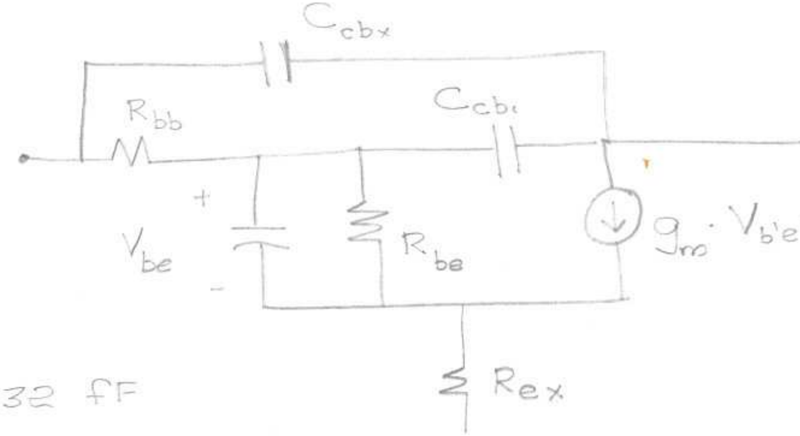
ECE 145C / 218C

HW 4 Solutions

1.



$C_{je} = 32$ fF

$\beta = 50$

$C_{cbi} = 2.2$ fF

$C_{cbx} = 6.0$ fF

$R_{bb} = 38\ \Omega$

$R_{ex} = 2.8\ \Omega$

$\tau = 0.28$ ps

$A_e = 0.6 \times 4.3 = 2.58\ \mu m^2$

$V_{ce} = 0.7V$.

Assumptions:

$C_{cbi} \approx 8.2$ fF, $C_{cbx} \approx 0$ fF

Voltage rail = 0 to -4.2 V.

$A_e^1 = 1.2\ \mu m^2$

$\Rightarrow \underline{C_{je}} = 32 \times \frac{1.2}{2.58} = \underline{14.88 \text{ fF}}$

Also $\underline{R_{ex}} = 2.8 \times \frac{2.58}{1.2} = \underline{6.02\ \Omega}$

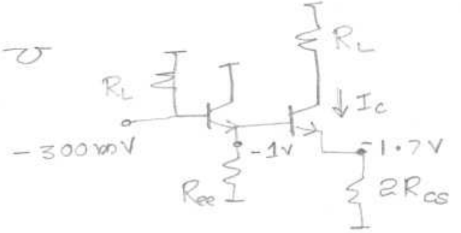
$C_{be} = C_{je} + g_m \tau_f = 14.88 \text{ fF} + (0.1846 \,\mho)\, 0.28 \text{ ps} = \underline{66.5 \text{ fF}}$

$I_c = 4 mA/\mu m^2 \times 1.2\ \mu m^2 = \underline{4.8\ mA}$

$R_{be} = \frac{\beta}{g_m} = 270.8\ \Omega$

$I_c \cdot R_L = 300\ mV \Rightarrow R_L = \frac{300}{4.8} = 62.5\ \Omega$.

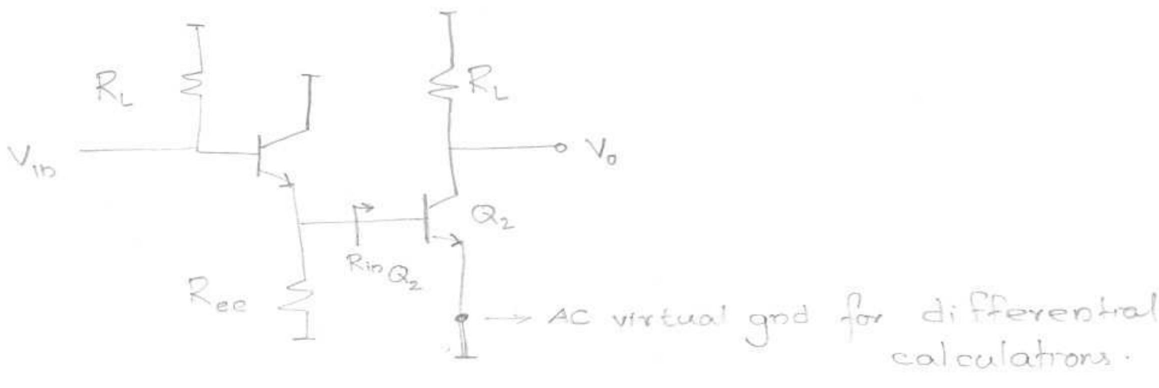
$g_m = \frac{I_c}{V_T} = \frac{4.8}{26} = 0.1846\ \mho$



$\Rightarrow r_e = \frac{1}{g_m} = \underline{5.417\ \Omega}$

$R_{ee} = \frac{(4.2-1)V}{4.8\ mA} = 666.7\ \Omega$. $R_{cs} = \frac{1}{2}\frac{(4.2-1.7)}{4.8 mA} = \underline{260.4\ \Omega}$

a) For AC differential gain, assume half circuit



$$R_{in\,Q_2} = R_{bb} + \beta\left(r_e + r_{ex}\right)$$

$$= 38 + 50\left(5.4 + 6\right)$$

$$= 608\ \Omega.$$

$$R_{in}\ Q_2 = R_{bb} + \beta\left(r_e + r_{ex} + R_{ee} \| R_{in\,Q_2}\right)$$

