ECE ECE145C (undergrad) and ECE218c (graduate)

Mid-Term Exam. Nov 5, 2007

Do not open exam until instructed to.

Open book

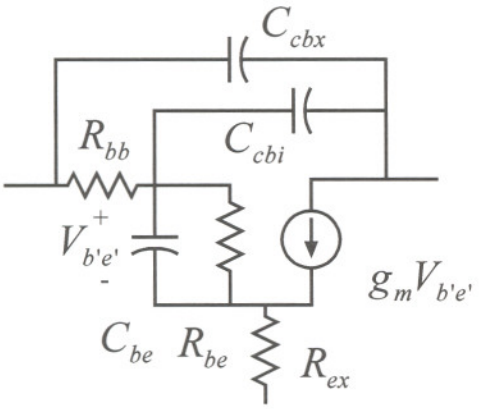
Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

Problem	Points Received	Points Possible
1a		10
1b		5
1c		20
1d		5
1e		10
1f		10
2a		10
2b		10
2c		10

Name: Solution

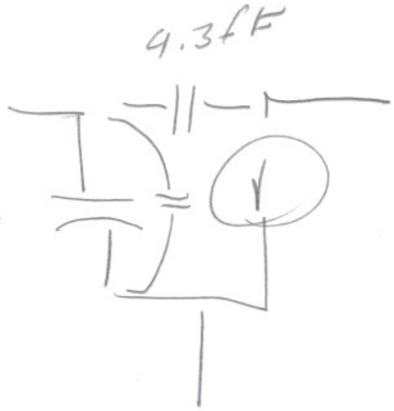
Problem 1, 70 points

You will be using the device model below:

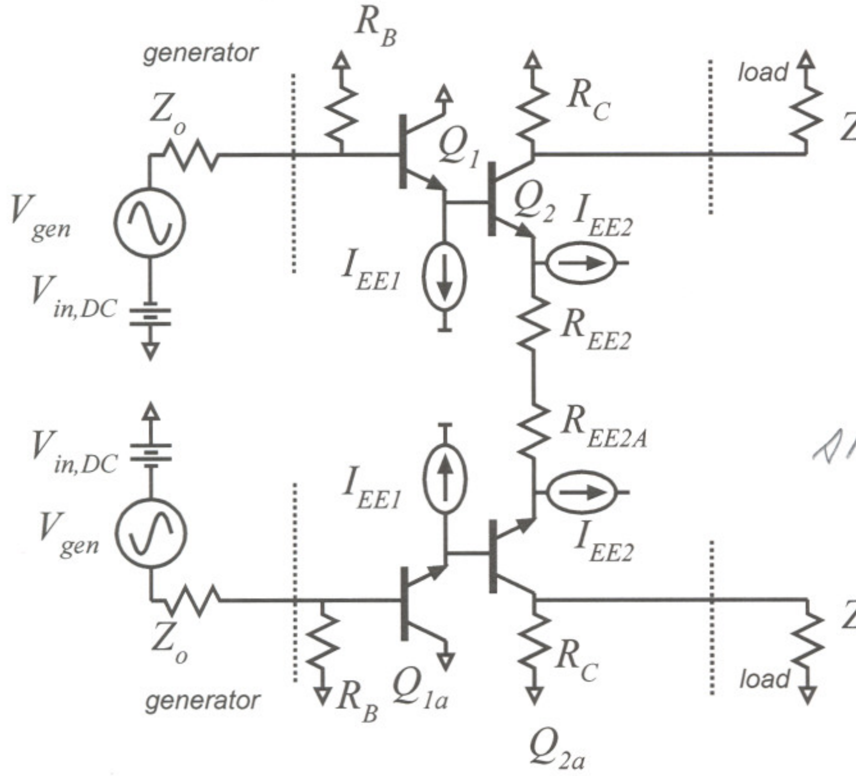


$C_{be} = C_{be,depl} + g_m \tau_f$. Cje=14 fF, beta=infinity, Ccbi=4.3 fF, Ccbx=0 fF, Rbb=0 Ohms, Rex=0 Ohms, and tau_f=0.44 ps. This transistor model has 5 um^2 emitter area.

The transistors have a Vbe=0.9 Volts when operating at 2.0 mA/um^2.



You will be working with the circuit below



Cdiff

All $I_C = I_E = 10mA$

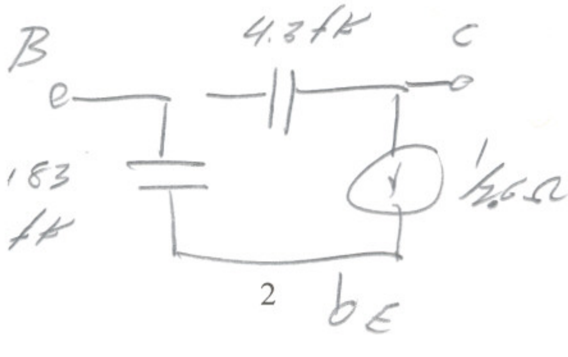
$g_m = 1/2.6\Omega$

$C_{diff} = g_m \tau_f$

$= \underline{169fF}$

$C_{be} = C_{je} + C_{diff}$

$= 183fF$



Part a: 10 points:
S-parameter design, gain definitions

Think carefully before answering---this is mostly a matter of getting details correct.

For differential input signals, the amplifier half-circuit is to have 50 Ohms input and 50 Ohms output impedance. Zo is to be 50 Ohms.

The amplifier is be designed for S21=10 dB at mid-band frequencies

All transistors are biased at 10 mA current

($V_{in,DC}$ is to be selected so that the amplifier input and output DC levels are identical---we will use this fact in part b)

Find the following:

$R_C =$ 50Ω $R_B =$ 50Ω.

