ECE 146A: Solutions to Problem Set 5

Problem 3.13 A dual band radio operates at 900 MHz and 1.8 GHz. The channel spacing in each band is 1 MHz. We wish to design a superheterodyne receiver with an IF of 250 MHz. The LO is built using a frequency synthesizer that is tunable from 1.9 to 2.25 GHz, and frequency divider circuits if needed (assume that you can only implement frequency division by an integer).

(a) How would you design a superhet receiver to receive a passband signal restricted to the band 1800-1801 MHz? Specify the characteristics of the RF and IF filters, and how you would choose and synthesize the LO frequency.

(b) Repeat (a) when the signal to be received lies in the band 900-901 MHz.

Solution 3.13(a) The center frequency is f_{RF} = 1800.5 MHz. For f_{IF} = 250 MHz, we have two possible choices for the LO frequency: $f_{LO} = f_{RF} + f_{IF}$ = 2050.5 MHz and $f_{LO} = f_{RF} - f_{IF}$ = 1549.5 MHz. The first choice for LO frequency falls within the tunable range of the frequency synthesizer. The second choice does not work, since it falls outside the tunable range, and cannot be obtained by frequency division by an integer from the tunable range.

Thus, we adopt the first choice, and have f_{LO} = 2050.5 MHz, $f_{RF} = f_{LO} - f_{IF}$ = 1800.5 MHz, and image frequency $f_{IM} = f_{LO} + f_{IF}$ = 2300.5 MHz.

The RF filter should be centered at 1800.5 MHz, passing the desired signal in 1800-1801 MHz while rejecting the image frequency at 2300.5 MHz. A bandwidth of 20 MHz comfortably does the trick, for example. The fixed IF filter at 250 MHz should pass the message, which falls in 249.5-250.5 MHz, and provide a sharp cut-off beyond.

(b) The center frequency is f_{RF} = 900.5 MHz. For f_{IF} = 250 MHz, we have two possible choices for the LO frequency: $f_{LO} = f_{RF} + f_{IF}$ = 1150.5 MHz and $f_{LO} = f_{RF} - f_{IF}$ = 649.5 MHz. Both choices fall outside the tunable range of the frequency synthesizer. The first choice cannot be obtained by integer frequency division (just falls outside the range obtained by dividing by two, which is 950-1125 MHz). The second choice can be obtained by dividing 1948.5 MHz (which does fall in the tunable range) by three.

Thus, we adopt the second choice, and have f_{LO} = 649.5 MHz, $f_{RF} = f_{LO} + f_{IF}$ = 900.5 MHz, and image frequency $f_{IM} = f_{LO} - f_{IF}$ = 399.5 MHz. The RF filter should be centered at 900.5 MHz, passing the desired signal in 900-901 MHz while rejecting the image frequency at 399.5 MHz. As before, a 20 MHz bandwidth would work. The IF filter specs are as in (a).

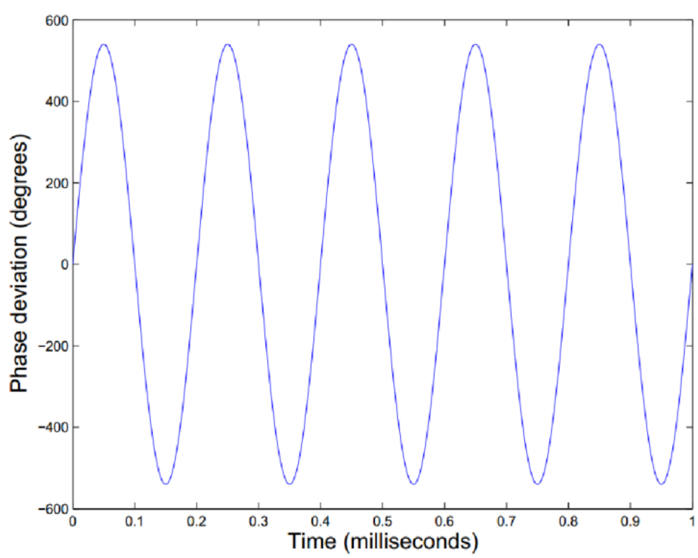

Figure 5 for Problem 3.14.

Problem 3.14 Figure 5 shows, as a function of time, the phase deviation of a bandpass FM signal modulated by a sinusoidal message.

(a) Find the modulation index (assume that it is an integer multiple of π for your estimate).

(b) Find the message bandwidth.

(c) Estimate the bandwidth of the FM signal using Carson's formula.

Solution 3.14: (a) For a sinusoidal message at frequency f_m, we have seen that the phase deviation of the FM signal is given by $\beta \sin 2\pi f_m t$, where $\beta = \frac{\Delta f_{max}}{f_m}$ is the modulation index. Thus, the maximum phase deviation is β. From the figure, we see that the maximum deviation is about 550 degrees. Since we assume β is a multiple of π, we set $\beta = 3\pi$ radians.