$$= 38 + 50\left(5.4 + 6 + 666 \| 608\right)$$

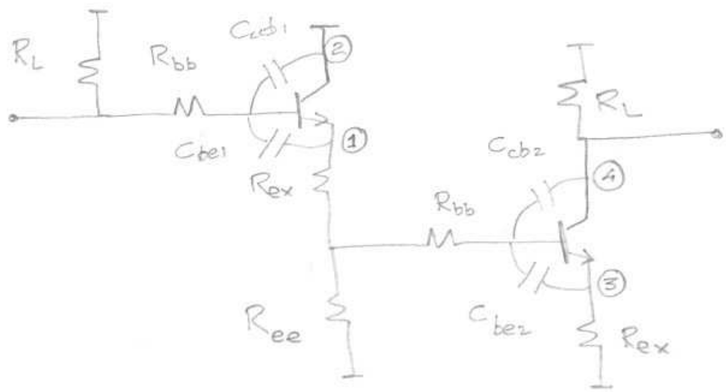
$$= 16.5\ k\Omega.$$

$$A_{V\,MB\,Q_1} = \frac{R_{EE} \| R_{in\,Q_2}}{R_{EE} \| R_{in\,Q_2} + r_e + r_{ex}} = 0.9654$$

$$A_{V\,MB\,Q_2} = \frac{-R_L}{r_{ex} + r_e} = -5.474$$

$$A_{V\,Total} = -5.285$$

b) High freq analysis using MOTC



$C_1 = C_{be1}$

$$R_{11}^{o} = \left[\left(1 - A_{v_1}\right)\left(R_{bb1} + R_L\right) + r_{e_1} \,\|\, \left(R_{ex} + R_{EE} \,\|\, R_{in\,Q_2}\right)\right] \,\|\, R_{BE1}$$

$$= \left[(1 - 0.965)(38 + 62.5) + 5.417 \,\|\, (6 + 666 \,\|\, 608)\right] \,\|\, 271$$

$$= \underline{8.5\ \Omega}.$$

$C_2 = C_{cb1}$

$$R_{22}^{o} = \left(R_{bb} + R_L\right) \,\|\, R_{in\,Q_1}$$

$$= \underline{100\ \Omega}$$

$$C_3 = C_{be2}, \quad R_{33}^{o} = \left[\left(1 - A_{2\,eff}\right)\left(R_{bb} + R_{EE} \,\|\, R_{out\,Q_1}\right) + R_{ex_2}\right] \,\|\, R_{be_2}$$

$$A_{2\,eff} = \frac{R_{ex2}}{R_{ex2} + r_{e_2}} \approx 0.5255$$

$$R_{out\,Q_1} = R_{ex1} + r_e + \frac{R_{bb} + R_L}{\beta} = 13.42$$

$$R_{33}^{o} = \underline{27.23\ \Omega}$$

$$C_4 = C_{b4}\ ,\quad R^{\circ}_{44} = \left(1 - A_{v_2}\right)\left(R_{ee} \,\|\, R_{in2} \,\|\, R_{out\,1}\right) + R_L$$

$$= \underline{139\ \Omega}.$$

$$R'_{22} = \left(R_{bb} + R_L\right) \,\|\, \left(R_{ex} + R_{EE} \,\|\, R_{in\,Q_2}\right)$$

$$= 85\,\Omega.$$

$$R'_{33} = \beta r_e \,\|\, \left[\left(1 - A_{g}\right)\left(R_{bb} + R_{EE} \,\|\, \left(R_{ex} + R_{bb1} + R_L\right)\right) + R_{ex2} \,\|\, r_{e_2}\right]$$

$$= 52\ \Omega$$

$$R^{1}_{44} = \left(1 - A_{v_2}\right)\left[R_{in\,Q_2} \,\|\, \left\{R_{bb_2} + R_{EE} \,\|\, \left(R_{ex} + R_{bb} + R_L\right)\right\}\right] + R_L$$

$$= 6.5 \times \left[608 \,\|\, \left\{38 + 666 \,\|\, \left(6 + 38 + 62.5\right)\right\}\right] + 62.5$$

$$= \underline{758\ \Omega}$$

$$R^{2}_{33} = \left[\left(1 - A_{v_{g/2}}\right)\left(R_{bb} + R_{EE} \,\|\, \left(R_{ex} + r_e\right)\right) + \left(R_{ex} + r_e\right)\right] \,\|\, \beta r_e$$

$$= \underline{22.8\ \Omega}$$

$$R^{2}_{44} = \left(1 - A_{v_2}\right)\left[\beta\left(r_e + r_{ex}\right) \,\|\, \left\{R_{bb} + R_{EE} \,\|\, \left(R_{ex} + r_e\right)\right\}\right] + R_L$$

$$= 350\ \Omega$$

$$R_{44}^{3} = R_{ex_2} \,\Big\|\, \left[R_{bb} + \left(R_{EE} \,\Big\|\, R_{ex} + r_e + \frac{R_{bb} + R_L}{\beta} \right) \right] + R_L$$