$R_{EE2} =$ 5.3Ω

$$A_v = A_{v1} A_{v2} = 3.16 = 1 \cdot A_{v2}$$

$$\Rightarrow A_{v2} = 3.16 = \frac{25\Omega}{R_{EE} + 2.6\Omega} \Rightarrow R_{EE} + 2.6 = \frac{25\Omega}{3.16} = 7.91\Omega$$

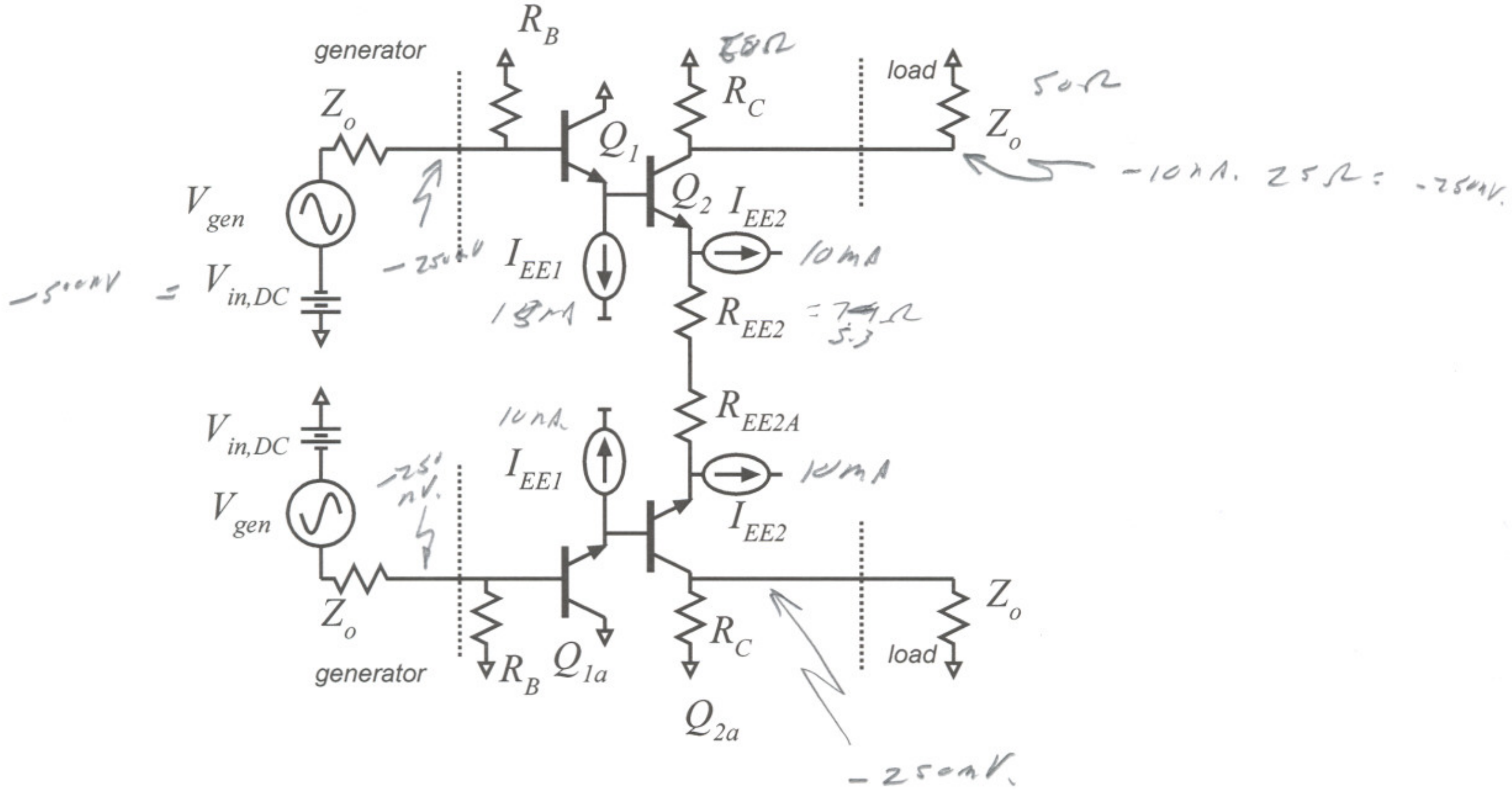
$$\Rightarrow R_{EE} = \underline{\underline{5.3\Omega}}$$

Part b, 5 points

DC bias

$V_{in,DC}$ is to be selected so that the amplifier input and output DC levels are identical (this represents the condition of a cascade of similar amplifiers)

Draw the DC node voltages on the circuit diagram



Part C, 20 points
bandwidth calculations

Using the method of time constants, find the coefficients a_1 and a_2 of the transfer function. **We will simplify the problem by NOW SETTING C_{cb1} to ZERO.**

The remaining capacitors are labeled $C_1 = C_{be1}, C_2 = C_{cb2}, \ C_3 = C_{be2}$

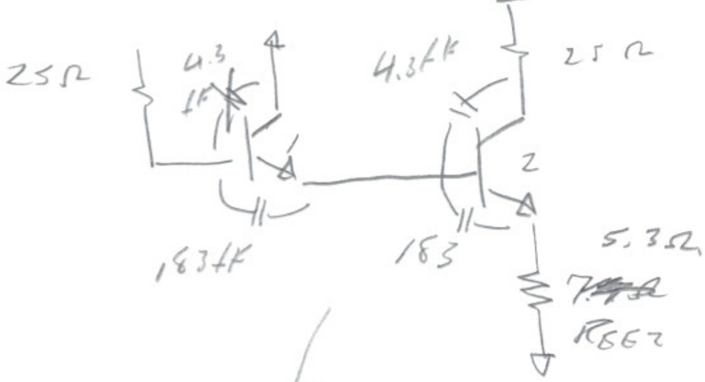
$R_{11}^0 =$ 2.6 Ω $\qquad$ $R_{22}^0 =$ 35.8 Ω

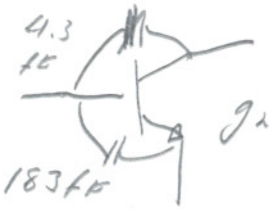
$R_{33}^0 =$ 7.9 Ω $\qquad$ $R_{22}^1 =$ 129 Ω

$R_{33}^1 =$ 30.3 Ω $\qquad$ $R_{33}^2 =$ 7.3 Ω

$a_1 =$ 1.11 ps. $\qquad$ $a_2 =$ $1.34\,(10^{-24})\ \mathrm{sec}^2$

You may use the degeneration approximation if you wish, but please make this clear and please be consistent.





