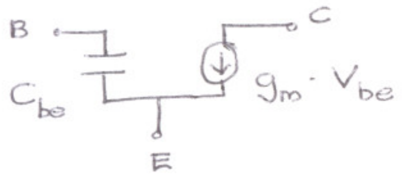
ECE 145C / 218C

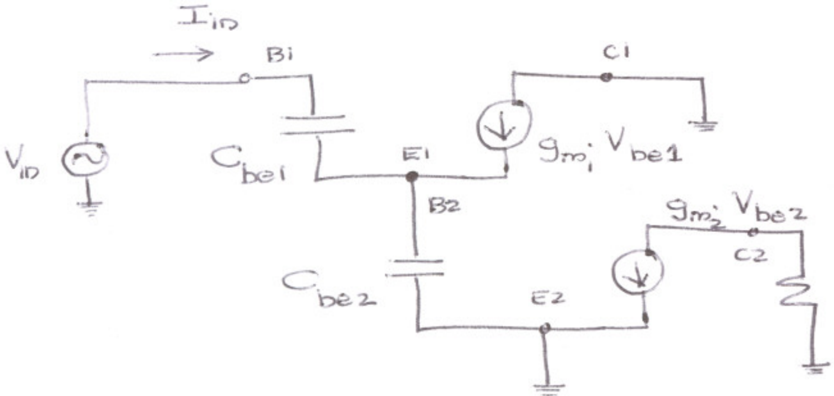
HW·3 Solution set *

1 a.



$$g_m = \frac{q \cdot I_c}{kT} = \underline{0.3846\ \mho}$$

$$\tau_f = 0.32\text{ps}; \quad C_{be} = g_m \cdot \tau_f = \underline{0.123\ \text{pF}}$$



$$R_{Leq} = R_{out\ term} \| R_{load}$$
$$= R_L = 25\Omega$$

$$I_{in} = (V_{in} - V_{be2}) \cdot j\omega C_{be1} \Rightarrow I_{in} + V_{be2} \cdot j\omega C_{be1} = V_{in} \cdot j\omega C_{be1}$$

$$V_{be2} = \frac{I_{in} + \dfrac{I_{in}}{j\omega C_{be1}} \cdot g_{m1}}{j\omega C_{be2}}$$

$$I_{in} + \left[I_{in} + \frac{I_{in} \cdot g_{m1}}{j\omega C_{be1}}\right]\frac{C_{be1}}{C_{be2}} = V_{in} \cdot j\omega C_{be1}$$

$$I_{in}\left\{1 + \frac{(g_{m1} + j\omega C_{be1})}{j\omega C_{be1}} \cdot \frac{C_{be1}}{C_{be2}}\right\} = V_{in} \cdot j\omega C_{be1}$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{-g_{m1}}{\omega^2 C_{be1} C_{be2}} - j\frac{(C_{be1} + C_{be2})}{\omega C_{be1} \cdot C_{be2}}.$$

Assuming $C_{be1} = C_{be2} = C_{be}$

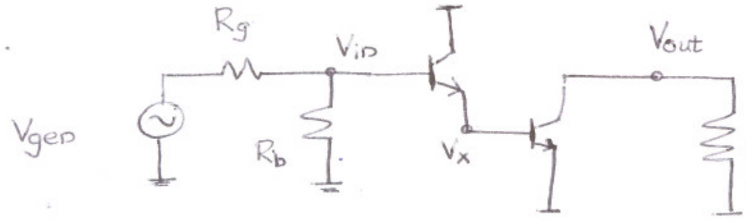
$g_{m1} = g_m$.

$$Z_{in} = \frac{-g_m}{\omega^2 C_{be}^2} - j\frac{2}{\omega C_{be}} \ \Omega$$

$$\approx -\frac{1}{g_m}\left(\frac{\omega_T}{\omega}\right)^2 - \frac{j}{g_m}\left(\frac{\omega_T}{\omega}\right) \ \Omega.$$

$$S_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

b).



