ECE 146B/UCSB: Digital Communication
Lab 3: Introduction to Channel Equalization

Reading: Chapter 4 (linear modulation), Chapter 6 (Section 6.1 on Gaussian random variables and Q function), and additional notes on equalization (an incomplete version of Chapter 7). We also state and use probability of error expressions from Chapter 6, but reading about how these are derived is not required at this point.

Lab Objectives: To understand the need for equalization in communication systems, and to implement linear MMSE equalizers adaptively.

Laboratory Assignment

0) Use your own code from Lab 1 or Lab 2 as a starting point. If you had difficulty in completing these labs, ask the TA for a template. As before, the transmit, channel, and receive filters are implemented at rate $4/T$. For simplicity, we consider BPSK signaling throughout this lab, and consider only real-valued signals. Generate $nsymbols = ntraining + npayload$ (numbers to be specified later) ± 1 BPSK symbols as in Lab 2, and pass them through the transmit, channel, and receive filters to get noiseless received samples at rate $4/T$.

1) Let us start with a trivial channel filter as before. Set $nsymbols = 200$. The number of rate $4/T$ samples at the output of the receive filter is therefore 800, plus tails at either end because the length of the effective pulse modulating each symbol extends over multiple symbol intervals. Take, say, 400 samples from somewhere in the middle of these samples, avoiding the tails. Divide these into segments of length equal to 2 symbol intervals, or 8 samples (you can do this by reshaping the column vector of 400 samples into an 8×50 matrix). Plot these overlapped intervals against time, expressed in units of the symbol interval (i.e., the x axis goes from 0 to 2, since you are plotting segments of the received waveform spanning two symbol intervals). You should get something like Figure 1, which is called an *eye diagram* because of its shape. The figure shows many different trajectories corresponding to the different symbol patterns that affect the received output over different two-symbol intervals. Because the cascade of the transmit and receive filter is approximately Nyquist, the eye is *open:* we can find a sampling time such that we can distinguish between +1 and -1 well, despite the influence of neighboring symbols.

2) Now introduce a non-trivial channel filter. In particular, consider a channel filter specified (at rate $4/T$) using the following matlab command:

$$\text{channel\_filter} = [-0.7, -0.3, 0.3, 0.5, 1, 0.9, 0.8, -0.7, -0.8, 0.7, 0.8, 0.6, 0.3]';$$

Generate an eye diagram again. You should get something that looks like Figure 2. Notice now that there is no sampling time at which you can clearly make out the difference between +1 and -1 symbols. The eye is now said to be *closed* due to ISI, so that we cannot make symbol decisions just by passing appropriately timed received samples through a thresholding device.

3) You are now going to evaluate probability of error without and with equalization. First, let us generate the noisy output of the receive filter. You will need to generate $nsymbols = ntraining + npayload$ (numbers to be specified later) ± 1 BPSK symbols as in lab 2, and pass them through the transmit filter, the dispersive channel, and the receive filter to get noiseless received samples at rate $4/T$. Since we are signaling along the real axis only, at the *input* to the receive filter, add iid $N(0, \sigma^2)$ real-valued

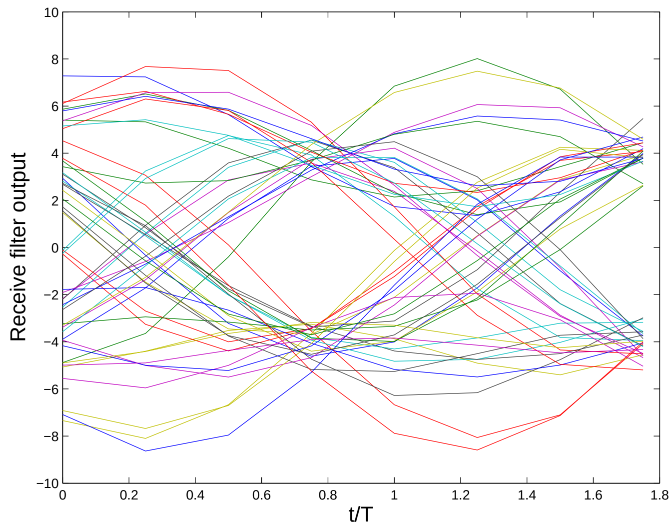

Figure 1: Eye diagram for a non-dispersive channel. The eye is open.

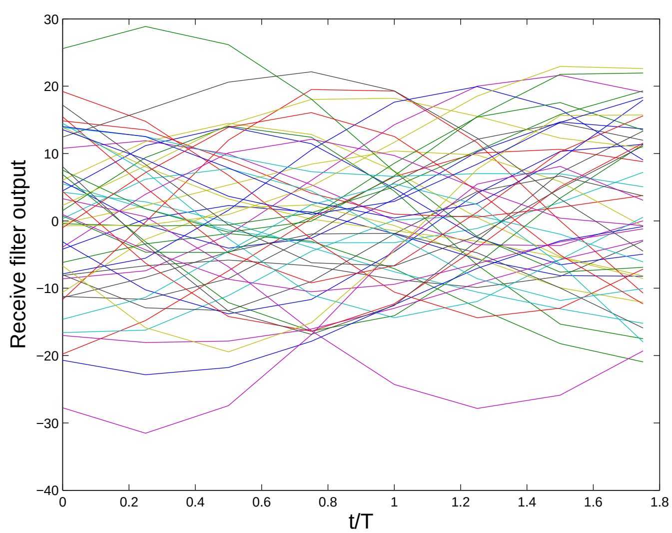

Figure 2: Eye diagram for a dispersive channel. The eye is closed.

noise samples (as in lab 2, choose $\sigma^2 = \frac{N_0}{2}$ corresponding to a specified value of $\frac{E_b}{N_0}$). Pass these (rate $4/T$) noise samples through the receive filter, and add the result to the signal contribution at the receive filter output.