$g_m = \frac{1}{2.6\,\Omega}$

$= G_E + g_m \tau_f = 183\,fF$

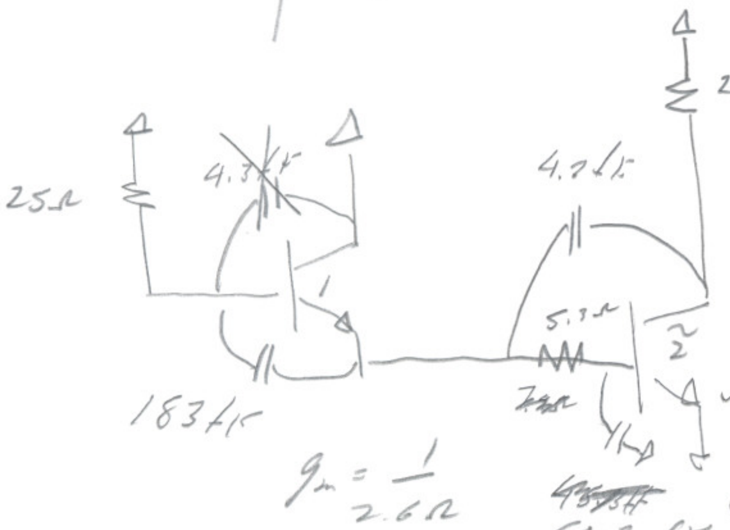
$1/2.6\,\Omega$

degen

$\tilde{G}_{BE2} = G_{BE2} \cdot \frac{2.6\,\Omega}{2.6\,\Omega + 5.3\,\Omega}$

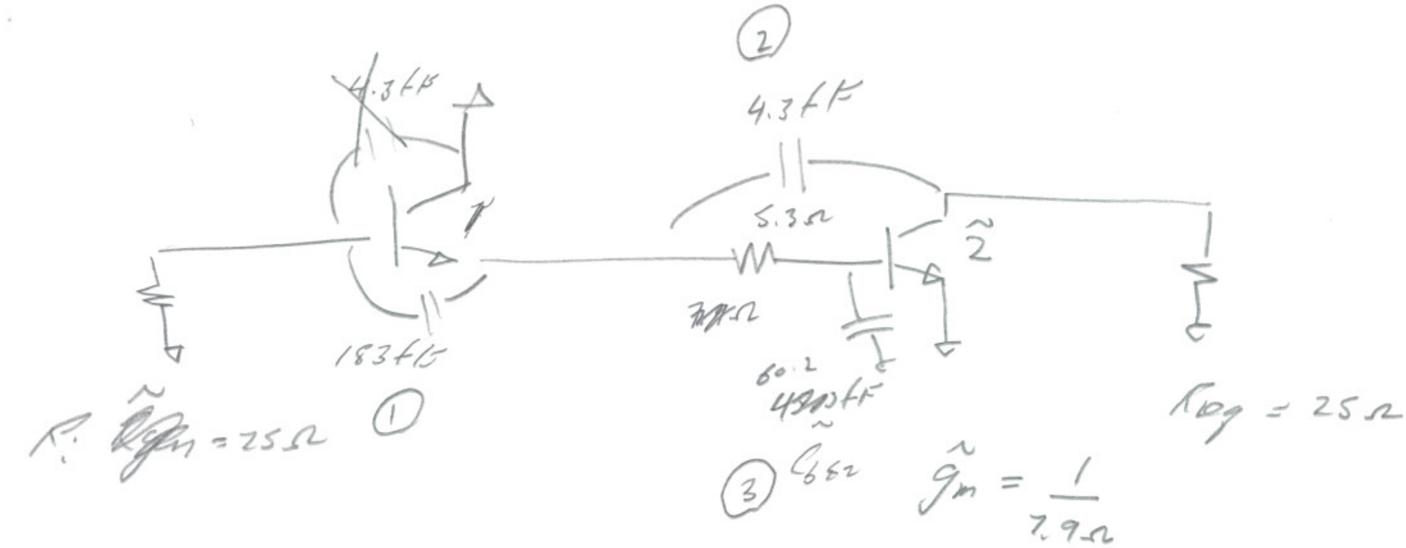
$= \cancel{45.3}\ fF$

$= 60.2\ fF$



$\tilde{g}_m = \frac{1}{7.9\,\Omega}$

$g_m = \frac{1}{2.6\,\Omega}$



$R_{11}^0 = 1/g_{m1} = 2.6\,\Omega$ $R_{11}^0 C_1 = 2.6\,\Omega \cdot 183\,\text{fF} = 476\,\text{fs}$

$R_{22}^0 = (1/g_{m1})(1 + \tilde{g}_{m2} R_{eq}) + R_{eq}$

$= 2.6\,\Omega\left(1 + \frac{25\,\Omega}{7.9\,\Omega}\right) + 25\,\Omega = 2.6\,\Omega(1 + 3.16) + 25\,\Omega$

$R_{22}^0 = \underline{35.8\,\Omega}$ $R_{22}^0 C_2 = 35.8\,\Omega \cdot 4.3\,\text{fF} = \underline{154\,\text{fs}}$

$R_{33}^0 = 1/g_{m1} + 5.3\,\Omega = 2.6\,\Omega + 5.3\,\Omega = 7.9\,\Omega$

$R_{33}^0 C_3 = 7.9\,\Omega \cdot 60.2\,\text{fF}$
$= \underline{476\,\text{fs}}$

$a_1 = 476 + 476 + 154\,\text{fs} = 1.11\,\text{ps}$

$R_{22}' = 25\,\Omega(1 + 3.16) + 25\,\Omega = 25\,\Omega(5.16) = 129\,\Omega$

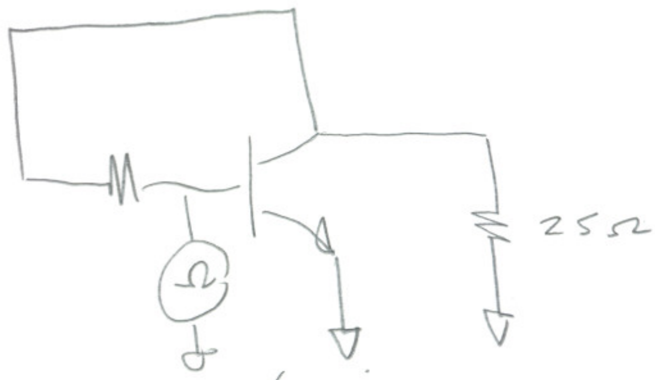
$C_1 R_{11}^0 R_{22}' C_2 = 476\,\text{fs}\,(129\,\Omega)(4.3\,\text{fF}) = 2.64 \cdot 10^{-25}\,\text{sec}^2$

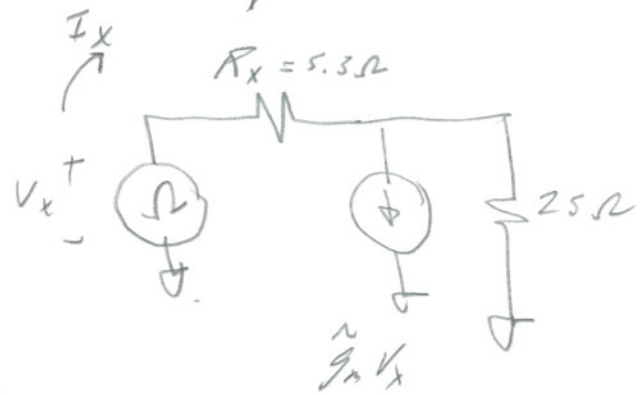
$R_{33}' = 25\,\Omega + 5.3\,\Omega = 30.3\,\Omega$

