ECE 146A: Communication Systems
Lab 2: Introduction to Complex Baseband Models
Carrier Phase Uncertainty and Wireless Multipath Channels

1 Objective

The goal of this lab is to explore modeling and receiver operations in complex baseband, and to practice writing Matlab code from scratch (i.e., without using prepackaged routines or Simulink) for simple computations. We consider two experiments: (1) modeling and undoing the effect of carrier phase mismatch between the receiver LO and the incoming carrier, (2) modeling the effect of a wireless multipath channel.

2 Equipment

Matlab is available in the workstations in the software lab.

3 Experiment 1: Modeling Carrier Phase Uncertainty

Consider a pair of independently modulated signals, $u_c(t) = \sum_{n=1}^{N} b_c[n]p(t-n)$ and $u_s(t) = \sum_{n=1}^{N} b_s[n]p(t-n)$, where the symbols $b_c[n], b_s[n]$ are chosen with equal probability to be +1 and -1, and $p(t) = I_{[0,1]}(t)$ is a rectangular pulse. Let $N = 100$.

(1.1) Use Matlab to plot a typical realization of $u_c(t)$ and $u_s(t)$ over 10 symbols. Make sure you sample fast enough for the plot to look reasonably "nice."

(1.2) Upconvert the baseband waveform $u_c(t)$ to get

$$u_{p,1}(t) = u_c(t)\cos 40\pi t$$

This is a so-called binary phase shift keyed (BPSK) signal, since the changes in phase due to the changes in the signs of the transmitted symbols. Plot the passband signal $u_{p,1}(t)$ over four symbols.

(1.3) Now, add in the Q component to obtain the passband signal

$$u_p(t) = u_c(t)\cos 40\pi t - u_s(t)\sin 40\pi t$$

Plot the resulting Quaternary Phase Shift Keyed (QPSK) signal $u_p(t)$ over four symbols.

(1.4) Downconvert $u_p(t)$ by passing $2u_p(t)\cos(40\pi t + \theta)$ and $2u_p(t)\sin(40\pi t + \theta)$ through crude lowpass filters with impulse response $h(t) = I_{[0,0.25]}(t)$. Denote the resulting I and Q components by $v_c(t)$ and $v_s(t)$, respectively. Plot v_c and v_s for $\theta = 0$ over 10 symbols. How do they compare to u_c and u_s? Can you read off the corresponding bits $b_c[n]$ and $b_s[n]$ from eyeballing the plots for v_c

and v_s?

(1.5) Plot v_c and v_s for $\theta = \pi/4$. How do they compare to u_c and u_s? Can you read off the corresponding bits $b_c[n]$ and $b_s[n]$ from eyeballing the plots for v_c and v_s?

(1.6) Figure out how to recover u_c and u_s from v_c and v_s if a genie tells you the value of θ (we are looking for an approximate reconstruction–the LPFs used in downconversion are non-ideal, and the original waveforms are not exactly bandlimited). Check whether your method for undoing the phase offset works for $\theta = \pi/4$, the scenario in (1.5). Plot the resulting reconstructions $\tilde{u}_c$ and $\tilde{u}_s$, and compare them with the original I and Q components. Can you read off the corresponding bits $b_c[n]$ and $b_s[n]$ from eyeballing the plots for $\tilde{u}_c$ and $\tilde{u}_s$?

4 Experiment 2: Modeling Wireless Multipath Channels

4.1 Background

We now provide a glimpse of wireless channel modeling using complex baseband. There are two key differences between wireless and wireline communication. The first, which is what we focus on now, is multipath propagation due to reflections off of scatterers adding up at the receiver. This addition can be constructive or destructive (as we saw in Example **??**), and is sensitive to small changes in the relative location of the transmitter and receiver which produce changes in the relative delays of the various paths. The resulting fluctuations in signal strength are termed *fading.* The second key feature of wireless, not considered here, is interference: wireless is a broadcast medium, hence the receiver can also hear transmissions other than the one it is interested in. We now explore the effects of multipath fading for some simple scenarios.