$$\approx 70$$

$$a_1 = R_{11}^{0} C_1 + R_{22}^{0} C_2 + R_{33}^{0} C_3 + R_{44}^{0} C_4$$

$$= (8.5\,\Omega)\, 66.5\,\mathrm{fF} + (100\,\Omega)\, 8.2\,\mathrm{fF} +$$

$$(27\,\Omega)\, 66.5\,\mathrm{fF} + (130\,\Omega)\, 8.2\,\mathrm{fF}$$

$$= 4\ \mathrm{ps.}$$

$$a_2 = R_{11}^{0} C_1 C_2 R_{22}^{1} + R_{11}^{0} C_1 C_3 R_{33}^{1} + R_{11}^{0} C_1 C_4 R_{44}^{1}$$

$$+ R_{22}^{0} C_2 C_3 R_{33}^{2} + R_{22}^{0} C_2 C_4 R_{24}^{2} + R_{33}^{0} C_3 C_4 R_{44}^{3}$$

$$= (8.5 \times 66.5\,\mathrm{fF} \times 8.2\,\mathrm{f} \times 85) + (8.5 \times 66.5\,\mathrm{f} \times 66.5\,\mathrm{f} \times 52)$$

$$+ (8.5 \times 66.5\,\mathrm{f} \times 8.2\,\mathrm{f} \times 358) + (100 \times 8.2\,\mathrm{f} \times 66.5\,\mathrm{f} \times 23)$$

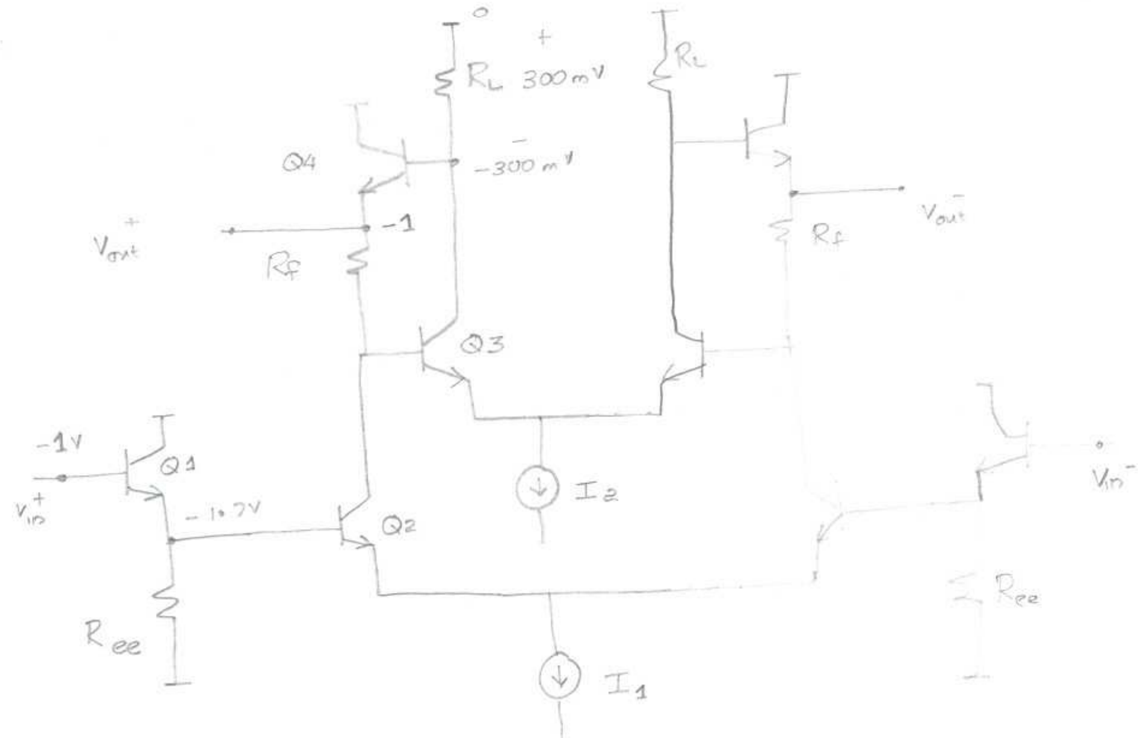
$$+ (100 \times 8.2\,\mathrm{f} \times 8.2\,\mathrm{f} \times 350) + (27 \times 66.5\,\mathrm{f} \times 8.2\,\mathrm{f} \times 70)$$

$$= 9 \times 10^{-24}\ \mathrm{sec}^2$$

$$\omega_n = \frac{1}{\sqrt{a_2}} = 3.3 \times 10^{11} \text{ rad/s} . \qquad f_d = \frac{\omega_d}{2\pi} \#$$

$$\xi = \frac{a_1}{2\sqrt{a_2}} = 0.68$$

2.



$V_{ee} = -5.2V$

$A_e = 1.2\ \mu m^2$, $I_{den} = 4\ mA/\mu m^2$, $I_c = 4.8\ mA$

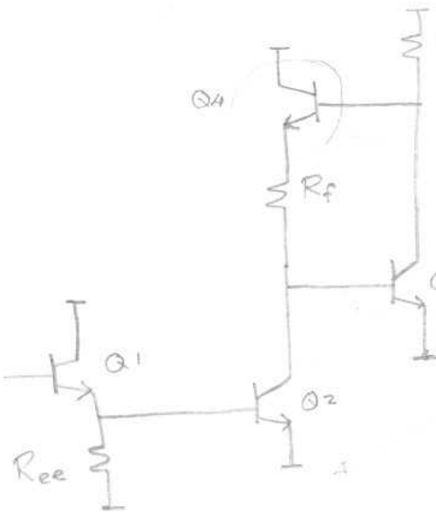
$I_c \cdot R_L = 300 \Rightarrow R_L = \dfrac{300\ mV}{4.8\ mA} = 62.5\ \Omega$

$R_{ee} = \dfrac{(5.2 - 1.7)V}{4.8\ mA} = 729\ \Omega$

Q_1 , Q_2 have no parasitic capacitances & have $\beta \to \infty$.