$C_1 R_{11}^0 R_{33}' C_3 = (476\,\text{fs})\,30.3\,\Omega \cdot 60.2\,\text{fF} = 8.7 \cdot 10^{-25}\,\text{sec}^2$

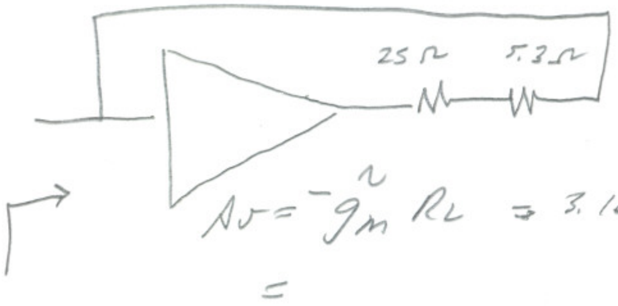
R

$R_{33}^{2} = 7.3\,\Omega$





$\tilde{g}_m = \frac{1}{7.9\,\Omega}$



$A_v = -\tilde{g}_m R_L = 3.16$

$$R = \frac{25\,\Omega + 5.3\,\Omega}{4.16} = \underline{\underline{7.28\,\Omega}}$$

$$C_2 R_{22}^{0} R_{33}^{2} C_3 = (476\,fF)(7.3\,\Omega)\cdot 60.2\,fF$$
$$= 2.1 \cdot 10^{-25}\ sec^2$$

a_2 = sum of 3 terms above = $1.34 \cdot 10^{-24}\ sec^2$

part D, 5 points

finding the poles

If the poles are widely separated, find the frequency of the lower pole by the separated pole approximation.

If the poles are complex, find the damped resonant frequency f_d and the damping factor ζ.

$$a_1 = 1.11 \cdot 10^{-12} \text{ sec}$$

$$a_2 = 1.34 (10^{-24}) \text{ sec}^2$$

try SPA : $\frac{a_2}{a_1} = 1.21 \cdot 10^{-12} \text{ sec}^2 \cong a_1 \Rightarrow$ SPA fails

try complex roots

$$1 + a_1 s + a_2 s^2 = 1 + \frac{2\zeta}{\omega_n} s + \frac{s^2}{\omega_n^2}$$

$$\omega_n = \frac{1}{\sqrt{a_2}} = 8.64 \cdot 10^{11} \text{ rad/sec} \Rightarrow f_n = 137.4 \text{ GHz}.$$

$$a_1 = 2\zeta/\omega_n \Rightarrow \zeta = \frac{a_1 \omega_n}{2} = \frac{a_1}{2\sqrt{a_2}} = 0.48$$

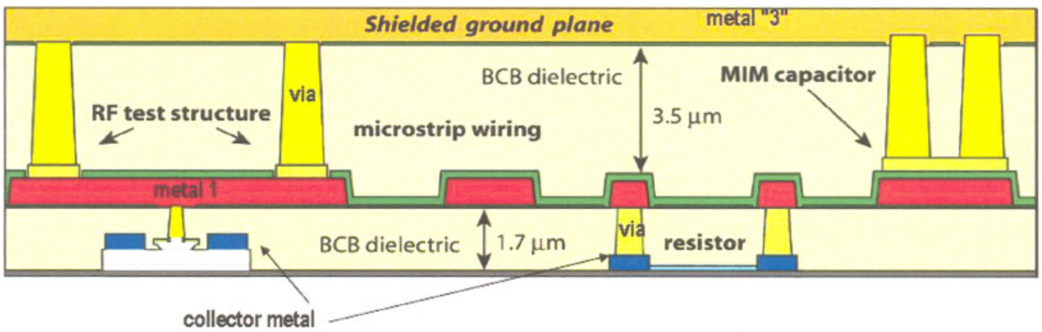
$$\boxed{\zeta \cong 0.5} \quad \boxed{f_n \cong 137 \text{ GHz}}$$

Part E, 10 points

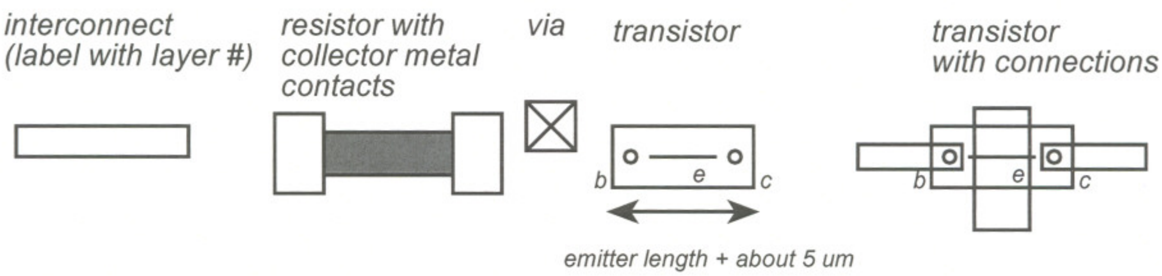
mask layout and transmission-line issues.

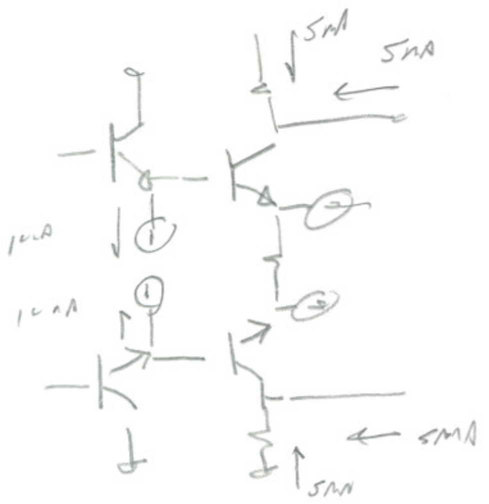
Resistors in the process have 50 Ohm sheet resistance and carry a maximum of 2 mA current per micron of width. Transmission lines are inverted microstrip, and have a characteristic impedance of $Z_o \sim 377\Omega * \varepsilon_r^{-1/2} * H/(H+W)$, where W is the conductor width, H= 5 micrometers and ε_r=3. Their propagation velocity is $v \sim c * \varepsilon_r^{-1/2}$, where c is the speed of light. Wires also carry maximum of 2 mA current per micron of width

The process cross section is like so:



Using notation like so, *carefully* draw a mask layout of the IC. Give lengths and widths of all your resistors and all your wires. Make sure that your layout includes the outline of the ground plane and shows the input and output connections. Indicate the current sources by resistors of large but unspecified length and of a width consistent with the current density limits given above.





wires, [resistor symbol] = 2mA/μm

5mA → 2.5μm minimum width

10mA → 5μm minimum width.