4) **Performance without equalization:** Let $\{r_k\}$ denote the output of the receive filter, and let $Z_n = r_{d+4(n-1)}$, $n = 1, 2, ..., nsymbols$ denote the best symbol rate decision statistics you can obtain by subsampling at rate $1/T$ the receive filter output. As in the solutions to earlier labs, choose the decision delay d equal to the location of the maximum of the overall response (which now includes the channel) to a single symbol. For $nsymbols = 10100$, compute the error probability of the decision rule $\hat{b}_n = sign(Z_n)$ as a function of E_b/N_0, where the latter ranges from 5 to 20 dB. Compare with the ideal error probability curve for BPSK signaling for the same range of E_b/N_0. This will establish that a simple one-sample per symbol decision rule does not work well for non-ideal channels, and motivate the equalization schemes discussed below.

Background on linear equalization

We now consider *linear equalization,* where the decision for symbol b_n is based on linear processing of a vector of samples $\mathbf{r}_n$ of length $L_{eq} = 2L+1$, where the entries of $\mathbf{r}_n$ are samples spaced by T/q, with the center sample being the same as the decision statistic in part 3: $q = 1$ corresponds to symbol-spaced sampling, and $q > 1$ corresponds to fractionally spaced sampling. We will consider two cases: $q = 1$ and $q = 2$.

$$\mathbf{r}_n = (r_{k+4(n-1)+d-(4/q)L}, r_{k+4(n-1)+d-(4/q)(L-1)}, ..., r_{k+4(n-1)+d}, r_{k+4(n-1)+d+(4/q)}, ..., r_{k+4(n-1)+d+(4/q)L})^T$$

The decision rule we use is

$$\hat{b}_n = sign(\mathbf{c}^T \mathbf{r}_n) \tag{1}$$

where $\mathbf{c}$ is a correlator whose choice is to be specified. Note that the decision rule in part 3 corresponds to the choice $\mathbf{c} = (0, .., 0, 1, 0, ..., 0)^T$, since it uses only the center sample.

The vector of samples $\mathbf{r}_n$ contains contributions from both the desired symbol b_n and from ISI due to $b_{n\pm1}$, $b_{n\pm2}$, etc. We will implement the linear minimum mean squared error (MMSE) equalizer, which minimizes the mean squared error (MSE)

$$MSE = E[(\mathbf{c}^T \mathbf{r}_n - b_n)^2]$$

This minimizes the MSE between the decision statistic and desired symbol by minimizing the contribution due to the noise and the ISI at the correlator output.

The MSE is a quadratic function of $\mathbf{c}$, which is minimized by the choice

$$\mathbf{c}_{MMSE} = \{E[\mathbf{r}_n \mathbf{r}_n^T]\}^{-1} E[b_n \mathbf{r}_n]$$

5) We use an adaptive implementation of the MMSE equalizer, which implements the above formula by replacing the statistical expectation by an empirical average. Assume that the first *ntraining* symbols are known training symbols, $b_1, ..., b_{ntraining}$. Define the $L_{eq} \times L_{eq}$ matrix

$$\hat{\mathbf{R}} = \frac{1}{ntraining} \sum_{n=1}^{ntraining} \mathbf{r}_n \mathbf{r}_n^T$$

and the $L_{eq} \times 1$ vector

$$\hat{\mathbf{p}} = \frac{1}{ntraining} \sum_{n=1}^{ntraining} b_n \mathbf{r}_n$$

The MMSE correlator is now approximated as

$$\hat{\mathbf{c}}_{MMSE} = (\hat{\mathbf{R}})^{-1} \hat{\mathbf{p}} \tag{2}$$

6) Now, the correlator obtained via (2) is used to make decisions, using the decision rule (1), on the unknown symbols $n = ntraining + 1, ..., nsymbols$.

7) Fix ntraining = 100 and npayload = 10000. For $L_{eq} = 3, 5, 7, 9$, and $q = 1, 2$, implement linear MMSE equalizers, and plot their error probabilities (for the payload symbols) as a function of E_b/N_0, in the range 5 to 20 dB. to the unequalized error probability and the ideal error probability found in part 3.
Hint: An efficient way to generate the statistics $\mathbf{c}^T \mathbf{r}_n$ is to pass an appropriate rate $2/T$ subsequence of the receive filter output through a filter whose impulse response is the time reverse of $\mathbf{c}$, and to then appropriately subsample at rate $1/T$ the output of the equalizing filter. This is much faster than correlating $\mathbf{c}$ with $\mathbf{r}_n$ for each n.

8) Comment on the performance of symbol-spaced versus fractionally spaced equalization. Comment on the effect of equalizer length of performance. What is the effect of increasing or decreasing the training period?

Lab Report: Your lab report should document the results of the preceding steps in order. Describe the reasoning you used and the difficulties you encountered.