Name ______________________________

ECE 147B

Practice Final Exam

Mar. 16, 2011

- You are allowed two, single-side 8 ½ x 11 sheets of self-prepared notes.
- Use **must** submit these "Cheat Sheets" with your exam; they will later be returned, as is.
- **No calculators, laptops, or other computing devices are allowed.**

____/16 Problem 1: G(s) vs G(z) (via ZOH) Bode analysis.

____/16 Problem 2: C(z) controller design.

____/16 Problem 3: P(z) time response.

____/16 Problem 4: C(z) controller analysis.

____/20 Problem 5: State space control and estimation.

____/18 Problem 6: State space block diagram analysis.

(102 total points possible.)

1) Given a plant with the continuous time (CT) transfer function (TF) below,

$$G(s) = \frac{2000}{(s+20)(s+20)}$$

in the standard ZOH (zero-order hold) configuration, with T=.03 sec.

a) Sketch a Bode plot for G(z). [You may plot magnitude either in dB or simply as a number, plotted in log scale. e.g., "10" could be either 20 dB or simply a magnitude of "10". Just label which convention you are using!]

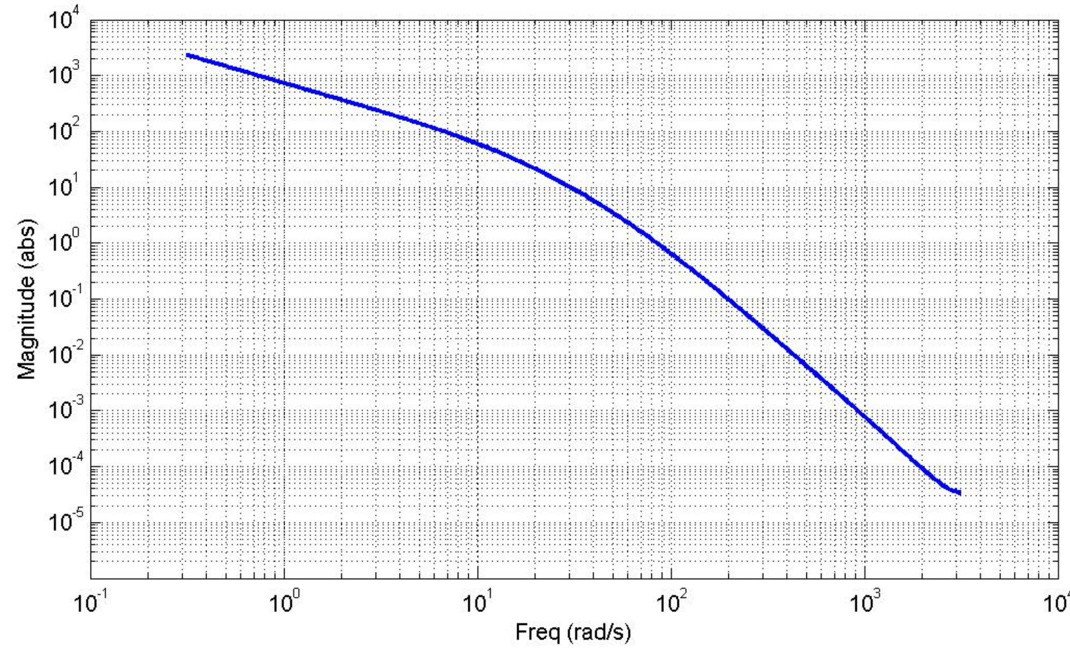
b) What is the approximate phase of G(z) at 20 rad/sec?