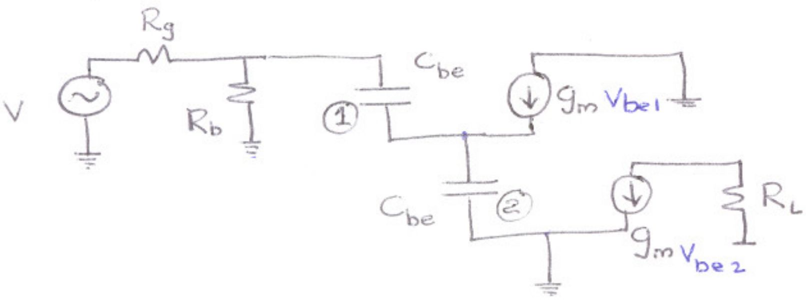
$$\left(R_{out\cdot term} \,\|\, R_{load}\right) = R_L = 25\,\Omega$$

$$V_{in} = V_{gen}\cdot\frac{R_b}{R_g + R_b} = \frac{V_{gen}}{2};$$

$$V_x = V_{in}\cdot\frac{R_{Leq}}{R_{Leq} + r_e} \approx V_{in}.$$

$$V_{out} = -V_x \cdot g_m \cdot R_L$$

$$\therefore \quad S_{21} = 2\frac{V_{out}}{V_{gen}} = -g_m \cdot R_L = \underline{-9.615}$$



$$R_{11}^{o} = (1 - A_V)\left(R_g \,\|\, R_b\right) + \frac{1}{g_m} \approx 0\times 25 + 2.6 = 2.6\ \Omega.$$

$$R_{22}^{o} = \frac{1}{g_m} = 2.6\ \Omega.$$

$$a_1 = R_{11}^o C_1 + R_{22}^o C_2$$

$$= (2\cancel{1}.6 + 2.6)\cdot 0.123\ \text{pF} = \cancel{3.7146}\ 629 \times 10^{-15}\ \text{s}$$

$$R_{22}' = R_g \parallel R_b = 25\ \Omega.$$

$$a_2 = R_{11}^o \cdot C_1 \cdot C_2 \cdot R_{22}' = (2\cancel{1}.6)(0.123\text{p})^2(25)$$

$$= \cancel{1.044 \times 10^{-23}} = 9.51 \times 10^{-25}\ s^2$$

$$A_{(s)} = \frac{A_{V\,MB}}{1 + a_1 s + a_2 s^2} = \frac{A_{V\,MB}}{1 + \frac{2\xi}{\omega_n} s + \frac{s^2}{\omega_n^2}}$$

$$= \frac{-9.6}{1 + (\cancel{3.715}\ 0.629 \times 10^{-12})s + (\cancel{1.044}\ 9.51 \times 10^{-25})s^2}$$

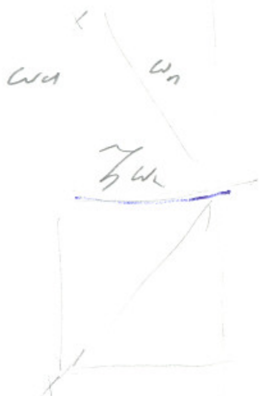
$$\therefore \omega_n = \cancel{3.035}\ 1.02 \times 10^{12}\ \text{rad/sec.}\ ; \quad f_n = 162\ \text{GHz.}$$

$$\xi = 0.\cancel{5}322$$

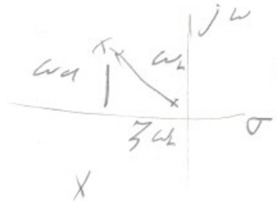
Pole freq $\omega_d = \omega_n \sqrt{1 - \xi^2} = \cancel{207}\ 965 \times 10^9\ \text{rad/sec.}$