Consider a passband transmitted signal at carrier frequency, of the form

$$u_p(t) = u_c(t)\cos 2\pi f_c t - u_s(t)\sin 2\pi f_c t = e(t)\cos(2\pi f_c t + \theta(t))$$

where

$$u(t) = u_c(t) + j u_s(t) = e(t)e^{j\theta(t)}$$

is the complex baseband representation, or complex envelope. In order to model the propagation of this signal through a multipath environment, let us consider its propagation through a path of length r. The propagation attenuates the field by a factor of $1/r$, and introduces a delay of $\tau(r) = \frac{r}{c}$, where c denotes the speed of light. Suppressing the dependence of τ on r, the received signal is given by

$$v_p(t) = \frac{A}{r} e(t-\tau)\cos(2\pi f_c(t-\tau) + \theta(t-\tau) + \phi)$$

where we consider relative values (across paths) for the constants A and ϕ. The complex envelope of $v_p(t)$ with respect to the reference $e^{j2\imath f_c t}$ is given by

$$v(t) = \frac{A}{r} u(t-\tau) e^{-j(2\pi f_c \tau + \phi)} \tag{1}$$

For example, we may take $A = 1$, $\phi = 0$ for a direct, or line of sight (LOS), path from transmitter to receiver, which we may take as a reference. Figure 1 shows the geometry of for a reflected path corresponding to a single bounce, relative to the LOS path. Follow standard terminology, θ_i denotes the angle of incidence, and $\theta_g = \frac{\pi}{2} - \theta_i$ the *grazing angle.* The change in relative amplitude and phase due to the reflection depends on the carrier frequency, the reflector material, the angle of incidence, and the polarization with respect to the orientation of the reflector surface. Since we do not wish to get into the underlying electromagnetics, we consider simplified models of relative amplitude and phase. In particular, we note that for grazing incidence ($\theta_g \approx 0$), we have $A \approx 1$, $\phi \approx \pi$.

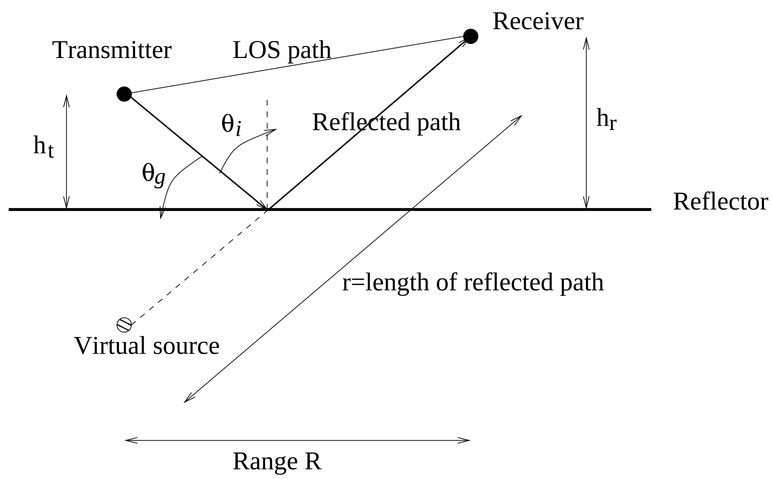

Figure 1: Ray tracing for a single bounce path. We can reflect the transmitter around the reflector to create a virtual source. The line between the virtual source and the receiver tells us where the ray will hit the reflector, following the law of reflection that the angles of incidence and reflection must be equal. The length of the line equals the length of the reflected ray to be plugged into (4).