Q_3 , Q_4 has same parameter values as calculated in problem # 1

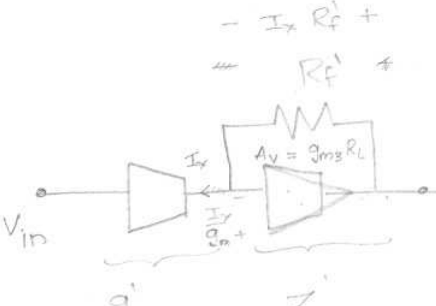
Eq. half circuit



- Let's assume to be ideal Voltage follower

$\leftarrow$ Virtual gnd for AC diff calc.

$\hookrightarrow$



$V_{out} = I_x R_f - \frac{I_x}{g_m}$

$\Rightarrow Z_T = \frac{V_{out}}{I_x}$

$R_{f'} = R_f + R_{ex_4}$

$$g_m' = \frac{g_{m2}}{1 + g_{m2} R_{ex}} \quad ; \quad Z_T = R_f \left(1 - \frac{1}{A_V}\right)$$

(since $\beta_1, \beta_2 \to \infty$)

$$= R_f \left(1 - \frac{1}{g_m R_L}\right)$$

$\therefore$ Total gain $A_V = g_m' \cdot Z_T'$

$$= \frac{g_{m2}}{1 + g_{m2} R_{ex}} \cdot \left(R_f \left(1 - \frac{1}{g_{m3} R_L}\right)\right)$$

Assuming $I_1 = I_2 \Rightarrow g_{m2} = g_{m3} = g_m$

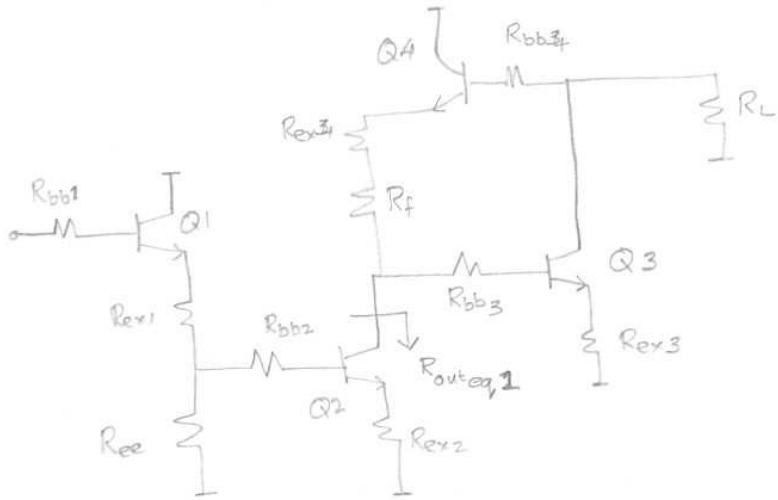
So for $A_V = 3$ (stage voltage gain)

$$\frac{0.1846}{1 + (0.1846 \times 6)} \cdot \left(R_f - \frac{R_f}{R_L \, 0.1846}\right) = 3$$

$\Rightarrow R_f' = 39.7\ \Omega$

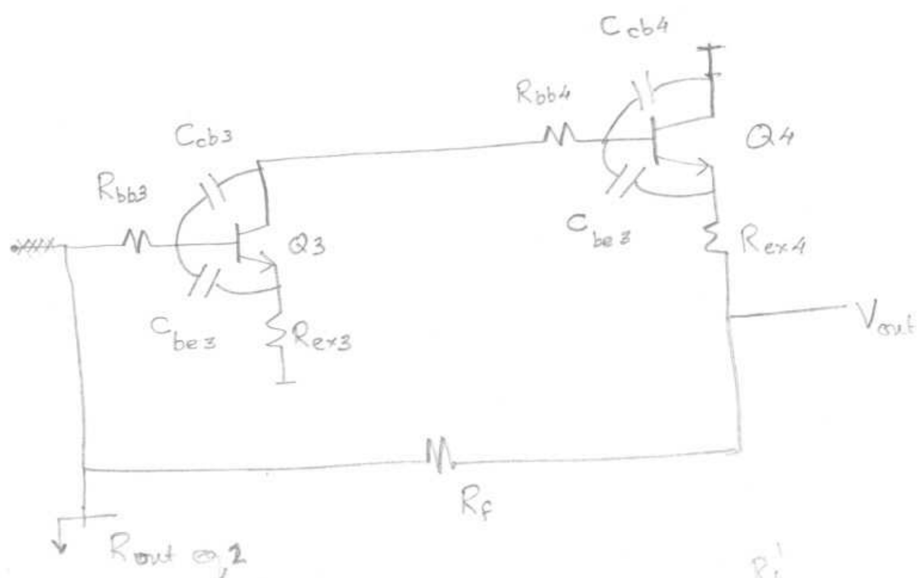
$\Rightarrow R_f = R_f' - R_{ex} = \underline{33.7\ \Omega}$

High freq Analysis.