$$f_d = \underline{153\ \text{GHz.}}$$

$$s_{p_{1,2}} = -\xi\omega_n \pm j\omega_d.$$



$$1 + a_1 s + a_2 s^2 = 1 + \frac{2\zeta s}{\omega_n} + \frac{s^2}{\omega_n^2}$$



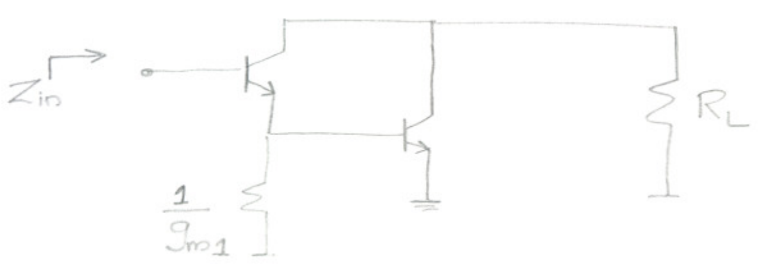
$(s - s_{p_1})(s - s_{p_2})$

$s_{p_{1,2}} = -\zeta\omega_n \pm j\omega_d$

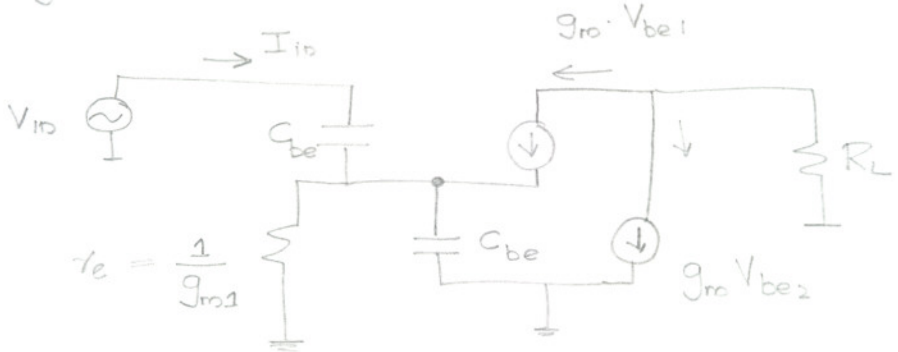
draw the root locus

guide ζ, ω_d, $f_d = \frac{\omega_d}{2\pi}$

2 a) SS circuit.



Neglecting imp term R;



$$g_{m1} = g_{m2} = g_m$$

$$C_{be1} = C_{be2} = C_{be}$$

$$V_{be1} = \frac{I_{in}}{j\omega C_{be}}$$

$$V_{be2} = I_{in} + g_m \cdot V_{be1} = \left(I_{in} + \frac{g_m I_{in}}{j\omega C_{be}}\right) \frac{r_e}{r_e + \frac{1}{j\omega C_{be}}} \cdot \frac{1}{j\omega C_{be}}$$

$$= I_{in} \frac{(g_m + j\omega C_{be}) \cdot r_e}{(1 + j\omega C_{be} r_e)(j\omega C_{be})}$$

$$I_{in} = (V_{in} - V_{be2})\, j\omega C_{be}$$

$$= \left[V_{in} - I_{in} \frac{(g_m + j\omega C_{be}) r_e}{(1 + j\omega C_{be} r_e)(j\omega C_{be})}\right] j\omega C_{be}$$

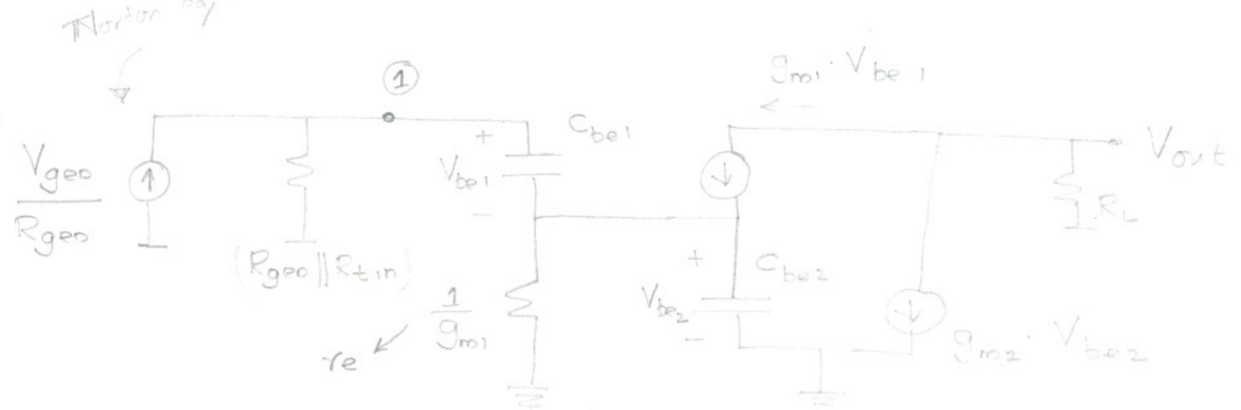
$$V_{in} \cdot j\omega C_{be} = I_{in}\left[1 + \frac{(g_m + j\omega C_{be})\, r_e \cdot j\omega C_{be}}{(1 + j\omega C_{be} r_e)(j\omega C_{be})}\right]$$

$$\frac{V_{in}}{I_{in}} = Z_{in} = \frac{1 + j\omega C_{be} r_e + g_m r_e + j\omega C_{be} r_e}{j^2 \omega^2 C_{be}^2 r_e + j\omega C_{be}}$$

$$= + \frac{(2 + 2j\omega C_{be} r_e)}{(j\omega C_{be} \mp \omega^2 C_{be}^2 r_e)} = \frac{+2\,(1 + j\omega C_{be} r_e)}{j\omega C_{be}\,(1 + j\omega C_{be} r_e)}$$

$$= -\frac{2j}{\omega C_{be}}$$

2b)