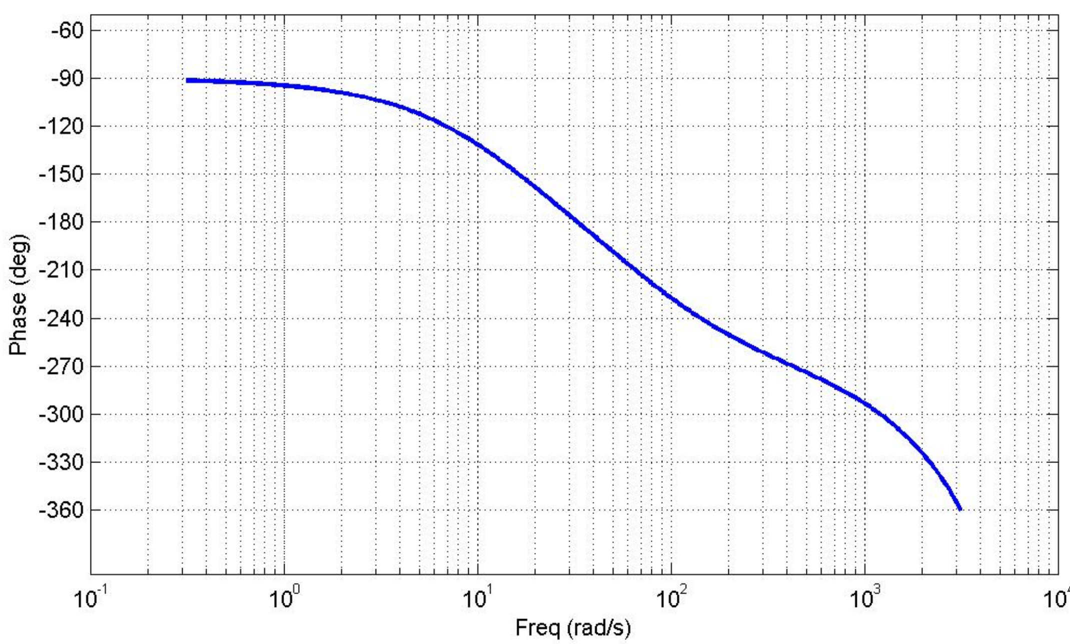
c) What is the approximate gain of G(z) at 20 rad/sec?

d) What is the DC gain of G(z)?

e) Assume we use a proportional controller, C(z) = 1, in the standard unity-feedback configuration. What would the steady state error be to a unit step input? (Note for study: always first check if the CLTF is stable…)

2) Below is a Bode plot shown below for some discrete-time plant, P(z). (See next page for parts 2a through 2d.





(Problem 2, continued…)

a) Solve for values of *a* and *b* (but NOT for *K*) for a DT controller of the form:

$$C(z) = K\left(\frac{z-a}{z-b}\right)$$

to achieve a phase margin of approximately 30 degrees at 30 rad/sec. (Note, you will later solve for *K* in part c.)

b) On the same axes as the Bode plot on the preceding page, sketch the gain and phase for the forward path, $C(z)P(z)$, using your values about for *a* and *b* and using $K=1$. **If you could not solve part a**, use your knowledge of control to sketch the general shape on both the phase and magnitude portions of the Bode plot on the preceding page.

c) Now, using your design from parts a and b, solve for the value of *K* that is required to set the crossover frequency to 30 rad/s. **Only if you could not solve parts a and b**, then instead solve for the value of *K* for a proportional controller, $C(z)=K$, (i.e., a controller with no pole or zero) and state whether or not the resulting closed loop system would be stable (and why).

d) For a system with crossover at 30 rad/s and a phase margin of 30 degrees, estimate *approximate* values for t_{peak} and the percent overshoot of the resulting, closed-loop response to a unit step input.

3) For each of the two DT TF's below, sketch the time response to a unit step input. You must clearly label some key points (in both time and magnitude), of course. Assume T=0.1 seconds in both cases.

a) $P(z) = \frac{5z^2-3}{z^2}$

b) $P(z) = \frac{2(1-e^{.02})}{(z-e^{.02})} \approx \frac{0.0396}{(z-0.9802)}$

4) For this discrete-time controller: $C(z) = \frac{z-.998}{z-.98}$, with sample time T=.001 s.

a) At approximately what frequencies (rad/s) are the equivalent CT pole and zero?

b) What is the DC gain of C(z)?

c) Below, sketch a Bode plot for C(z). Include both magnitude and phase, and label carefully, as always.

d) What type of controller is this, and what is its function? (i.e., in general, why would one use this type of controller?)