(b) The phase deviation is sinusoidal with frequency f_m. From the plot, $1/f_m = 0.2$ ms, so that $f_m = 5$ KHz, which is the message bandwidth.

(c) By Carson's formula, $B_{FM} \approx 2(\beta + 1)f_m = 2(3\pi + 1)5 = 104$ KHz.

Problem 3.15 The input $m(t)$ to an FM modulator with $k_f = 1$ has Fourier transform

$$M(f) = \begin{cases} j2\pi f & |f| < 1 \\ 0 & else \end{cases}$$

The output of the FM modulator is given by $u(t) = A\cos(2\pi f_c t + \varphi(t))$

where f_c is the carrier frequency.

(a) Find an explicit time domain expression for $\varphi(t)$ and carefully sketch $\varphi(t)$ as a function of time. (b) Find the magnitude of the instantaneous frequency deviation from the carrier at time $t = \frac{1}{4}$.

(c) Using the result from (b) as an approximation for the maximum frequency deviation, estimate the bandwidth of $u(t)$.

Solution 3.15

$$\phi(t) = \phi(0) + 2\pi k_f \int_0^t m(\tau)d\tau$$

$$\mathcal{F}[m(t)] = M(f) = j2\pi f I_{[-1,1]}(f)$$

$$\Rightarrow \mathcal{F}[\int_0^t m(\tau)d\tau] = \frac{M(f)}{j2\pi f} = I_{[-1,1]}(f)$$

$$\Rightarrow \int_0^t m(\tau)d\tau = \mathcal{F}^{-1}I_{[-1,1]}(f) = 2sinc(2t)$$

$$\Rightarrow \phi(t) = \phi(0) + 2\pi k_f\,(2sinc(2t))$$

a) FM waveform phase $\Rightarrow \phi(t) = 4\pi sinc(2t) + K$

b) The instantaneous frequency deviation from the carrier at time t=1/4 will be

$$f(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

$$\Rightarrow f(\frac{1}{4}) = -\frac{16}{\pi} = 5.09$$

(b) Using Carson's formula, we get the following estimate for the bandwidth of output u:

$$B_{FM} \approx 2\Delta f_{max} + 2W \approx 2(5.09)+2(1)= 12.18$$

Problem 3.16 Let $p(t) = I_{[-\frac{1}{2},\frac{1}{2}]}(t)$ denote a rectangular pulse of unit duration. Construct the signal

$$m(t) = \sum_{n=-\infty}^{\infty} (-1)^n p(t-n)$$

The signal $m(t)$ is input to an FM modulator, whose output is given by

$$u(t) = 20\cos(2\pi f_c t + \varphi(t))$$

$$\Phi(t) = 20\pi \int_{-\infty}^{t} m(\tau)d\tau + a$$ where_

and a $\phi(0) = 0.$ is chosen such that

(a) Carefully sketch both $m(t)$ and $\varphi(t)$ as a function of time.

(b) Approximating the bandwidth of $m(t)$ as $W \approx 2$, estimate the bandwidth of $u(t)$ using Carson's formula.

(c) Suppose that a very narrow ideal BPF (with bandwidth less than 0.1) is placed at $f_c + \alpha$. For which (if any) of the following choices of α will you get nonzero power at the output of the BPF: (i) $\alpha = .5$, (ii) $\alpha = .75$, (iii) $\alpha = 1$.

Solution 3.16 (a) See Figure 1.

(b) We see from the figure that the instantaneous frequency deviation $f(t) = \frac{1}{2\pi}\frac{d\phi}{dt}$ takes values -10 and 10, so that $\Delta f_{max} = 10$. Using Carson's formula, we get the following estimate for the bandwidth of the VCO output u:

$$B_{FM} \approx 2\Delta f_{max} + 2W \approx 24$$

(c) The message $m(t)$ is periodic with period 2, and hence so is $\varphi(t)$ and the complex envelope $e^{j\varphi(t)}$. We expect to find Fourier series components for the complex envelope at nf_m, where $f_m = \frac{1}{2}$ is the fundamental frequency and n takes integer values, so that the spectrum of the passband signal has discrete components at $f_c + nf_m$. Thus, we may get nonzero powers at $f_c + \alpha$ for $\alpha = 0.5, 1$, but we will get zero power for $\alpha = 0.75$.

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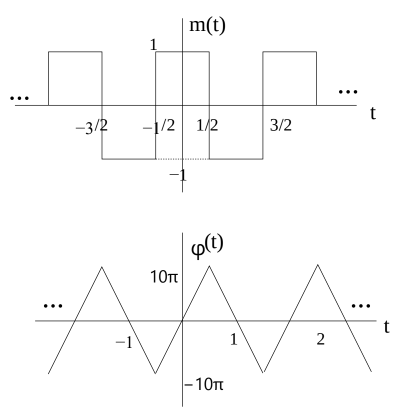

Figure 1: Sketch of message $m(t)$ and phase deviation $\varphi(t)$ for Problem 3.16(a).