Kirchoff current eq @ 1

$$\frac{V_{gen}}{R_{gen}} = \frac{(V_{be1} + V_{be2})}{R_{gen} \| R_{tin}} + V_{be1}\, j\omega C_{be1}$$

$$= \frac{(V_{be1} + V_{be2})}{R_g/2} + V_{be1} \cdot j\omega C_{be}. \quad (1)$$

$$(V_{be1} \cdot j\omega C_{be1} + g_m \cdot V_{be1}) \frac{r_e}{r_e + \frac{1}{j\omega C_{be}}} \cdot \frac{1}{j\omega C_{be}} = V_{be2}$$

$$V_{be1} \frac{(g_m + j\omega C_{be}) r_e}{1 + j\omega C_{be} \cdot r_e} = V_{be2} = (\pm) V_{be1}$$

$$V_{out} = \left(g_{m1} \cdot V_{be1} + g_{m2} \cdot V_{be2}\right) R_L$$

$$= g_m \cdot R_L \left(V_{be1} + V_{be2}\right)$$

$$= g_m R_L \left[V_{be1} \right] \cdot 2$$

$$= g_m R_L V_{be1} \cdot 2$$

$$= g_m R_L V_{be1} \cdot 2$$

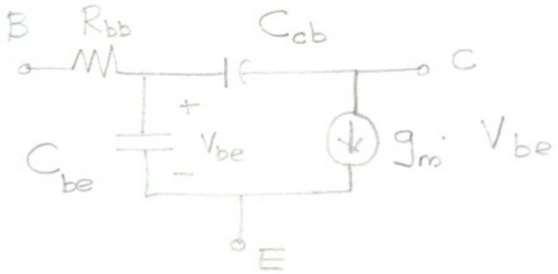
$$\frac{V_{gen}}{R_{gen}} = \frac{(V_{be1} + V_{be1})}{R_g / 2} + V_{be1} \cdot j\omega C_{be}$$

$$= V_{be1} \left(\frac{4}{R_g} + j\omega C_{be} \right)$$

$$\frac{V_{out}}{V_{gen}} = \frac{g_m R_L V_{be1} \cdot 2}{V_{be1} \cdot R_{gen} \left(4 + j\omega C_{be} \cdot R_{gen}\right)}$$

$$= \frac{- g_m \cdot R_L / 2}{\left[1 + j\omega C_{be} \frac{R_{gen}}{4} \right]}$$

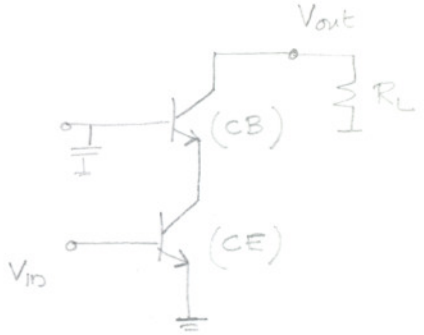
3.