5) For a continuous-time state space system,

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

with the following A,B,C, and D matrices:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 \\ -100 & -20 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & 0 \end{bmatrix} \qquad D = [0]$$

a) Write out the differential equation(s) describing the dynamic system.

b) Write a continuous-time transfer function from input U(s) to output Y(s).

c) Solve for the vector *K* for a state space controller, *u*=-*BK*, such that the resulting poles are the solution to the characteristic equation: $s^2 + 25s + 400 = 0$.
(Note that this controller is similar to a PD control, since feedback involves gains on both position and velocity. You may use any clever method to solve for the appropriate values in K by hand!) **If you cannot solve by hand, write appropriate MATLAB commands.**

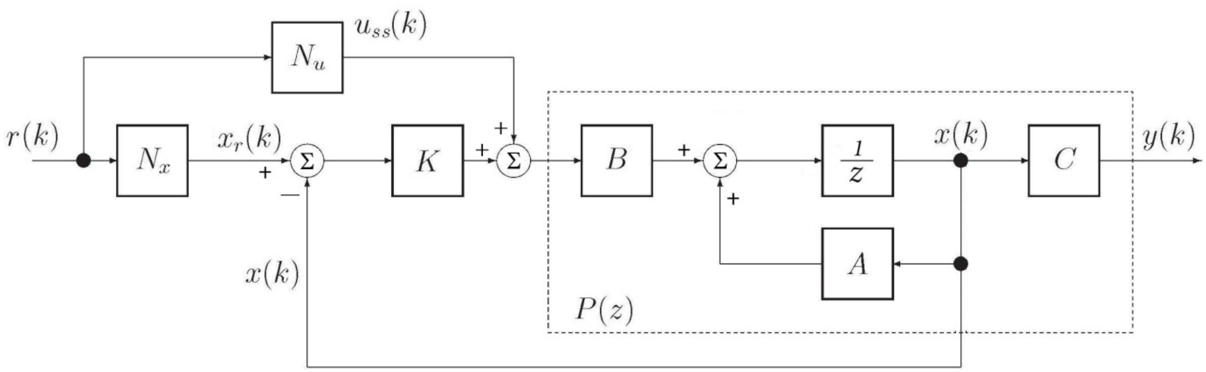
d) Assume you now wish to solve for a vector *L* for a state space estimator. What dimensions must *L* have? (i.e., how many rows and columns must *L* have?) [Recall that the roots of "*A*-*LC*" gives estimator poles, so *LC* must be the same size as *A*…]

e) EXTRA CREDIT: Assume all estimator gains in L are just 1 (where L is of appropriate size, from part d). The poles are solutions to what characteristic equation??

6) Below is a block diagram for a state space feedback configuration we have discussed in class this year. Assume the state space matrices have the following dimensionalities:

A (nxn) ; B (nx1) ; C (mxn) ; [D is just all zeros…]

Also assume *r(k)* and *y(k)* match one another in size. (i.e., that *r(k)* gives a desired set of references for the outputs, *y(k)*.)



a) What is the required dimensionality of N_x?

b) What is the required dimensionality of N_u?

c) What is the function of N_x?

d) What is the function of N_u?

e) Assume the system reaches a steady state (ss). We want the following to be true:

$$x_r = N_x r \approx x_{ss} \quad \text{and} \quad Cx_{ss} = y_{ss} \approx r$$

What must N_x be for this to be true, assuming C=5. (i.e., C is just a scalar here. Also assume N_u is removed from the block diagram, or equivalently set to all zeroes.)

f) Now, assume all the state space matrices are simply scalar values, and that:

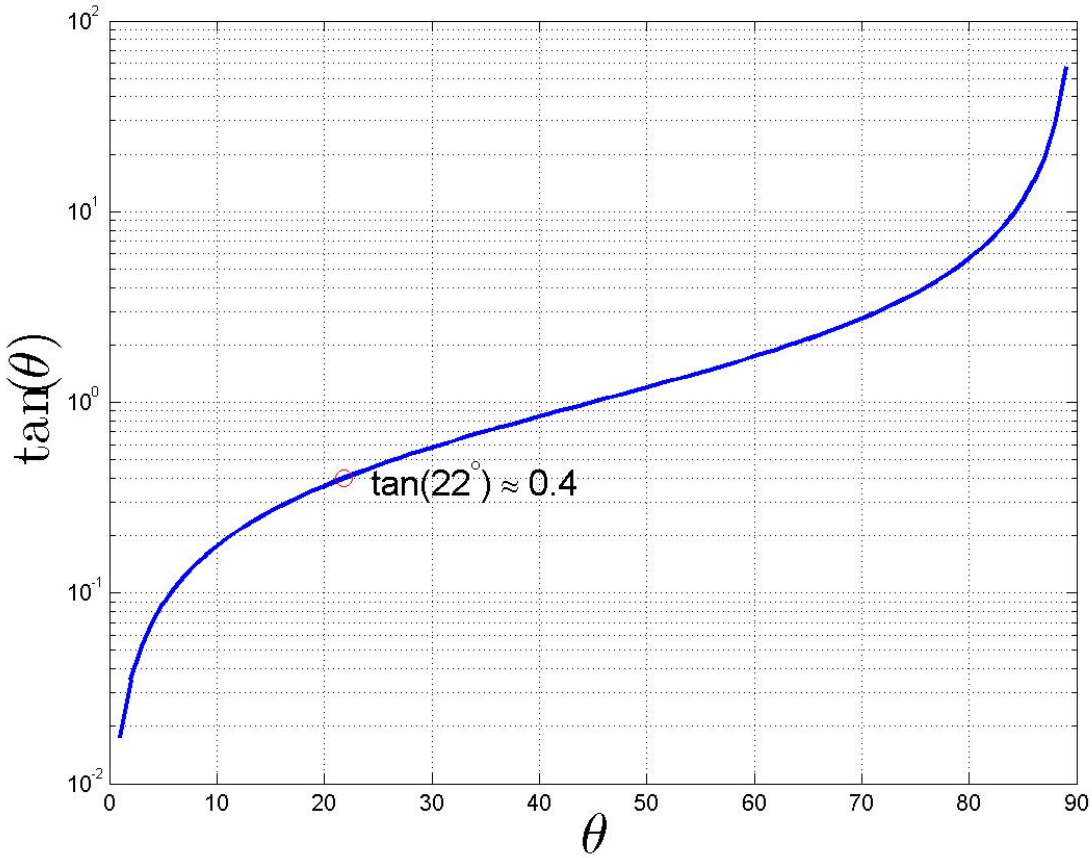
$$C=5, \; K=100.$$

For your solution in part "e" (with N_u removed from the block diagram, or equivalently set to all zeroes) assume the steady state response of y_{ss} to a unit step in the reference is $y_{ss} = 0.8$. What values of N_u must be used with your solution for N_x in part "e" above to yield zero steady state error? (Hint: Block diagram algebra/manipulation may be useful, to separate the N_u and N_x terms from any loops; i.e., so you have some function of K, N_u, and N_x preceding the rest of the block diagram as a total, effective "gain" block.)

End of practice exam.

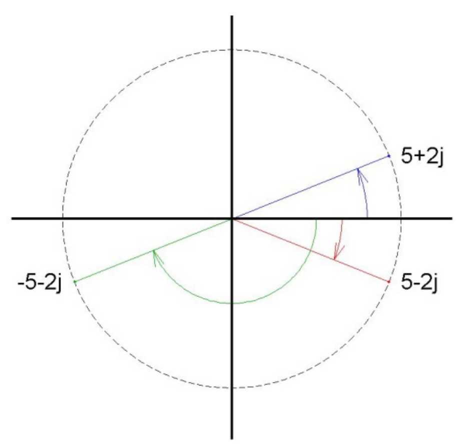
Appendix A: The tangent function.