Generalizing (1) to multiple paths of length $r_1, r_2, ...$, the complex envelope of the received signal is given by

$$v(t) = \sum_i \frac{A_i}{r_i} u(t - \tau_i) e^{-j(2\pi f_c \tau_i + \phi_i)} \tag{2}$$

where $\tau_i = \frac{r_i}{c}$, and A_i, ϕ_i depend on the reflector characteristic and incidence angle for the ith ray. This corresponds to the complex baseband channel impulse response

$$h(t) = \sum_i \frac{A_i}{r_i} e^{-j(2\pi f_c \tau_i + \phi_i)} \delta(t - \tau_i) \tag{3}$$

This is in exact correspondence with our original multipath model (**??**), with $\alpha_i = \frac{A_i}{r_i} e^{-j(2\pi f_c \tau_i + \phi_i)}$. The corresponding frequency domain response is given by

$$H(f) = \sum_i \frac{A_i}{r_i} e^{-j(2\pi f_c \tau_i + \phi_i)} e^{-j2\pi f \tau_i} \tag{4}$$

Since we are modeling in complex baseband, f takes values around DC, with $f = 0$ corresponding to the passband reference frequency f_c.

Channel delay spread and coherence bandwidth: Let τ_{min} and τ_{max} denote the minimum and maximum of the delays $\{\tau_i\}$. The difference $\tau_d = \tau_{max} - \tau_{min}$ is called the *channel delay* *spread.* The reciprocal of the delay spread is termed the *channel coherence bandwidth,* $B_c = \frac{1}{\tau_d}$. A baseband signal of bandwidth W is said to be *narrowband* if $W\tau_d = W/B_c \ll 1$, or equivalently, if its bandwidth is significantly smaller than the channel coherence bandwidth.

We can now infer that, for a narrowband signal around the reference frequency, the received complex baseband signal equals a delayed version of the transmitted signal, scaled by the complex channel gain

$$h = H(0) = \sum_i \frac{A_i}{r_i} e^{-j(2\pi f_c \tau_i + \phi_i)} \tag{5}$$

Two ray model: Suppose our propagation environment consists of the LOS ray and the single reflected ray shown in Figure 1. Then we have two rays, with $r_1 = \sqrt{R^2 + (h_r - h_t)^2}$ and $r_2 = \sqrt{R^2 + (h_r + h_t)^2}$. The corresponding delays are $\tau_i = r_i/c$, $i = 1, 2$, where c denotes the speed of propagation. The grazing angle is given by $\theta_g = \tan^{-1} \frac{h_t + h_r}{R}$. Setting $A_1 = 1$ and $\phi_1 = 0$, once we specify A_2 and ϕ_2 for the reflected path, we can specify the complex baseband channel.

We now apply ray tracing for modeling a lamppost based network.