50Ω = 1□ = 2.5μm · 2.5μm.

$$50\Omega = \frac{377\Omega}{\sqrt{3}} \cdot \frac{H}{H+W}$$

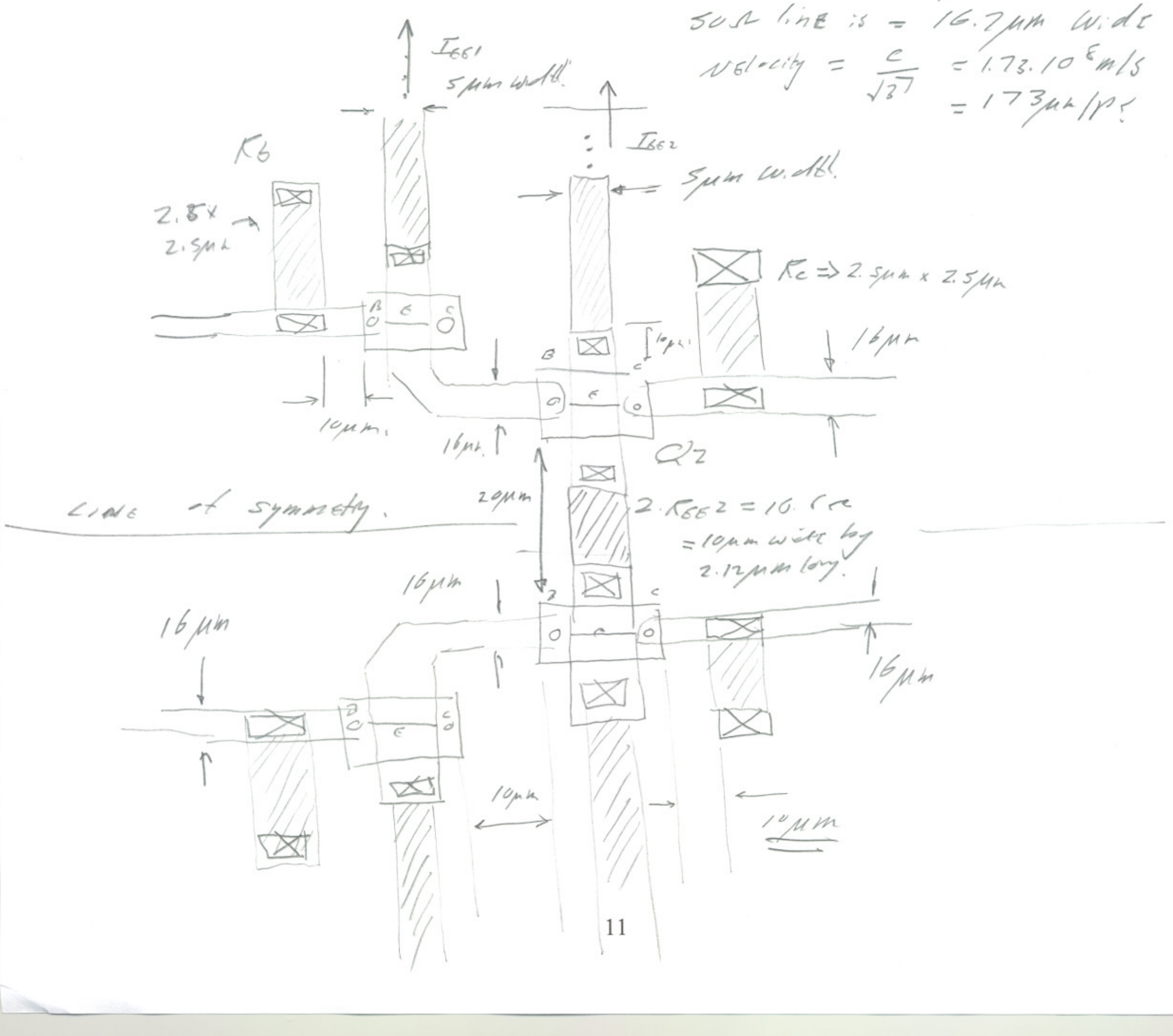
$$= 217\Omega \cdot \frac{H}{H+W}$$

$H = 5\mu m$

50Ω line is = 16.7μm wide

$$\text{velocity} = \frac{c}{\sqrt{3}} = 1.73 \cdot 10^{8}\, m/s$$

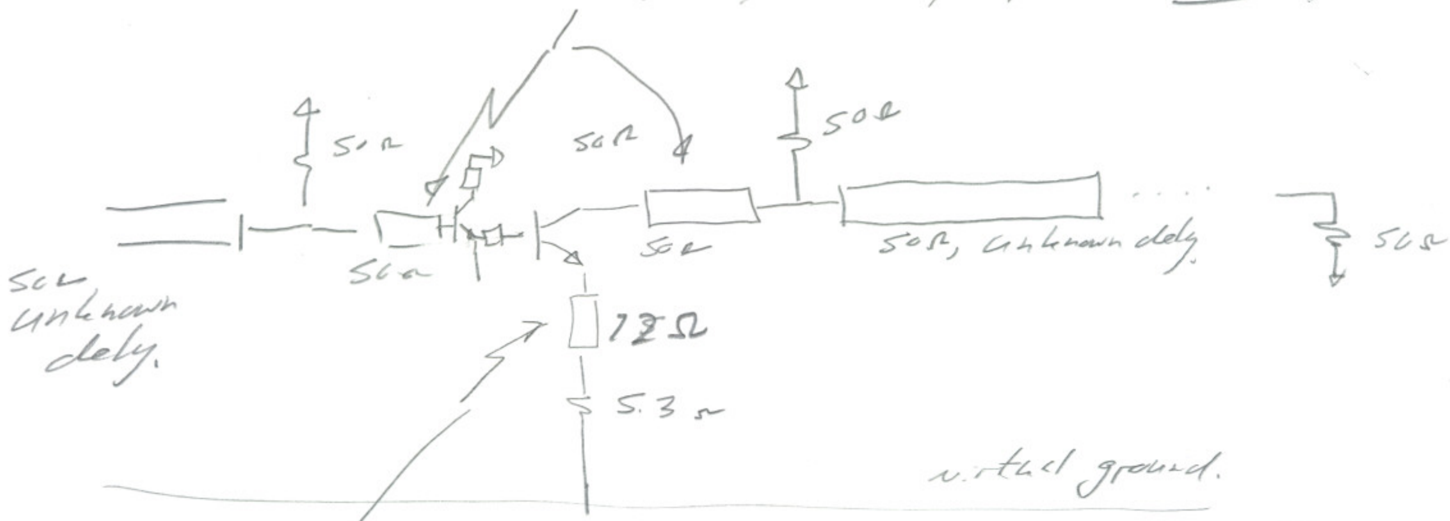
$$= 173\mu m/ps$$



Part F, 10 points
transmission-line issues.

