} = 4.3\,fF.$

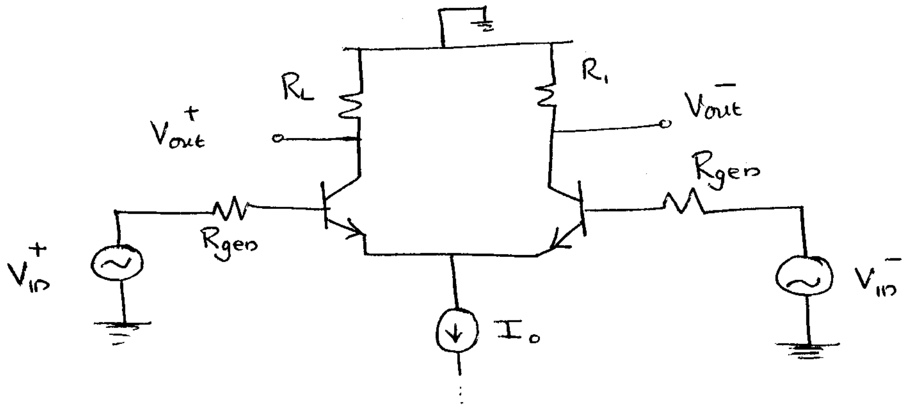
$R_{ex} = 6.67\,\Omega \approx 0.$

$\therefore$ Simplified model

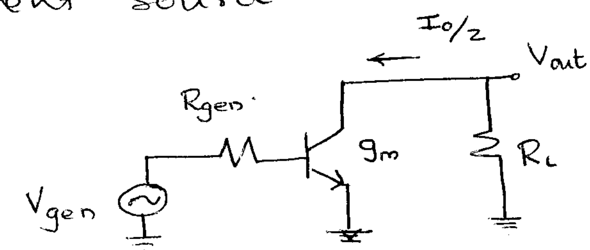


A) $I_o = 3\,mA$, $I_o \cdot R_L = 300\,mV \Rightarrow R_L = 100\,\Omega$.

$R_{gen} = R_L = 100\,\Omega$.



Using symmetry split ~~into~~ half circuit. The emitter node is at virtual gnd assuming ideal current source.



Small signal model.

$$g_m = \frac{I_c/2}{V_T} = 57\,mS.$$

Assuming $\beta \to \infty$, $Z_{in} \to \infty$ $\therefore V_{be} \approx V_{in}$

$$V_{out} = g_m \cdot V_{be} \cdot R_L \Rightarrow \frac{V_{out}}{V_{in}} = A_V = g_m \cdot R_L = 57\times 10^{-3} \times 100 = \underline{5.7}$$

Under limiting conditions, full current I_o flows through one transistor. $\Rightarrow V_{out} = I_o \cdot R_L = \underline{-300\,mV}$.

$$\tilde{g}_m = \frac{I_o}{V_T} = .12$$

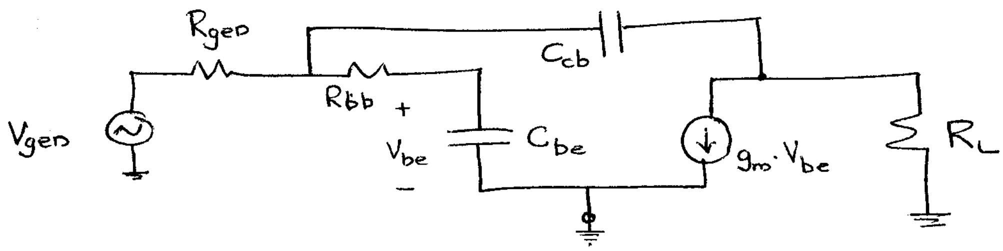
$$V_{in} = \frac{V_{out}}{\tilde{g}_m \cdot R_L} \approx 25\,mV$$

We need differential voltage of $\underline{50\,mV}$ for full switching.

High frequency analysis using MOTC. ⑦

$$\frac{V_o}{V_{gen}} = \left(Av_{midband}\right) \cdot \frac{\left(1 + s\,\tau_{zero}\right)}{\left(1 + a_1 \cdot s + a_2 s^2\right)}.$$

Using symmetry we will get half circuit as:



$$\tau_{zero} = \frac{-C_{cb}}{g_m} \approx -74 \cdot fs$$

$$a_1 = R_{11}^{o}\, C_1 + R_{22}^{o}\, C_2$$

$$C_1 = C_{be} \Rightarrow R_{11}^{o} = R_{bb} + R_{gen} = 130\,\Omega$$

$$C_2 = C_{cb} \Rightarrow R_{22}^{o} = R_{gen}\left(1 + g_m R_L\right) + R_L = 100\left(1 + 5\cdot 7\right) + 10$$

$$= 770\,\Omega$$

$$\therefore a_1 = (130\,\Omega)\cdot(42\,fF) + (770\,\Omega)\cdot(4\cdot 3\,fF)$$

$$= \underline{\underline{8\cdot 9\ ps.}}$$

0 since $V_{be} = 0$

$$a_2 = R_{11}^{o} C_1\, C_2\, R_{22}^{1} \quad ; \quad R_{22}^{1} = \left(R_{bb} \| R_{gen}\right)\left(1 - A_v\right) + R_L$$

$$= (30 \| 100)\cdot(1 - 0) + 100 = \underline{\underline{123\,\Omega.}}$$

$$\therefore a_2 = (130\,\Omega)\cdot(42\,fF)\cdot(4\cdot 3\,fF)\cdot(123\,\Omega)$$

$$= 2\cdot 9 \times 10^{-24}\ sec^2$$

Since $a_2 \ll a_1$ $\quad f_{3dB} \approx \dfrac{1}{2\pi a_1} = \underline{\underline{18\ GHz.}}$

$$A(s) = \frac{-5\cdot 7\left[1 - \left(74 \times 10^{-15}\right) s\right]}{1 + \left(8\cdot 9 \times 10^{-12}\right)\cdot s + \left(2\cdot 9 \times 10^{-24}\right)\cdot s^2}$$

b). The given circuit is a differential amplifier which we discussed earlier preceeded by an emitter follower [Voltage follower].

$V_{in} = \frac{V_{gen}}{2}$ as $R_{gen} = R_{bias}$.

$A_{V\,\text{emitter follower}} \approx 1$ (Assuming infinite β).

$A_{V\,\text{diff. amp}} \approx -g_m \cdot R_L = -\frac{I_{cs}}{2V_T} \cdot R_L$

$\therefore V_{out} = -V_{in} \cdot A_{V_{EF}} \cdot A_{\text{diff amp}} = -\frac{V_{gen}}{2} \cdot 1 \cdot \frac{I_{cs}}{2V_T} \cdot R_L$

$$\therefore A_V = -\frac{I_{cs} \cdot R_L}{4V_T}$$

Assuming the values used in (a) part

$A_V = -2.85$