$$R_{bb} \approx \frac{10}{g_m} = 26\ \Omega$$

$$C_{cb} \approx \frac{C_{be}}{10} = 0.1\ C_{be}$$

$V_{b2} = 2.5\ V.$



Gain of volt. divider = $\frac{1}{2}$

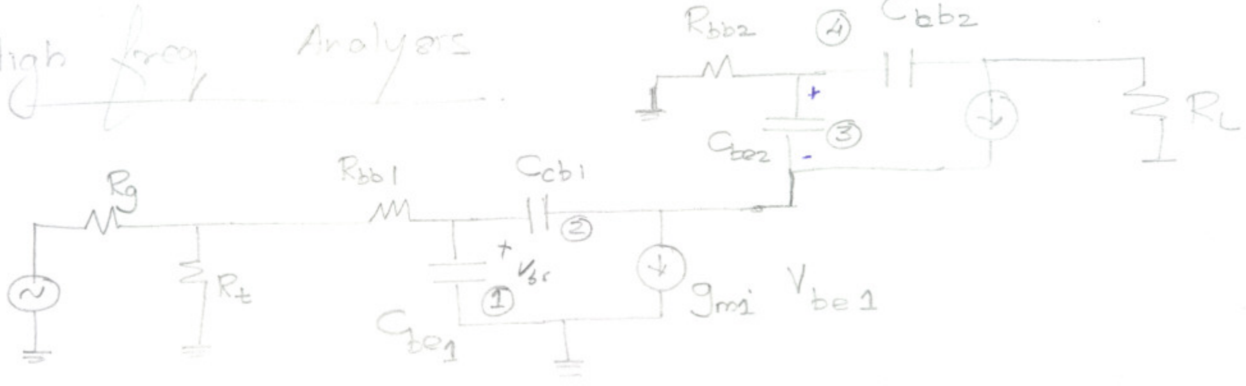
$$\text{Gain of CE stage} = g_{m1} \cdot \left(\frac{1}{g_{m2}}\right) \approx -1.$$

$$\text{Gain of CB stage} = \frac{(R_L)}{(1/g_{m2})} = g_m R_L$$

$$\therefore\ S_{21} = \frac{2\,V_{out}}{V_{gen}} = g_m R_L = \underline{-9.6}$$

$$A_{V\,\text{Mid band}} = -9.6$$

High freq Analysis



$R_{bb1} = R_{bb2} = R_{bb}$

$C_{be1} = C_{be2} = C_{be}$

$C_{cb1} = C_{cb2} = C_{cb}$

$g_{m1} = g_{m2} = g_m.$

$$R_{11}^{o} = R_{bb} + R_g/2 = 26 + 25 = 51\ \Omega$$

$$R_{22}^{o} = \left(R_{bb} + R_g/2\right)(1 - \underbrace{A_V}_{-1}) + \frac{1}{g_{m2}}$$

$$= 51 \times 2 + 2.6 = 104.6\ \Omega$$

$$R_{33}^{o} = R_{bb2} + \frac{1}{g_{m2}} = 26 + 2.6 = 28.6\ \Omega.$$

$$R_{44}^{0} = R_{bb} + R_L = 26 + 25 = 51\,\Omega.$$

$$a_1 = \sum_{i=1}^{4} R_{ii}^{0}\, C_i = (51 + 28.6) \times 0.123 \times 10^{-12}$$
$$+ (104.6 + 51) \times \frac{0.123}{10} \times 10^{-11}$$
$$= 1.17 \times 10^{-11}\ \text{sec}$$

$$a_2 = R_{11}^{0} C_1 C_2 R_{22}^{1} + R_{11}^{0} C_1 C_3 R_{33}^{1} + R_{11}^{0} C_1 C_4 R_{44}^{1}$$
$$+ R_{22}^{0} C_2 C_3 R_{33}^{2} + R_{22}^{0} C_2 C_4 R_{44}^{2} + R_{33}^{0} C_3 C_4 R_{44}^{3}$$

$$R_{22}^{1} = \frac{1}{g_{m2}} = 2.6\,\Omega.$$

$$R_{33}^{1} = R_{bb_2} + \frac{1}{g_{m2}} = 25 + 2.6 = 27.6\,\Omega.$$

$$R_{44}^{1} = (1 + g_m R_L) R_{bb} + R_L = 10.6 \times 26 + 25 = \cancel{51\,\Omega}\ 300.6\,\Omega$$

$$R_{33}^{2} = \left(R_{bb} + \frac{R_g}{2}\right) \| \frac{1}{g_m} + R_{bb} = (51 \| 2.6) + 26$$
$$= 28\,\Omega$$

$$R_{44}^{2} = R_{bb2}(1 + g_m R_L) + R_L = 300.6\,\Omega$$

$$R_{44}^{3} = R_{bb} + R_L = 51\,\Omega$$

$$a_2 = 7.968 \times 10^{-22}\ \text{sec}^2.$$

$$A(s) = \frac{-9.6}{1 + 1.17 \times 10^{-11} s + 7.968 \times 10^{-22} s^2}$$

$$\omega_n = 35.4 \times 10^{9}\ \text{rad/s}, \quad \xi = 0.2 \qquad \omega_d = \omega_n \sqrt{1 - \xi^2}.$$

$$s = -\xi \omega_n \pm j\omega_d$$