From your hand-drawn layout, draw a schematic diagram of the circuit which gives the characteristic impedance and delay time (length/velocity) of *each and every* interconnect wire you drew on your layout. Comment on the quality (or lack thereof) of the correspondence between your actual layout and the desired IC design.

50Ω, delay = 10μm/(173μm/ps) = $\underline{58\text{ fs}}$



20μm length total ⇒ 10μm length in half-circuit.

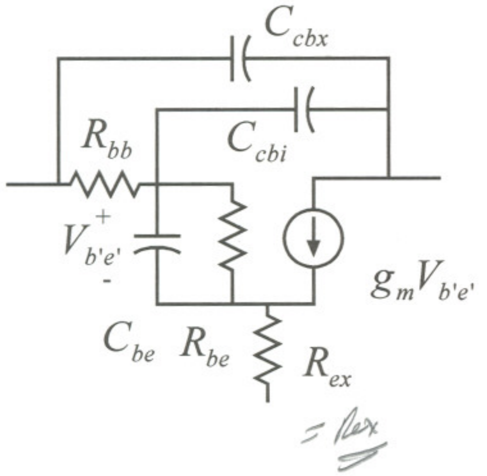
width is that of R_{BE} = 10μm

so $Z_0 = 217\Omega \cdot \frac{5\mu m}{5\mu m + 10\mu m} = \underline{72\Omega}$

$\text{delay} = \frac{20\mu m}{173\mu m/ps} = \underline{116\text{ fs}}$

Problem 2, 30 points

You will be using the device model below:

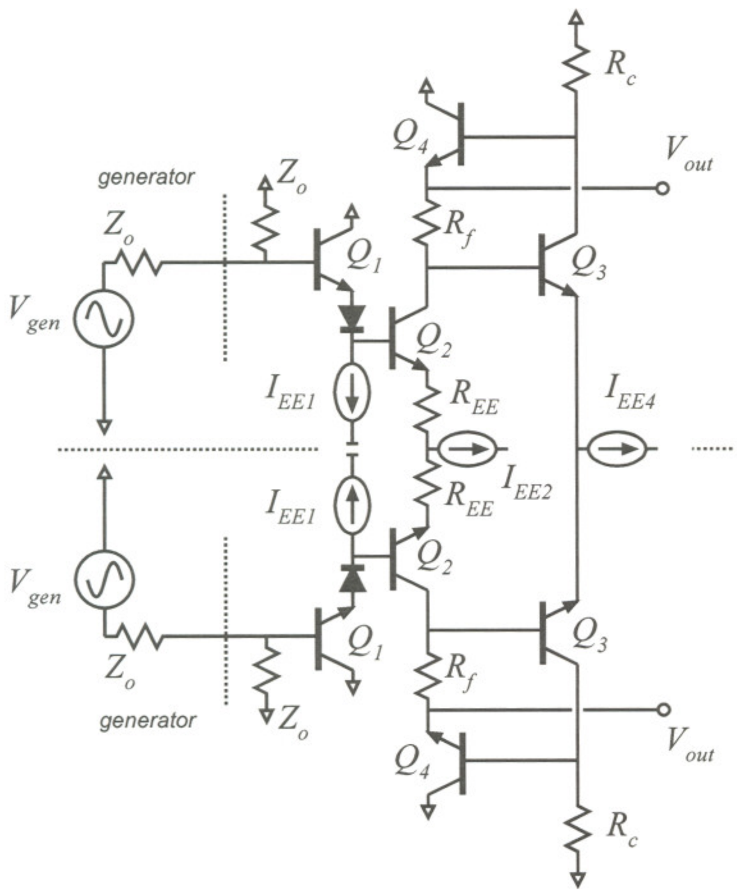


$C_{be} = C_{be,depl} + g_m \tau_f$. **Cje=6 fF**, beta=infinity, **Ccbi=3 fF**, Ccbx=0 fF, **Rbb=15 Ohms**, Rex=0 Ohms, and **tau_f=0.25 ps**. This transistor model has **1 um^2** emitter area.

The transistors have a Vbe=0.9 Volts when operating at **10.0 mA/um^2**.

(these #s for 250 nm generation InP HBT--except the HBT really has 5-10 Ohm-um^2 emitter resistivity)

You will be working with the circuit below--the dotted line indicates virtual ground for the differential half-circuit. This is a standard Cherry-Hooper drawn floorplan-style.



$\frac{KT}{q} \ln \frac{2mA}{10mA} = 42mV$, so $V_{bb} = 0.9V - 42mV$
$= 0.858V$

Part a: 10 points:
DC bias

≡ approximate as 0.9V ≡

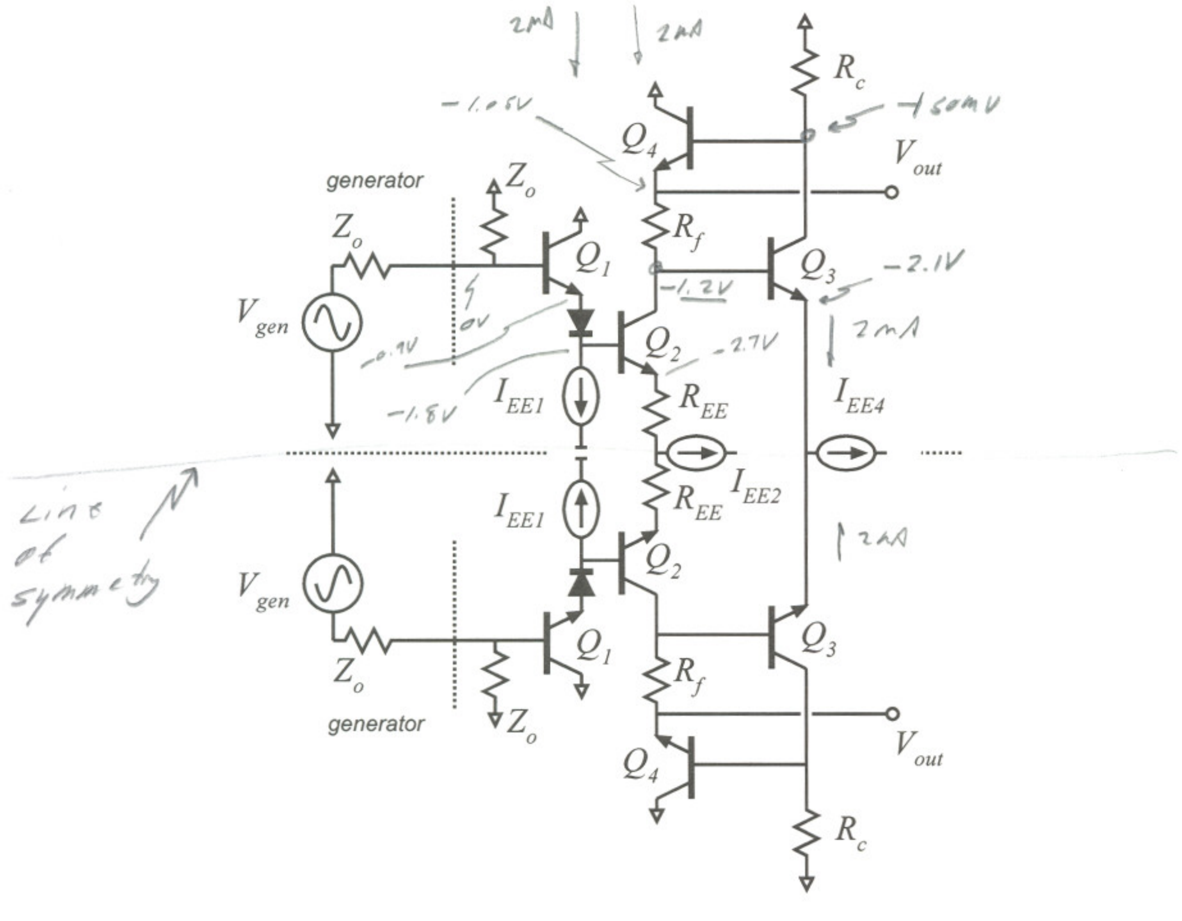
All transistors are biased at 2 mA emitter or collector current. Zo is 50 Ohms. The voltage drops across R_f and R_c are both 150 mV. The DC input voltage is Zero volts.