Assume $\beta \to \infty$ for Q_1 & Q_2, $R_{out\,eq,2} \to \infty$

So we can reduce the eq ckt to



$$Av_{eq3} = \frac{R_{ex}}{R_{ex} + \frac{1}{g_m}} \approx .52$$

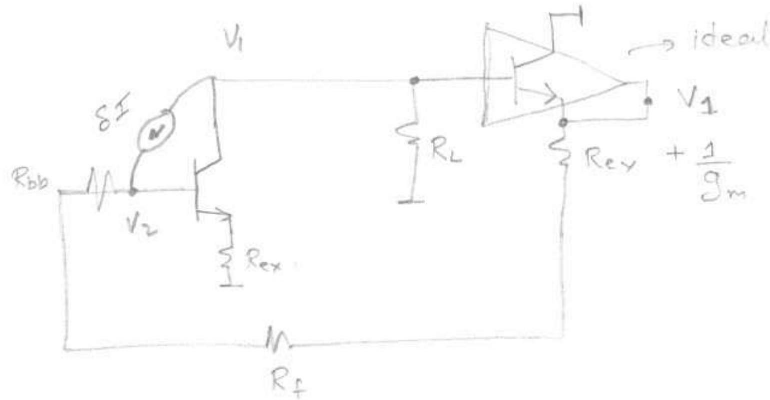
$$C_1 = C_{be3} \ ; \ R_{11}^{o} = \left(\overbrace{R_{bb} + R_F}^{R_f'} + R_{ex4} + r_{e4}\right)\left(1 - Av_3\right) + R_{ex3}$$

$$= (38 + 40 + 5.42)(1 - 0.522) + 6$$

$$= 46\ \Omega.$$

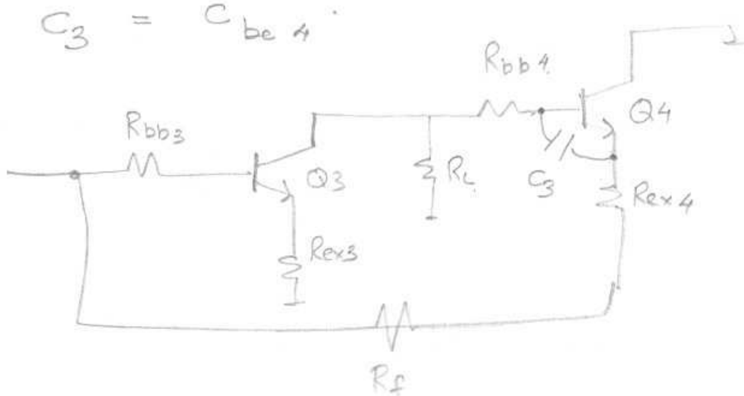
$C_2 = C_{cb3}$;

R°_{22} ##



$$\frac{V_2 - V_1}{\delta I} = R_{bb} + R_f + R_{ex} + \frac{1}{g_m} = R_{22} = 84\ \Omega$$

$C_3 = C_{be4}$



Let's assume Q_4 to be ideal voltage follower

so $Av_{Q_4} \approx 1$

$$R^{\circ}_{33} \approx \left[(1 - Av_{Q_4})(R_{bb4} + R_L) + R_{ex4} + \beta(r_{e_3} + R_{ex_3}) + R_f' + R_{bb3} \right] \Big\| \frac{1}{g_{m4}}$$

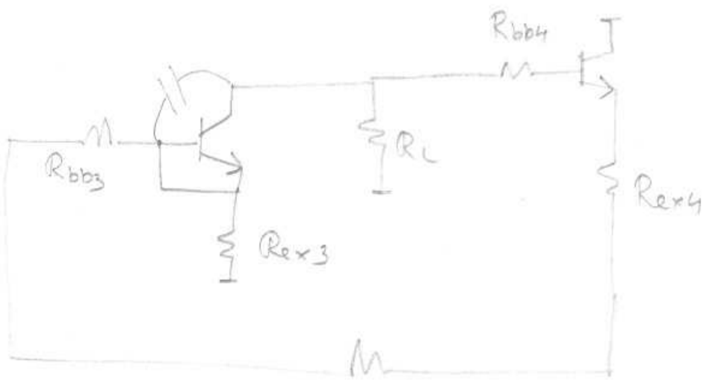
$$\approx 5\ \Omega$$

$C_4 = C_{cb4}$; $R^{\circ}_{44} = R_{bb4} + R_L = 100\ \Omega$

$$a_1 = (66.5\,f)\left[(45.87 + 105)\right] + (8.2f)(\cancel{530}^{\,5} + 83.5)$$

$$\approx \underline{14\ ps}.$$

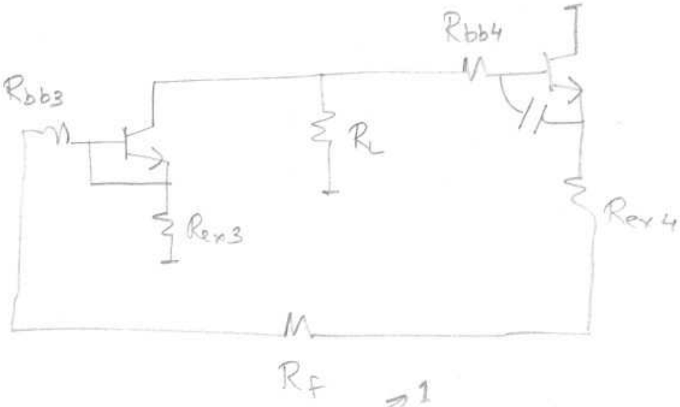
$\underline{R_{22}^{1}}$



$$R_{22}' = R_{ex3} \parallel \left(R_{bb3} + R_f + R_{ex4} + \frac{1}{g_{m4}}\right) + R_L$$

$$\approx 70\,\Omega.$$

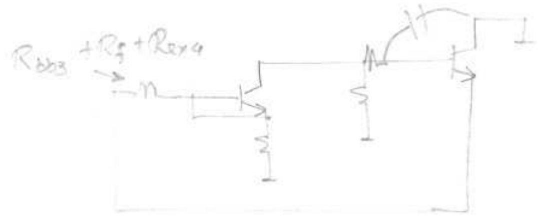
R_{33}'