4.2 Modeling a lamppost based broadband network

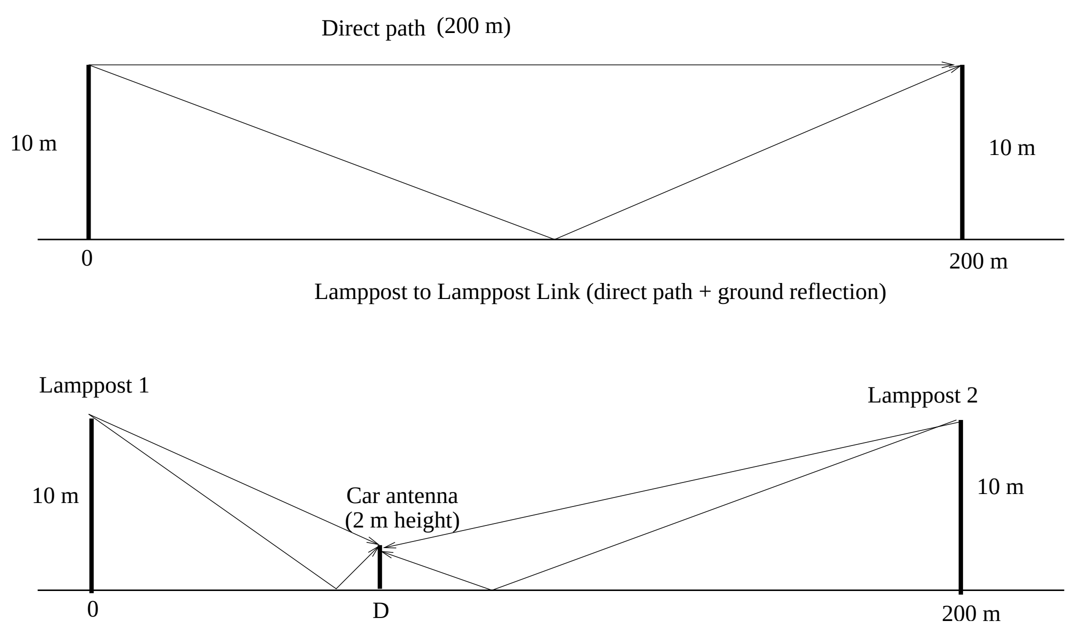

Figure 2: Links in a lamppost-based network.

Consider a lamppost-based network supplying broadband access using unlicensed spectrum at 5 GHz. Figure 2 shows two kinds of links: lamppost-to-lamppost for backhaul, and lamppost-to-mobile for access, where we show nominal values of antenna heights and distances. We explore simple channel models for each case, consisting only of the direct path and the ground reflection. For simplicity, assume throughout that $A_1 = 1$, $\phi_1 = 0$ for the direct path, and $A_2 = 0.98$, $\phi_2 = \pi$ for the ground reflection (we assume a phase shift of π for the reflected ray even though it may not be at grazing incidence, especially for the lamppost to mobile link).

(2.1) Find the delay spread and coherence bandwidth for the lamppost-to-lamppost link. If the message signal has 20 MHz bandwidth, is it "narrowband" with respect to this channel?
(2.2) Repeat item (2.1) for the lamppost-to-car link when the car is 100 m away from each lamppost.

4.3 Fading and diversity for the backhaul link

First, let us explore the sensitivity of the lamppost to lamppost link to variations in range and height. Fix the height of the transmitter on lamppost 1 at 10 m, Vary the height of the receiver on lamppost 2 from 9.5 to 10.5 m.
(2.3) Letting h_{nom} denote the nominal channel gain between two lampposts if you only consider the direct path and h the net complex gain including the reflected path, plot the normalized power gain in dB, $20 \log_{10} \frac{|h|}{|h_{nom}|}$, as a function of the variation in the receiver height. Comment on the sensitivity of channel quality to variations in the receiver height.
(2.4) Modeling the variations in receiver height as coming from a uniform distribution over $[9.5, 10.5]$, find the probability that the normalized power gain is smaller than -20 dB? (i.e., that we have a fade in signal power of 20 dB or worse).
(2.5) Now, suppose that the transmitter has two antennas, vertically spaced by 25 cm, with the lower one at a height of 10 m. Let h_1 and h_2 denote the channels from the two antennas to the receiver. Let h_{nom} be defined as in item (2.3). Plot the normalized power gains in dB, $20 \log_{10} \frac{|h_i|}{|h_{nom}|}$, $i = 1, 2$. Comment on whether or not both gains dip or peak at the same time.
(2.6) Plot $20 \log_{10} \frac{\max(|h_1|,|h_2|)}{|h_{nom}|}$, which is the normalized power gain you would get if you switched to the transmit antenna which has the better channel. This strategy is termed *switched diversity.*
(2.7) Find the probability that the normalized power gain of the switched diversity scheme is smaller than -20 dB.
(2.8) Comment on whether, and to what extent, diversity helped in combating fading.

4.4 Fading on the access link

Consider the access channel from lamppost 1 to the car. Let $h_{nom}(D)$ denote the nominal channel gain from the lamppost to the car, *ignoring the ground reflection.* Taking into account the ground reflection, let the channel gain be denoted as $h(D)$. Here D is the distance of the car from the bottom of lamppost 1, as shown in Figure 2.
(2.9) Plot $|h_{nom}|$ and $|h|$ as a function of D on a dB scale (an amplitude α is expressed on the dB scale as $20 \log_{10} \alpha$). Comment on the "long-term" variation due to range, and the "short-term" variation due to multipath fading.

5 Lab Report

- Answer all questions and print out the most useful plots to support your answers.
- Write a paragraph about any questions or confusions that you may have experienced with this lab.