Find the following:

$R_C =$ 75 Ω $\qquad R_F =$ 75 Ω

$I_{EE1} =$ 2 mA. $\qquad I_{EE2} =$ 4 mA $\qquad I_{EE3} =$ 4 mA.

Draw all node voltages and branch currents on the circuit diagram.



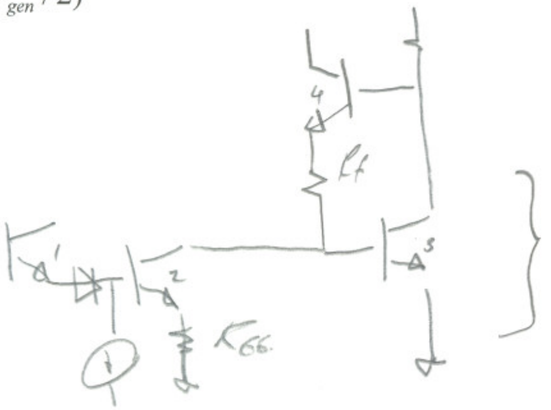
$R_C = \frac{150mV}{2mA} = 75\Omega = R_f$

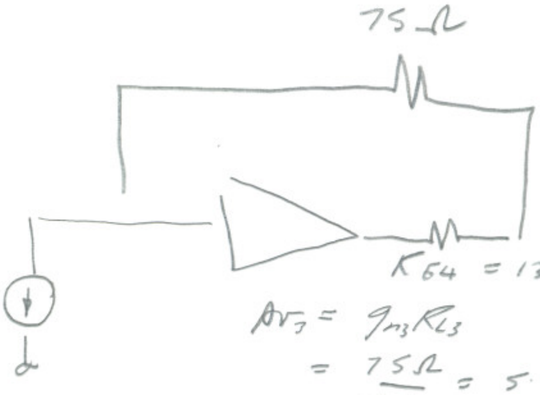
Part b, 10 points

Circuit gains

$r_6 = 13\Omega$

Find the value of Ree required for 10 dB circuit gain----gain being defined as $V_{out}/(V_{gen}/2)$





$R_{E4} = 13\Omega$

$A_{v3} = g_{m3}R_{L3} = \frac{75\Omega}{13\Omega} = 5.77$

$g_{mA} = \frac{1}{r_{E2} + R_{E6}}$

$V_{out} = Z_T \cdot I_i$

$\frac{V_i}{I_i} = \frac{(R_f + R_{out})}{1 + A_v}$

$V_i = \frac{R_f + R_{out}}{1 + A_v} I_i$

$V_{out} - V_i = -R_f I_i$

$Z_T = R_f - \frac{R_f + R_{out}}{1 + A_v}$

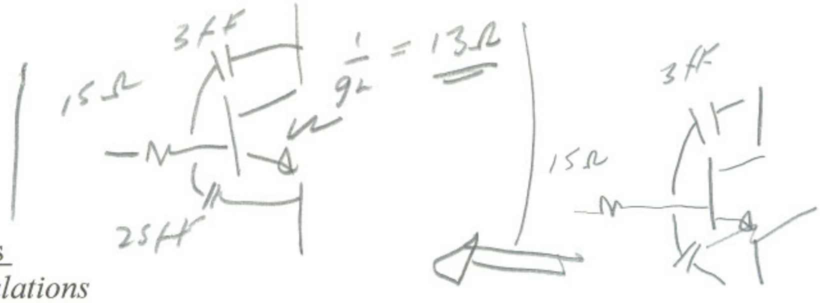
$= 75\Omega - \frac{75\Omega + 13\Omega}{6.77} = 75\Omega - 12.99\Omega$

$= 62\Omega$

$A_v = 3.16 = \frac{62\Omega}{r_{E2} + R_{E6}} \rightarrow r_{E2} + R_{E6} = 19.6\Omega$