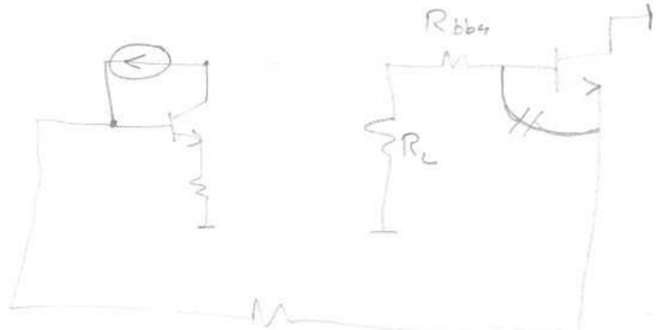
$$R_{33}' = \left[(R_L + R_{bb4})(1 - \overset{\nearrow 1}{A_v}) + (R_{ex4} + R_f + R_{bb3} + \cancel{\beta} R_{ex3})\right] \parallel \frac{1}{g_{m4}}$$

$$\approx 1/g_m = \underline{5.4}$$

$$R_{44}^{1} = R_{bb4} + R_L = 100\ \Omega.$$



R_{22}^{3} #



$$R_{feed} = R_{bb3} + R_{ex4} + R_f$$

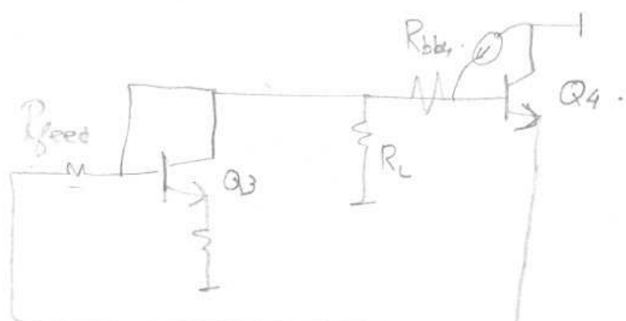
$$R_{22}^{3} = \left(R_{bb4} + R_{feed}\right) \parallel \left[\beta\left(r_{e3} + r_{ex3}\right) + R_L\right]$$

$$= 120 \parallel \left(50 \times 11 + 62.5\right)$$

$$= 120 \parallel 612.5$$

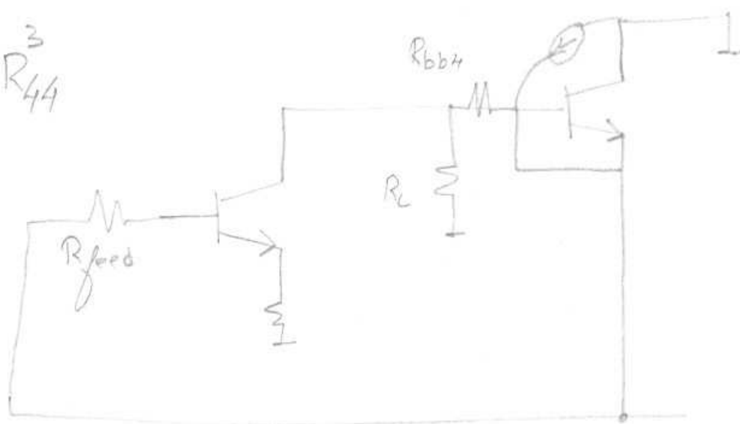
$$\approx 110\ \Omega.$$

R_{44}^{2}



$$R_{44}^{2} = R_{bb4} + R_L \parallel \left(R_{feed} + \frac{1}{g_{m4}}\right) = 74\ \Omega$$

R^3_{44}



$$R^3_{44} \approx R_L + R_{bb4} = 100\,\Omega .$$

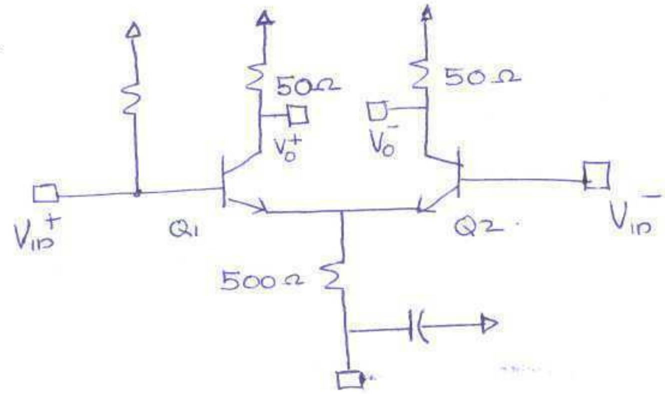
$$a_2 = R^0_{11} C_1 C_2 R^1_{22} + R^0_{11} C_1 C_3 R^1_{33}$$
$$R^0_{11} C_1 C_4 R^1_{44} + R^0_{33} C_2 C_3 R^3_{22}$$
$$R^0_{22} C_2 C_4 R^2_{44} + R^0_{33} C_3 C_4 R^3_{44}$$
$$= 21 \times 10^{-24}\ \mathrm{sec}^2$$

$$\omega_n = \frac{1}{\sqrt{a_2}} \approx 235 \times 10^9\ \mathrm{rad/sec} \rightarrow \frac{f_D}{2\pi} = \frac{\omega_D}{2\pi}$$