Note: This Appendix is likely to appear in the actual Final Exam, too, so ***learn to use it now****.*



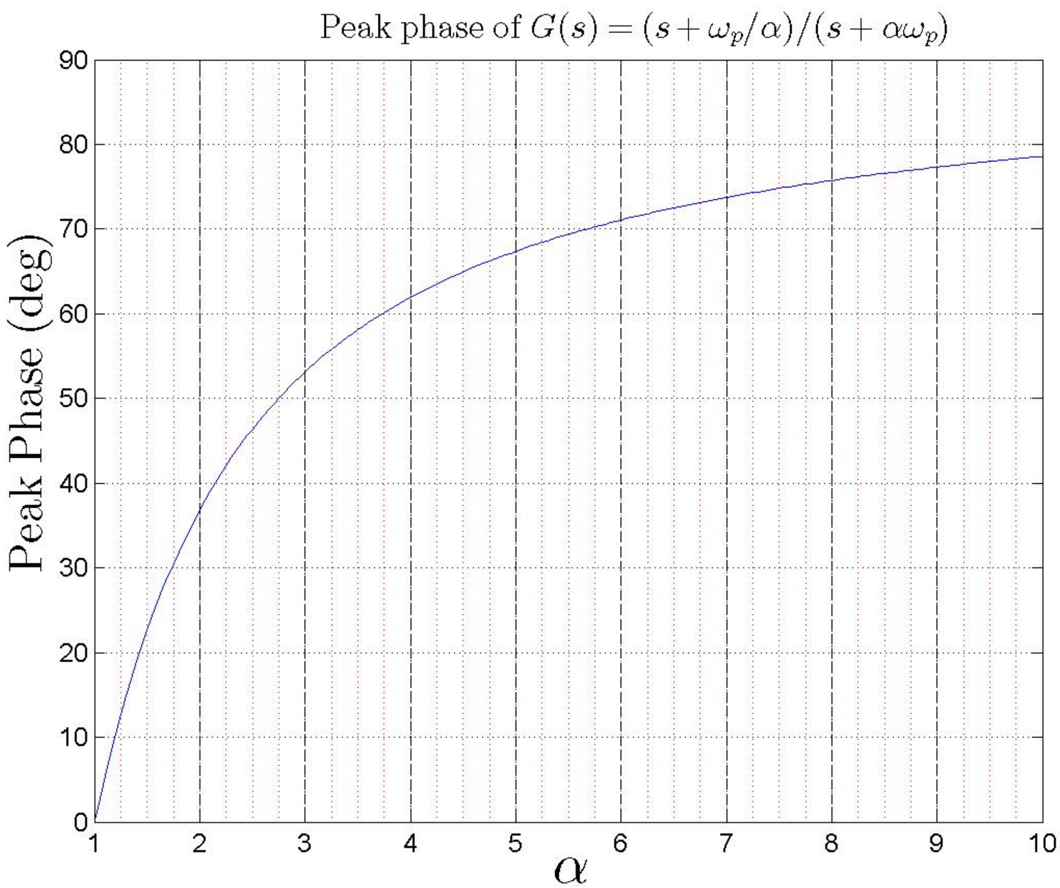
The examples and figure below illustrate how to use semilog plot above:

- The complex number 5+2j has phase: $\text{atan}\left(\frac{2}{5}\right) \approx 22^\circ$.
- The complex number 5-2j has phase: $\text{atan}\left(\frac{-2}{5}\right) \approx -22^\circ$.
- The complex number -5-2j has phase: $\text{atan}\left(\frac{-2}{-5}\right) \approx -180^\circ + 22^\circ = -158^\circ$.



Appendix B: Lead and Lag phase shift prediction.

The figure below is similar to Figure 2.17 in FPW (page 41), except that the definition of "alpha" below is different. (Below, it is the square root of the value in Fig. 2.17 – which makes design a bit easier, since you can avoid taking a square root by using the figure below…)



Example usage. Say we desire a continuous-time compensator to provide 30 degrees of phase. Find 30 degrees on the y axis, and this corresponds to alpha=1.75 (approximately). Say we wish this peak phase to occur at 500 rad/s, then:

$$G(s) = \frac{(s + \frac{500}{1.75})}{(s + 500 \cdot 1.75)} = \frac{(s + 500\frac{4}{7})}{(s + 500\frac{7}{4})} = \frac{(s + \frac{2000}{7})}{(s + \frac{3500}{4})} \approx \frac{(s + 300)}{(s + 900)}$$

For discrete-time controller design, this can then be converted using Tustin with pre-warp.