$R_{E6} = 19.6 - r_{E2} = 19.6 - 13\Omega$

$= 6.62\Omega$



Part C, 10 points

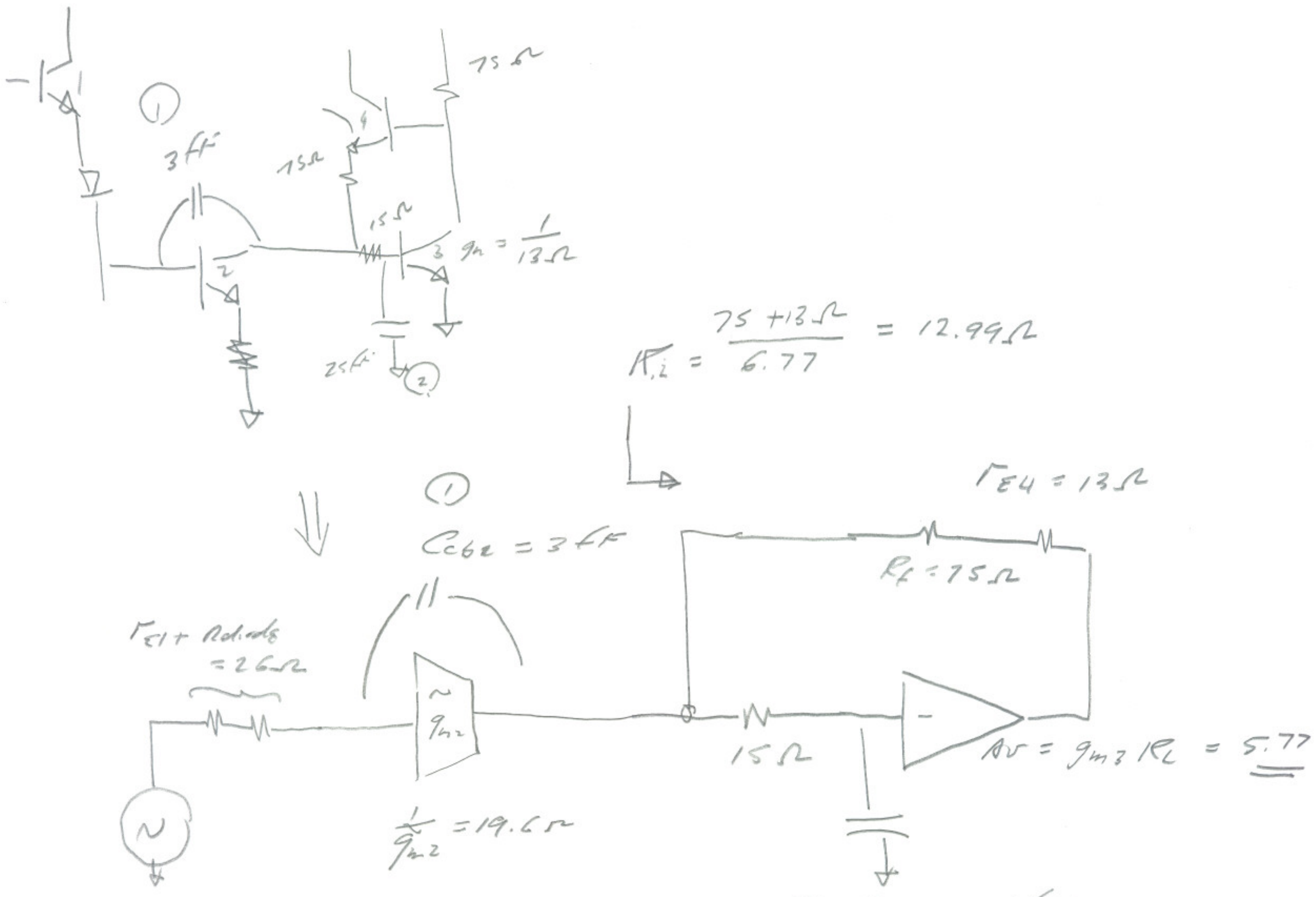
bandwidth calculations

$6fF + g_m \tau_f = 6fF + \frac{0.25ps}{13\Omega} = 6fF + 19fF = 25fF$

considering only the base-emitter capacitance of Q3 and the base-collector capacitance of Q2, determine the circuit first and second-order time constants

$a_1 =$ 549 fs $\qquad a_2 =$ $7 \cdot 10^{-26}$ sec^2

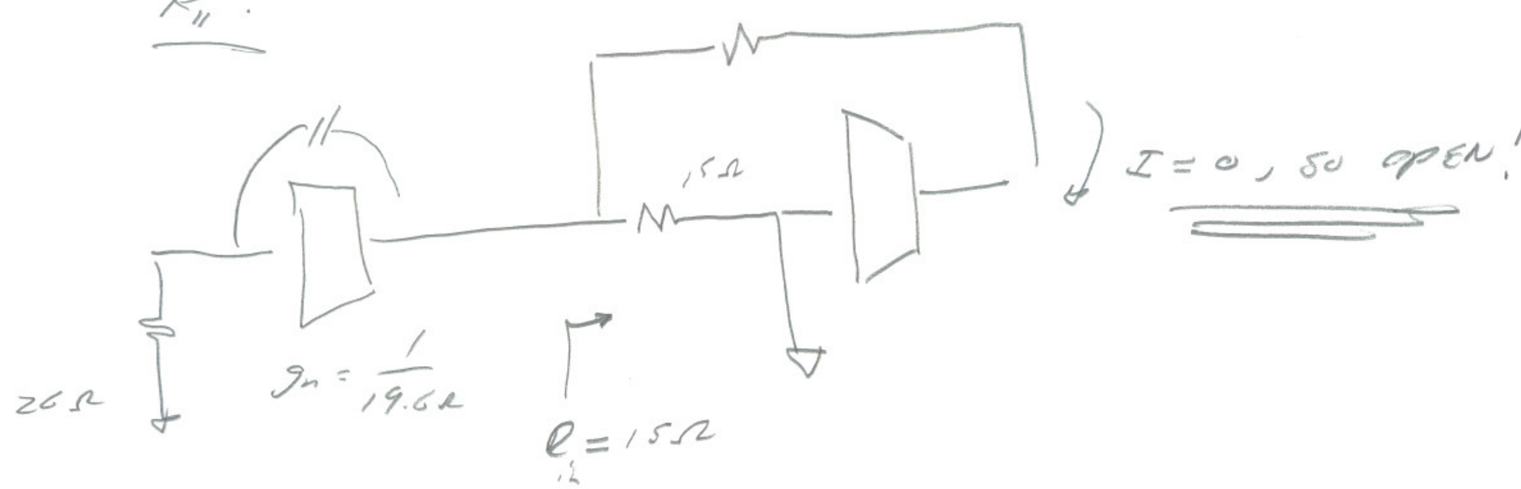
and from this estimate the circuit bandwidth (and--if relevant---damping factor.)



$$R_{11}^{0} = 26\left(1 + \frac{12.99}{19.6}\right) + 12.99\Omega = 56.2\Omega$$

$$R_{22}^{0} = \frac{15\Omega + 75\Omega + 13\Omega}{6.77} = 15.2\Omega$$

$\underline{R_{11}^{2}:}$



$$R_{11}^{2} = 26Ω\left(1 + \frac{15Ω}{19.6Ω}\right) + 15Ω = 60.9Ω$$

$$a_1 = R_{11}^{0} C_1 + R_{22}^{0} C_2 = 56.2Ω \cdot 3fF + 15.2Ω \cdot 25fF$$
$$= 169fs + 380fs = \underline{549fs}$$

$$a_2 = R_{11}^{2} C_1 C_2 R_{22}^{0} = 60.9Ω \cdot 3fF \cdot 380fs =$$
$$= \underline{6.94 \cdot 10^{-26}\ sec^2}$$