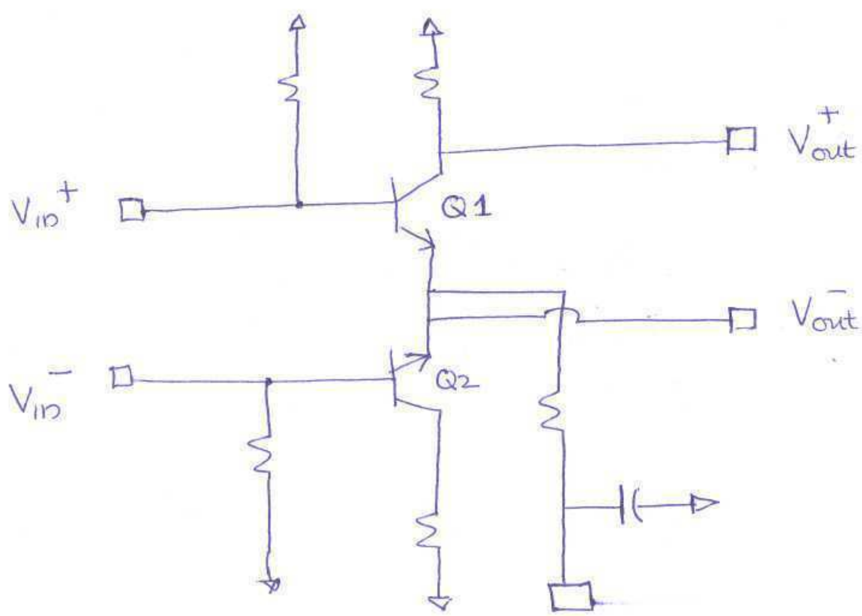
$$f_D = \frac{\omega_D}{2\pi}$$

$$\zeta = \frac{a_1}{2\sqrt{a_2}} \approx 1.3$$

4.



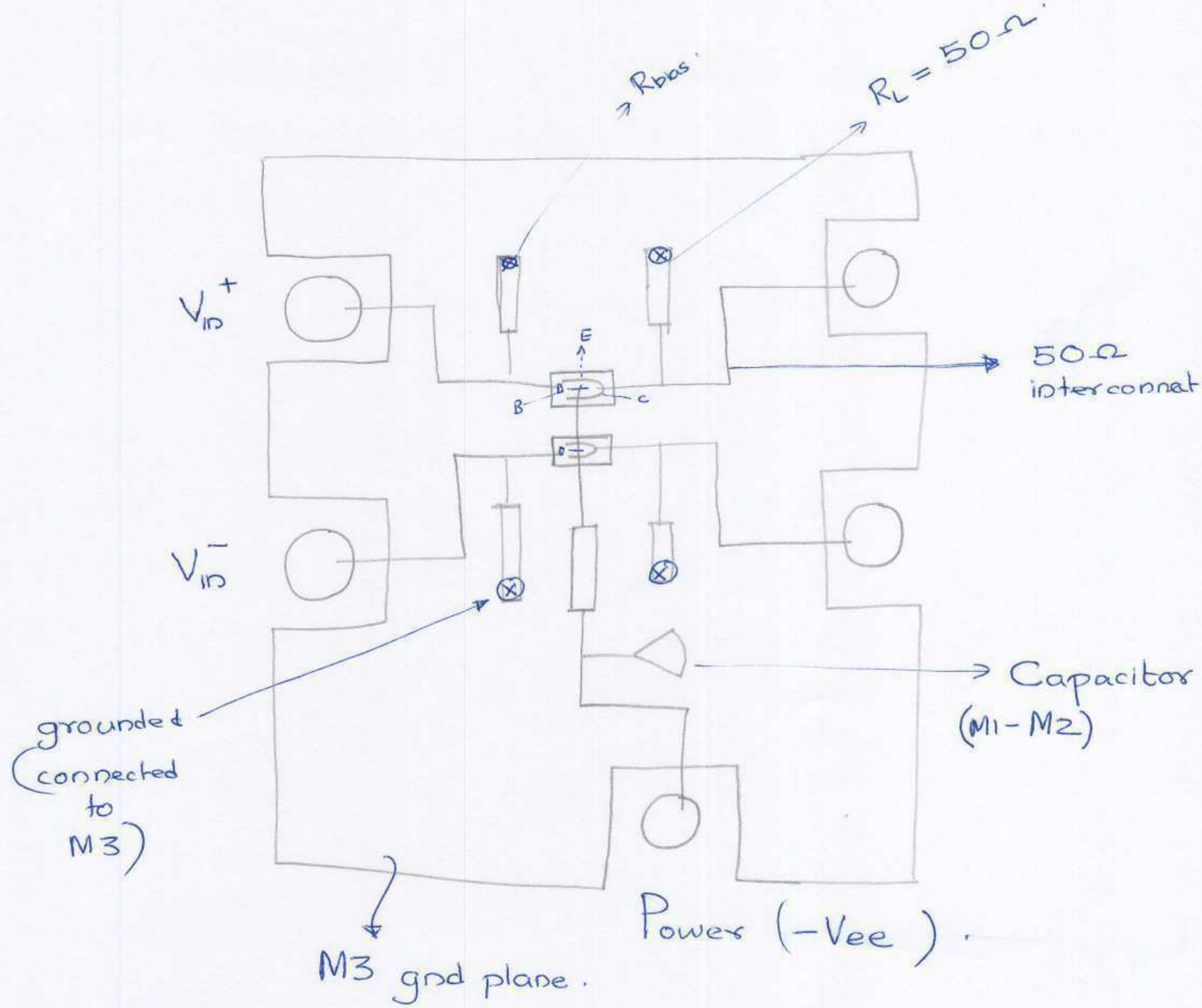
For ease of layout let's redraw it as



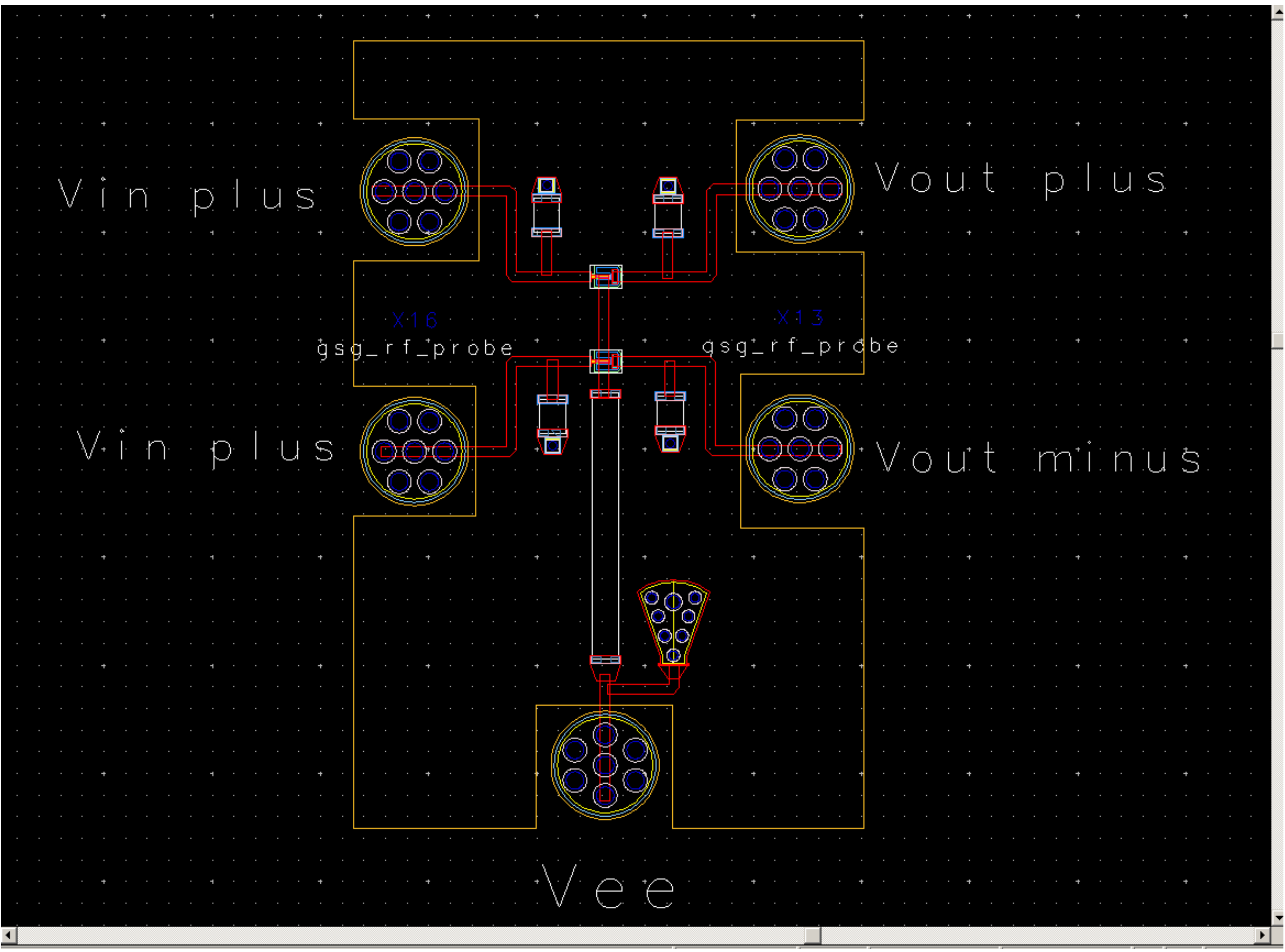
The mask is given in next page.

Concerns.

1. Make sure that all the nodes are well connected.
2. Design the signal carrying interconnect to be $50\,\Omega$ using LINCALC.
3. Pads with M1 — M3 contact is used for signal input & output as well as power.
4. Top metal layer (M3) acts as continous ground plane.
5. Size the R ~~us~~ assuming $R_{sheet} = 50\,\Omega/\square$.
6. C is made using M1 — M2 layers.
7. Resistors are grounded on one side by contact to M3.
8. All signal carrying lines (both half circuits) should be symmetric. ~~same~~



Rbias
RL = 50 Ω
Vin+
E
B
C
50 Ω interconnect.
Vin−
Capacitor (M1 - M2)
grounded (connected to M3)
Power (−Vee)
M3 gnd plane.


Vin plus
Vout plus
X16
gsg_rf_probe
X13
gsg_rf_probe
Vin plus
Vout minus
Vee